Advanced Program Representations

Goal:

- more effective analysis
- faster analysis
- easier transformations

Approach:

more directly capture important program properties

- e.g. data flow, independence

Examples

CFG:

+ simple to build
+ complete
+ no derived info to keep up to date during transformations

– computing info is slow and/or ineffective
  - lots of propagation of big sets/maps

Def/use chains

Def/use chains directly linking defs to uses & vice versa

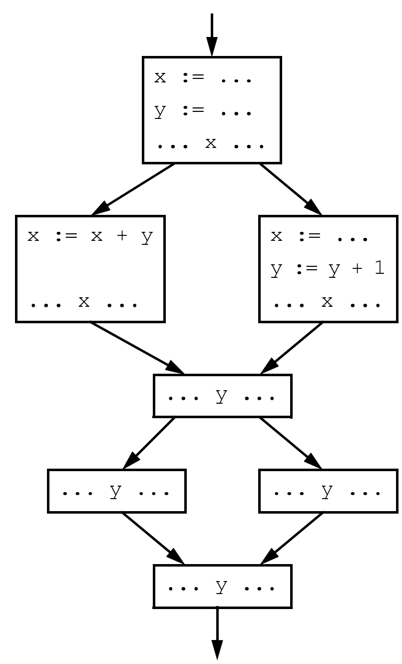
+ directly captures data flow for analysis
  - e.g. constant propagation, live variables easy

– ignores control flow
  - misses some optimization opportunities, since it assumes all paths taken
  - not executable by itself, since it doesn’t include control dependence links
  - not appropriate for some optimizations, such as CSE and code motion

– must update after transformations
  - but only ever remove edges, not add

– space-consuming, in worst case: O(N^2) edges per variable

Example



Static single assignment (SSA) form

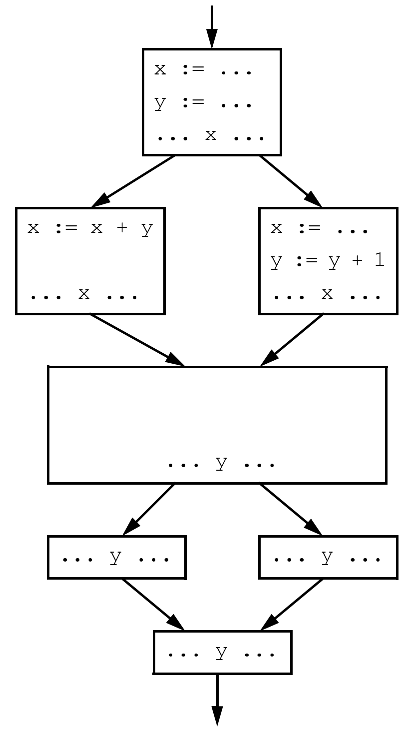
[Alpern, Rosen, Wegman, & Zadeck, two POPL 88 papers]

Invariant: at most one definition reaches each use

Constructing equivalent SSA form of program:

1. Create new target names for all definitions
2. Insert **pseudo-assignments** at merge points reached by multiple definitions of same source variable: x_m := $\phi(x_1, ..., x_n)$
3. Adjust uses to refer to appropriate new names

Example



Comparison

+ lower worst-case space cost than def/use chains: O(*EV*)
+ algorithms simplified by single assignment property:
  - variable has a unique meaning independent of program point
  - can treat variable, its defining stmt, & its value synonymously
  - can have single global table mapping var to info, not one per program pt. that must be propagated, copied, etc.
+ transformations not limited by reuse of variable names
  - can reorder assignments to same source variable, without changing meaning in SSA version

– still not executable by itself
  - and ϕ-functions require an oracle!

– still must update/reconstruct after transformations

– inverse property (static single use) not provided
  - **dependence flow graphs** [Pingali *et al.*] and **value dependence graphs** [Weise *et al.*] fix this, with single-entry, single-exit (SESE) region analysis

Very popular in research compilers, analysis descriptions

Implementing ϕ-functions

Semantics of x_m := $\phi(x_1, ..., x_n)$:
set x_m to x_i, if control last came from predecessor i

How to implement (generate code for) this?

