

Graduate Programming Languages: Reference Sheet for Lecture 3

$$\begin{aligned}
 s & ::= \text{skip} \mid x := e \mid s; s \mid \text{if } e \text{ } s \text{ } s \mid \text{while } e \text{ } s \\
 e & ::= c \mid x \mid e + e \mid e * e \\
 (c & \in \{\dots, -2, -1, 0, 1, 2, \dots\}) \\
 (x & \in \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{z}_1, \mathbf{z}_2, \dots, \dots\})
 \end{aligned}$$

- Semantics for expressions

1. Informal idea; the need for *heaps*
2. Definition of heaps
3. The evaluation *judgment* (a relation form)
4. The evaluation *inference rules* (the relation definition)
5. Using inference rules
 - *Derivation trees* as interpreters
 - Or as *proofs* about expressions
6. *Metatheory*: Proofs about the semantics

- Then semantics for statements

– ...

$$H(x) = \begin{cases} c & \text{if } H = H', x \mapsto c \\ H'(x) & \text{if } H = H', y \mapsto c' \\ 0 & \text{if } H = \cdot \end{cases}$$

$$\boxed{H ; e \Downarrow c}$$

$$\begin{array}{c} \text{CONST} \\ \hline H ; c \Downarrow c \end{array} \quad \begin{array}{c} \text{VAR} \\ \hline H ; x \Downarrow H(x) \end{array} \quad \begin{array}{c} \text{ADD} \\ \hline H ; e_1 \Downarrow c_1 \quad H ; e_2 \Downarrow c_2 \\ \hline H ; e_1 + e_2 \Downarrow c_1 + c_2 \end{array} \quad \begin{array}{c} \text{MULT} \\ \hline H ; e_1 \Downarrow c_1 \quad H ; e_2 \Downarrow c_2 \\ \hline H ; e_1 * e_2 \Downarrow c_1 * c_2 \end{array}$$

$$\boxed{H_1 ; s_1 \rightarrow H_2 ; s_2}$$

$$\begin{array}{c} \text{ASSIGN} \\ \hline H ; e \Downarrow c \\ \hline H ; x := e \rightarrow H, x \mapsto c ; \text{skip} \end{array} \quad \begin{array}{c} \text{SEQ1} \\ \hline H ; \text{skip}; s \rightarrow H ; s \end{array} \quad \begin{array}{c} \text{SEQ2} \\ \hline H ; s_1 \rightarrow H' ; s'_1 \\ \hline H ; s_1; s_2 \rightarrow H' ; s'_1; s_2 \end{array}$$

$$\begin{array}{c} \text{IF1} \\ \hline H ; e \Downarrow c \quad c > 0 \\ \hline H ; \text{if } e \text{ } s_1 \text{ } s_2 \rightarrow H ; s_1 \end{array} \quad \begin{array}{c} \text{IF2} \\ \hline H ; e \Downarrow c \quad c \leq 0 \\ \hline H ; \text{if } e \text{ } s_1 \text{ } s_2 \rightarrow H ; s_2 \end{array} \quad \begin{array}{c} \text{WHILE} \\ \hline H ; \text{while } e \text{ } s \rightarrow H ; \text{if } e \text{ } (s ; \text{while } e \text{ } s) \text{ skip} \end{array}$$