CSE 505: Programming Languages

Lecture 11 — Types

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The Big Picture

Building Sweet Skills:

- ▶ Defining languages and semantics
  - ▶ Grammars and inductive definitions
  - ▶ Abstract vs. concrete syntax
- ▶ Formal proofs and structural induction
- ▶ Invariants, determinism, equivalence
- ▶ Lambda calculus
  - ▶ Small, simple model of computation

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Onward and Upward!

- ▶ Today: TYPES!
- ▶ What do they do?
- ▶ Why do we want them?
- ▶ How do we formalize them?
- ▶ What makes a good type system?

Types

Types are a major new topic worthy of a lifetime's study

- ▶ Continue to use Lambda Calculus as our core model
- ▶ But will soon enrich with other common primitives

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Today:

- ▶ Motivation for type systems
- ▶ What a type system is designed to do and not do
  - ▶ Vocab: definition of stuckness, soundness, completeness, etc.
- ▶ The Simply-Typed Lambda Calculus
  - ▶ A basic and natural type system
  - ▶ Starting point for more expressiveness later

Introduction to Types

Aren't more powerful PLs are *always* better?

- ▶ Turing Complete so we can do anything (λ-Calculus / x86)
- ▶ Super flexible (e.g., higer order functions)
- ▶ Conveniences to keep programs short (concision is king!)

Introduction to Types

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If so, types are taking us in the wrong direction!

- ▶ Type systems restrict which programs we can write :(
- ▶ If types are any good, must help in some other PL dimension...

Why types?

Why types? Let's think about it...

This is a game called "read the professor's mind."

Why types? (Part $1/\infty$)

1. Catch "stupid" mistakes early, even before testing!
   - Example: "if" applied to "mkpair"
   - Stupid, too-clever occasionally indistinguishable (ptr xor)
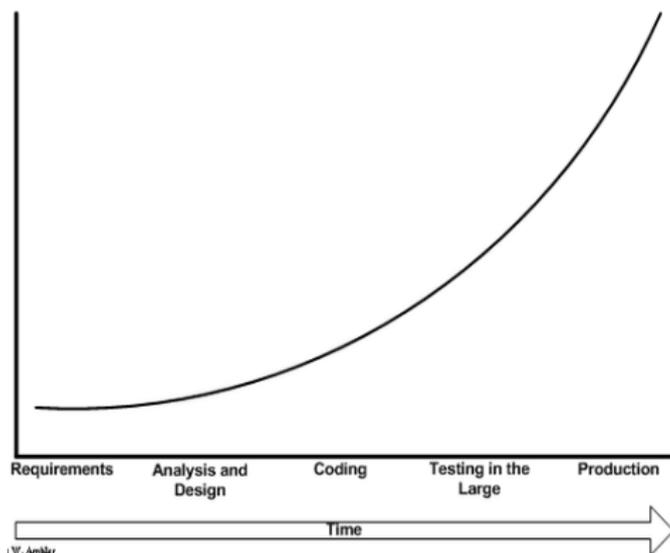
| Requirements | Analysis and Design | Coding | Testing in the Large | Production |

Time

W. Ambler

Why types? (Part 2/∞)

2. (Safety) Prevent getting stuck (e.g., $x\ v$)
   ▶ Ensure execution never gets to a "meaningless" state
   ▶ But "meaningless" depends on the semantics
   ▶ PLs typically have some type errors, others run-time errors

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You're gonna do it anyway...

> As our system has grown, a lot of the logic in our Ruby system
> sort of replicates a type system... I think it may just be a
> property of large systems in dynamic languages, that eventually
> you end up rewriting your own type system, and you sort of do
> it badly. Youre checking for null values all over the place.
> Theres lots of calls to Rubys kind_of? method, which asks, Is
> this a kind of User object? Because thats what were expecting.
> If we dont get that, this is going to explode. It is a shame to
> have to write all that when there is a solution that has existed
> in the world of programming languages for decades now.
>
> – Alex Payne (Twitter)

Why types? (Part $3/\infty$)

3. Help our compiler friends out a bit
   - ▶ "filter" between AST and compiler/interpreter
   - ▶ Strengthen compiler assumptions, help optimizer
   - ▶ Don't have to check for impossible states
   - ▶ Orthogonal to safety (e.g. C/C++)

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4. Enforce encapsulation (an *abstract type*)
   - Clients can't break invariants
   - Hide implementation (now can change list/pair)
   - Requires safety, meaning no "stuck" states that corrupt run-time (e.g., C/C++)
   - Can enforce encapsulation without static types, but types are a particularly nice way

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5. Syntactic overloading
   - Have symbol lookup depend on operands' types
   - Only modestly interesting semantically
   - Late binding (lookup via *run-time* types) more interesting

What is a type system?