- along each predecessor edge i, insert x_m := x_i
- delete ϕ statement

If register allocator assigns x_m, x_1, ..., x_n to the same register, then these move instructions will be deleted

- x_m, x_1, ..., x_n usually have non-overlapping lifetimes, so this kind of register assignment is legal

Common subexpression elimination

At each program point, compute set of **available expressions**: map from expression to variable holding that expression

- e.g. `{a+b → x, -c → y, *p → z}`

More generally, can have map from expensive expression to equivalent but cheaper expression

- subsumes CSE, constant prop, copy prop., ...

CSE transformation using AE analysis results: if `a+b→x` available before `y := a+b`, transform to `y := x`

Specification

All possible available expressions $AvailableExpr = Expr \times Var$

- $Expr$ = set of all right-hand-side expressions in procedure (or maybe all possible expressions)
- Var = set of all variables in procedure

[is this a function from *Expr* to *Var*, or just a relation?]

Domain AV = Pow($AvailableExpr$)

$ae_1 \leq_{AV} ae_2 \quad \Leftrightarrow$

$T_{AV} \quad =$

$\perp_{AV} \quad =$

$ae_1 \cap_{AV} ae_2 \quad \Leftrightarrow$

height(AV) $=$

Flow functions

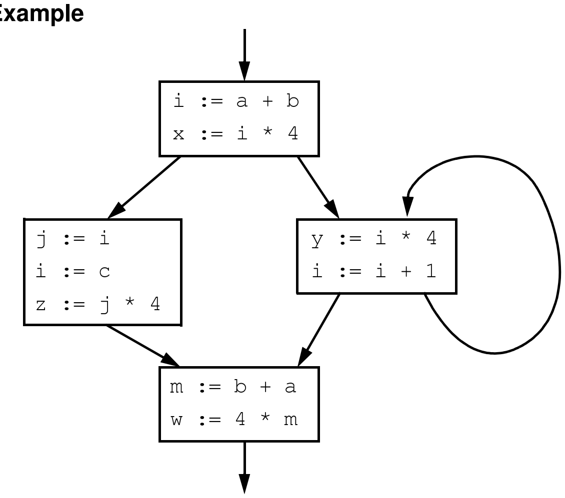
What direction to do analysis?

Initial conditions?

$AE_{X\ :=\ Y\ op\ Z}$(in) =

$AE_{X\ :=\ Y}$(in) =

Example



Exploiting SSA form

Problem: previous available expressions overly sensitive to name choices, operand orderings, renamings, assignments, ...

A solution:

Step 1: convert to SSA form

- distinct values have distinct names
  - ⇒ can simplify flow functions to ignore assignments

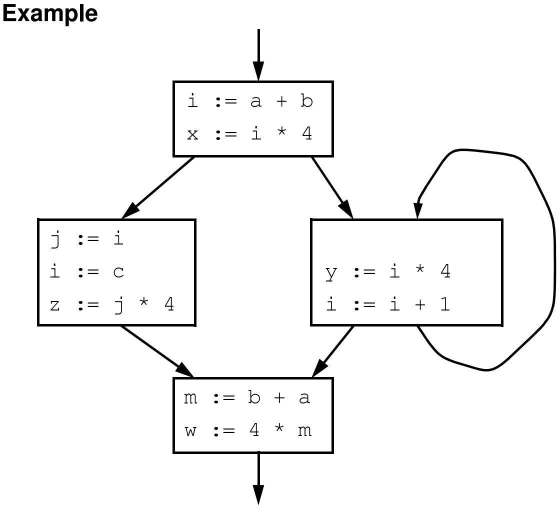
$AE^{SSA}{}_{X\ :=\ Y\ op\ Z}(\text{in}) =$

