CSE 505 Programming Languages, Homework 2
Due: Friday, October 23

This assignment is mostly in Coq, with some additional work on paper. We recommend completing the problems in the associated Coq file, then completing the written portion of the assignment. Please complete the written portion of the assignment using LaTeX and upload a PDF file.

Written Assignment (20 points total)

We will extend IMP with the ability to push and pop variables:

- A heap now maps a variable to a number and a list (i.e., a stack) of numbers. The initial heap maps each variable to 0 and the empty-list.
- There are two new statement forms: PushVar x and PopVar x. The former changes the heap H such that x maps to the same number and a list that is like the old list except the number is added to the beginning. The latter is the inverse of the former and has no effect if the list for x is empty. Both PushVar x and PopVar x "become Skip" in one step.

Please complete the following:

1. (10 points) Define a formal small-step operational semantics for this new language. Be sure to give a precise definition for heaps, explaining your notation. Define your semantics as a set of inference rules and typeset the rules using the `mathpartir` package as described below.
2. (10 points) Prove the following property for IMP extended with PushVar x and PopVar x:

   > If a statement s has no While loops and from the empty heap s can step to heap h' and statement s', then for all variables v, the length of the stack for v in h' does not exceed the number of PushVar v statements in s (the original statement).

The semantics will be very similar to the IMP semantics you defined in the Coq file, and to the formal IMP semantics discussed in lecture. The proof will be closely related to a proof you do in the Coq part of the homework, but should be given in English.

On Typesetting Inference Rules

Inference rules can be typeset using the `mathpartir` package included with the homework. For example:

$$\frac{A \qquad B}{s \to s'}\ \text{AB} \qquad\qquad \frac{B \qquad C}{s \to s'}\ \text{BC}$$

$$\frac{B \qquad A}{s \to s'}\ \text{BA} \qquad\qquad \frac{C \qquad s \to s'}{s \to s'}\ \text{CD}$$

was generated from:

```
\begin{mathpar}
  \inferrule*[Right=AB]{ A \and B }{ s \to s' } \and
  \inferrule*[Right=BC]{ B \and C }{ s \to s' } \\
  \inferrule*[Right=BA]{ B \and A }{ s \to s' } \and
  \inferrule*[Right=CD]{ C \and s \to s' }{ s \to s' } \\
\end{mathpar}
```