Step 2: do **copy propagation**

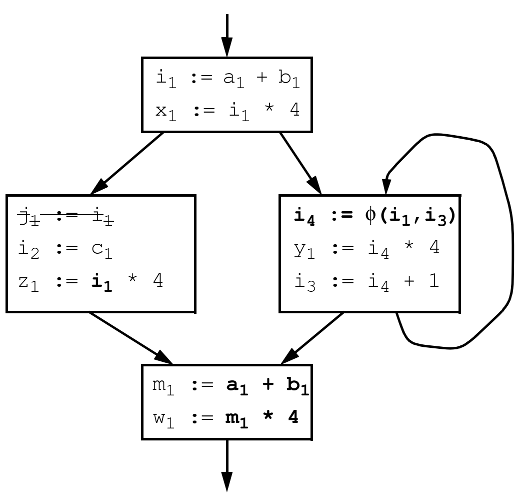
- same values (usually) have same names
  - ⇒ avoid missed opportunities

Step 3: adopt canonical ordering for commutative operators
⇒ avoid missed opportunities

Example



After SSA conversion, copy propagation, & operand order canonicalization:



SSA form and pointers

What about pointers?

```
x := 5;
y := 7;
p := new int;
q := test1 ? &x : (test2 ? &y : p);
*q := 9;
// what are the unique SSA names for x & y here? *p?
x := x + 1;
// what does q point to here?
```

SSA wishes to assign a unique name for each variable (memory location?) at each point

- dynamic memory allocations introduce many "anonymous variables"
- pointer stores don't definitely update any variable, but may update many
- SSA gives different names to the same variable, but `&` creates a pointer to all of them

Some solutions

Option 1: don't use SSA invariant for pointed-to memory

- heap memory, variables that have their addresses taken

Option 2: insert copies between SSA vars and real vars before and/or after may-use/may-def operations

- pointers point to real, non-SSA variable
- insert `var := `var_i *before* any may-use/-def of `var`
- insert var_j `:=`ι`(`var_i`, var)` *after* any may-def of `var`
  - ι`(`var_i`, var)` uses oracle to return either var_i or `var`

x_1 := 5;

y_1 := 7;

p_1 := new int;

q_1 := *test1* ? &x : (*test2* ? &y : p_1);

  x := x_1;

  y := y_1;

*q_1 := 9;

  x_2 := $\iota(x_1, x)$;

  y_2 := $\iota(y_1, y)$;

x_3 := x_2 + 1;

Loop-invariant code motion

Two steps: analysis & transformation

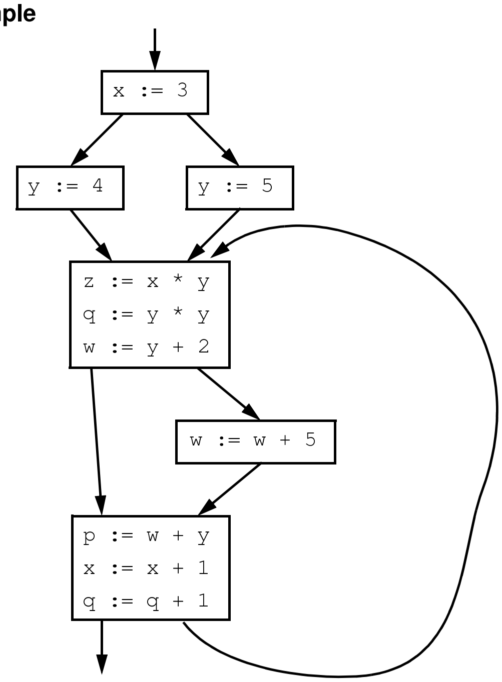
Step 1: find invariant computations in loop

- invariant: computes same result each time evaluated

Step 2: move them outside loop

- to top: **code hoisting**
  - if used within loop
- to bottom: **code sinking**
  - if only used after loop

Example



Detecting loop-invariant expressions

An expression is invariant w.r.t. a loop *L* iff:

(base cases:)

- it's a constant
- it's a variable use, **all of whose defs are outside *L***

(inductive cases:)

- it's a **pure** computation all of whose args are loop-invariant
- it's a variable use **with only one reaching def**, and the rhs of that def is loop-invariant

Computing loop-invariant expressions

Option 1:

- repeat iterative dfa
  until no more invariant expressions found
  - to start, optimistically assume all expressions loop-invariant

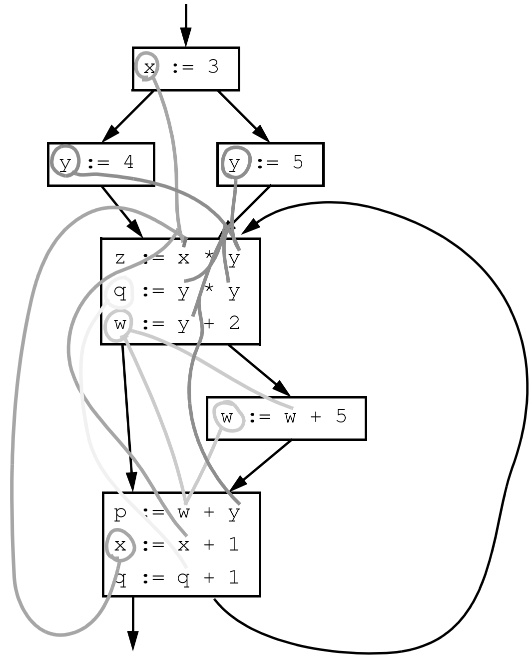
Option 2:

- build def/use chains,
  follow chains to identify & propagate
  invariant expressions

Option 3:

- convert to SSA form,
  then similar to def/use form

Example using def/use chains



Loop-invariant expression detection for SSA form

SSA form simplifies detection of loop invariants,
since each use has only one reaching definition

An expression is invariant w.r.t. a loop *L* iff:

(base cases:)

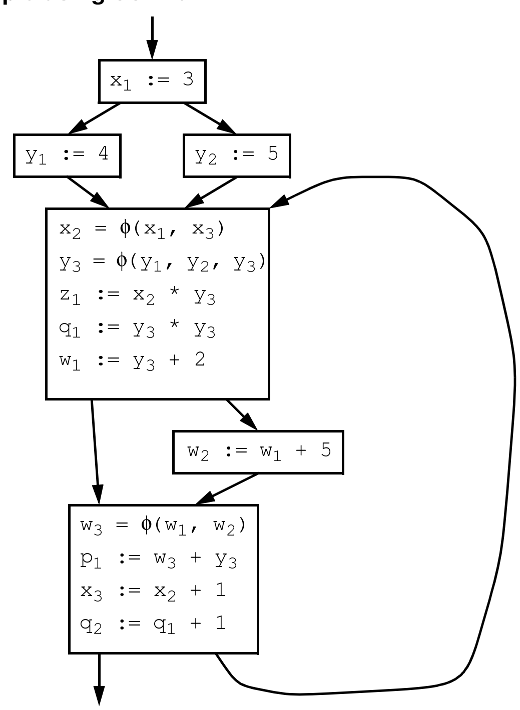
- it's a constant
- it's a variable use **whose <u>single def</u> is outside *L***

(inductive cases:)

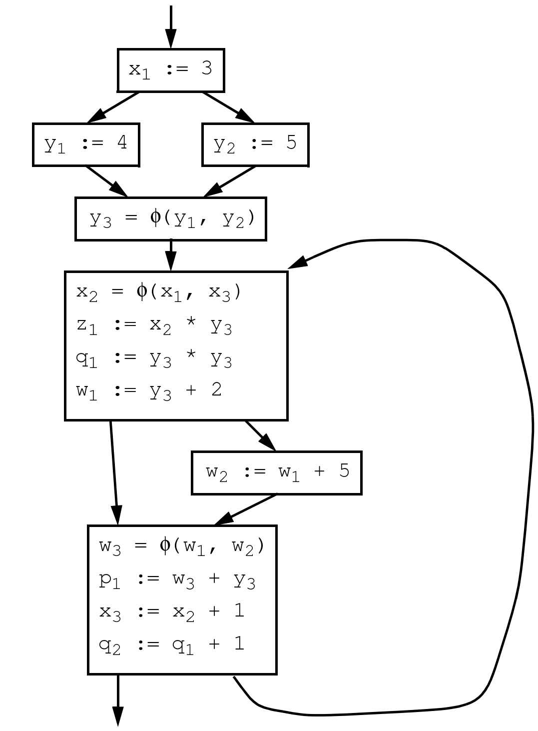
- it's a pure computation
  all of whose args are loop-invariant
- it's a variable use
  whose <u>single def</u>'s rhs is loop-invariant

ϕ functions are *not* pure

Example using SSA form



Example using SSA form & preheader



Code motion

When find invariant computation *S:* `z := x op y`,
want to move it out of loop (to loop preheader)

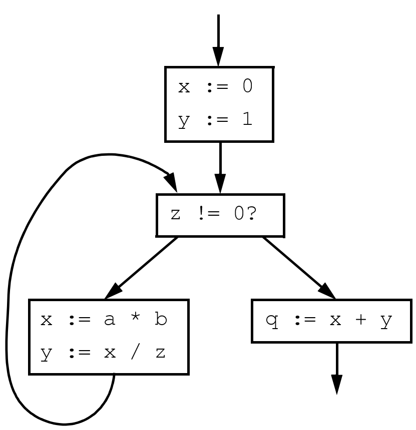
- preserve relative order of invariant computations, to preserve data flow among moved statements

When is this legal?

Condition #1: domination restriction

To move *S:* `z := x op y`,
S must **dominate** all loop exits
[*A* dominates *B* when all paths to *B* first pass through *A*]

- otherwise may execute *S* when never executed otherwise
- if *S* is pure, then can relax this condition, at cost of possibly slowing down program



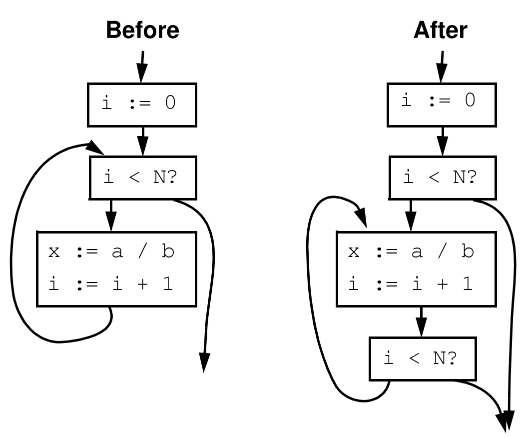
Avoiding domination restriction

Requirement that invariant computation dominates exit is strict

- nothing inside a conditional branch can be moved
- nothing after a loop exit test can be moved
  - what happens in a while loop? a for loop?

Can be circumvented through other transformations such as **loop normalization**

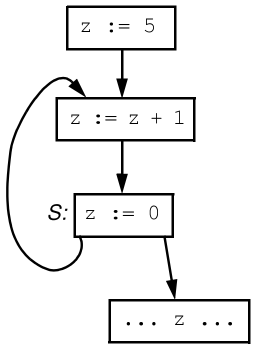
- move loop exit test to bottom (while-do ⇒ if-do-while)



Condition #2: data dependence restriction

To move *S:* `z := x op y`,
S must be the only assignment to `z` in loop, and
no use of `z` in loop is reached by any def other than *S*

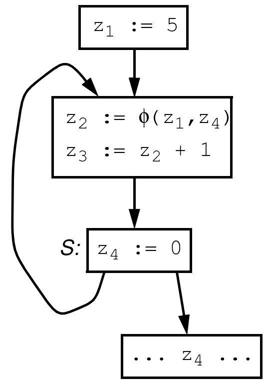
- otherwise may reorder defs/uses and change outcome



Avoiding data dependence restriction

Restriction unnecessary if in SSA form

- implementation of ϕ functions as moves will cope with reordered defs/uses



More refined representations

Problem: control-flow edges in CFG overspecify evaluation order

Solution: introduce more refined notions w/ fewer constraining edges that still capture required orderings

- side-effects occur in proper order
- side-effects occur only under right conditions

Some ideas:

- explicit control dependence edges,
  control-equivalent regions,
  control-dependence graph (PDG)
- operators as nodes (Click, VDG, Whirlwind, etc.)
  - *computable* ϕ-function operator nodes
- control dependence via data dependence (VDG)

Control dependence graph

Program dependence graph (PDG):
data dependence graph + control dependence graph (CDG)
[Ferrante, Ottenstein, & Warren, TOPLAS 87]

Idea: represent controlling conditions directly

- complements data dependence representation

A node (basic block) *Y* is **control-dependent** on another *X* iff *X* determines whether *Y* executes, i.e.

- there exists a path from *X* to *Y* s.t. every node in the path other than *X* & *Y* is **post-dominated** by *Y*
- *X* is not post-dominated by *Y*

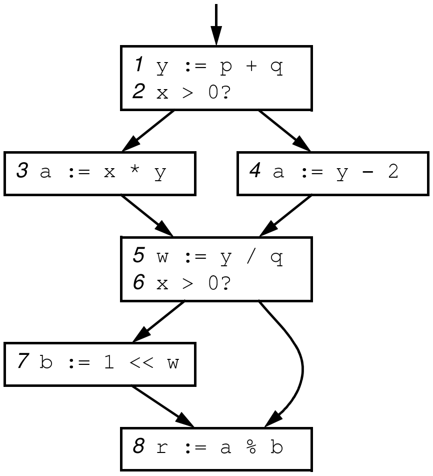
Control dependence graph:
Y proper descendant of *X* iff *Y* control-dependent on *X*

- label each child edge with required branch condition
- group all children with same condition under **region** node

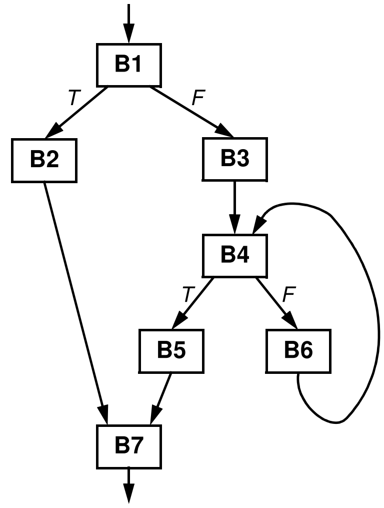
Two sibling nodes execute under same control conditions $\Rightarrow$ can be reordered or parallelized, as data dependences allow

(Challenging to "sequentialize" back into CFG form)

Example



An example with a loop



Operators as nodes

Before: nodes in CFG were simple assignments
- could have operations on r.h.s.
- used variable names to refer to other values

Alternative: treat the operators themselves as the nodes
- refer directly other other nodes for their operands

```
Node ::= Constant              // 0 operands
       | Var                   // 0 operands
       | &Var                  // 0 operands
       | Unop                  // 1 operand
       | Binop                 // 2 operands
       | * (ptr deref)         // 1 operand
       | . (field deref)       // 1 operand
       | [] (array deref)      // 2 operands
       | φ                     // n operands
       | Fn()                  // n operands
       | Var:= (var assn)      // 1 operand
       | *:= (ptr assn)        // 2 operands
```

Flow of data captured directly in operand dataflow edges
Also have control flow edges sequencing these nodes
- or some more refined control dependence edges

Example

```
p := &r;
x := *p;
a := x * y;
w := x;
x := a + a;
v := y * w;
a := v * 2;
```

Improvements

Bypass variable stores and loads

- i.e., build def/use chains

Treat variable names as (temporary) labels on nodes

- a variable reference implemented by an edge from the node with that label
- a variable assignment shifts the label

The nodes themselves become
the subscripted variables of SSA form

Each computation has its own name (i.e., itself)

More improvements

"Value numbering":
merge all nodes that compute the same result

- same operator
- same data operands (recursively)
- same control dependence conditions
- operator is pure

Implements (local) CSE

Can do this bottom-up as nodes are initially constructed

- "hash consing"

In face of possibly cyclic data dependence edges,
an optimistic algorithm can get better results [Alpern *et al.* 88]

Would like to support algebraic identities, too, e.g.

- commutative operators
- `x+x` = `x*2`
- associativity, distributivity

Another example

```
y := p + q;
if m > 1 then
  a := y * x;
  b := a;
else
  b := x - 2;
  a := b;
endif
if m < 1 then
  d := y * x;
else
  d := x - 2;
endif
w := a / r;
u := b / r;
t := d / r;
if m > 1 then
  c := y * x;
else
  c := x - 2;
endif
z := c / r;
```


The example, in SSA form

```
y := p + q;
if m > 1 then
  a_1 := y * x; b_1 := a_1;
else
  b_2 := x - 2; a_2 := b_2;
a_3 := φ(a_1,a_2);
b_3 := φ(b_1,b_2);
if m < 1 then
  d_1 := y * x;
else
  d_2 := x - 2;
d_3 := φ(d_1,d_2);
w := a_3 / r;
u := b_3 / r;
t := d_3 / r;
if m > 1 then
  c_1 := y * x;
else
  c_3 := x - 2;
c_3 := φ(c_1,c_2);
z := c_3 / r;
```

An improvement

ϕ-functions were treated poorly

- impure, since don't know when they're the same
  - even if they have the same operands and are in the same control equivalent region!

Fix: give them an additional input: the selector value (now called select nodes, sometimes written as γ)

- e.g., a boolean, for a 2-input ϕ
- e.g., an integer, for an *n*-input ϕ

ϕ-functions now are pure!

Value dependence graphs

[Weise, Crew, Ernst, & Steensgaard, POPL 94]

Idea: represent all dependences, including control dependences, as data dependences

+ simple, direct dataflow-based representation of all "interesting" relationships
  - analyses become easier to describe & reason about

– harder to sequentialize into CFG

Control dependences as data dependences:

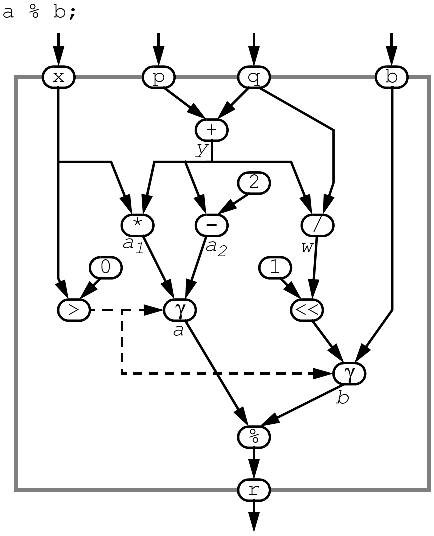
- control dependence on order of side-effects
  $\Rightarrow$ data dependence on reading & writing to global Store
  - optimizations to break up accesses to single Store into separate independent chunks (e.g. a single variable, a single data structure)
- control dependence on outcome of branch
  $\Rightarrow$ a select node, taking test, then, and else inputs
  $\Rightarrow$ demand-driven evaluation model

Loops implemented as tail-recursive calls to local procedures

Apply CSE, folding, etc. as nodes are built/updated

Example, after store splitting

```
y := p + q;
if x > 0 then a := x * y else a := y - 2;
w := y / q;
if x > 0 then b := 1 << w;
r := a % b;
```



Sequentialization

How to generate code from a soup of operators and edges?

Need to *sequentialize* back into a regular CFG

Must find an ordering that respects dependences (data and control)

Hard with arbitrary graph

- can get cycles with full PDG, VDG transforms
- may need to duplicate code to get a legal schedule

Sequentialization via placement

A solution, due to Click: treat as placement problem
- – limits transformations/optimizations possible
- + simpler to implement

Start from original (empty) CFG

Goal: assign each operation to
the least-frequently-executed basic block
that respects its data dependences
- ϕ-nodes tied to their original merge point

Hoist operations out of loops where possible
Push operations into conditionals where possible

Example

```
i := 0;
while ... do
  x := i * b;
  if ... then
    w := c * c;
    y := x + w;
  else
    y := 9;
  end
  print(y);
  i := i + 1;
end
```


Example, in SSA form

i_1 := 0;
while ... do
  i_3 := $\phi(i_1, i_2)$;
  x := i_3 * b;
  if ... then
    w := c * c;
    y_1 := x + w;
  else
    y_2 := 9;
  end
  y_3 := $\phi(y_1, y_2)$;
  print(y_3);
  i_2 := i_3 + 1;
end