What **isn't** a type system?

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Appel's Axiom:

> *The difference between a program analysis and a type*
> *system is that when a type system rejects a program it's*
> *the programmer's fault.*

What is a type system?

Er, uh, you know it when you see it. Some clues:

- ► A decidable (?) judgment for classifying programs
    - ► E.g., $e_1 + e_2$ has type int if e_1, e_2 have type int (else *no type*)

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    - ▶ Often a more natural framework for *flow-sensitive* properties
    - ▶ Types often more natural for *higher-order programs*

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This is a CS-centric, PL-centric view.

- Foundational type theory has more rigorous answers :)
- Type systems have a long history...

# Roots in Paradox
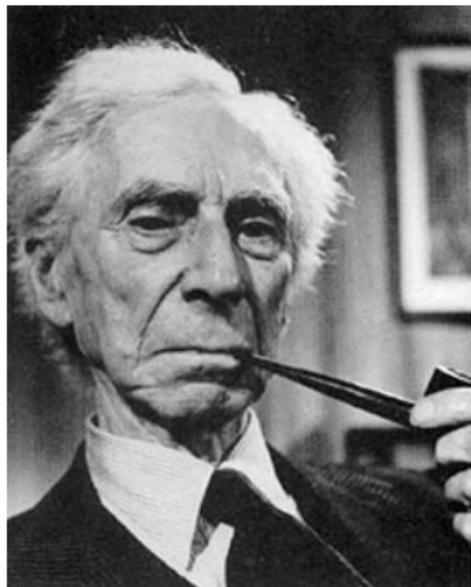
Roots in Paradox

Let $R = \{x \mid x \notin x\}$, then
$R \in R \iff R \notin R$
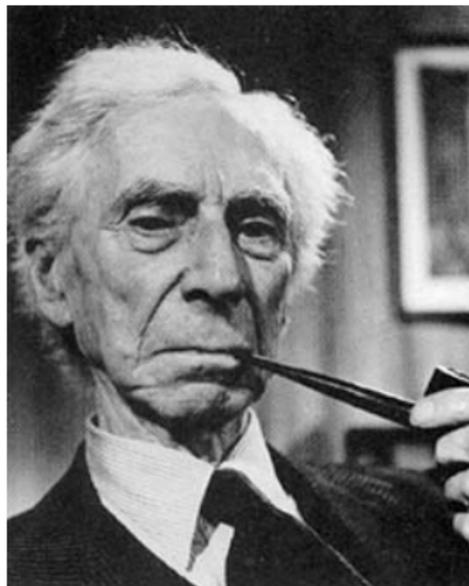
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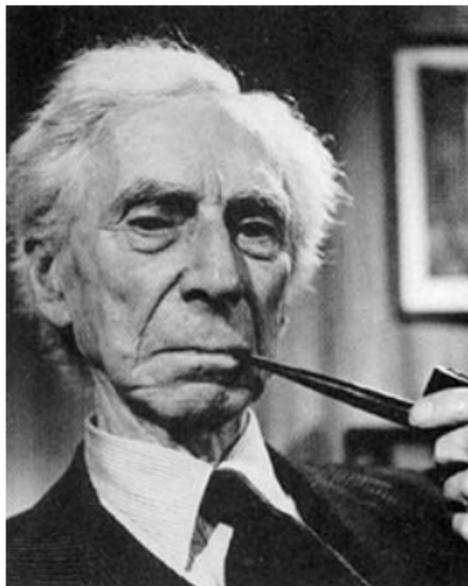
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The barber is a man in a town who shaves exactly those men who do not shave themselves. All men in this town are clean shaven.

Who shaves the barber?

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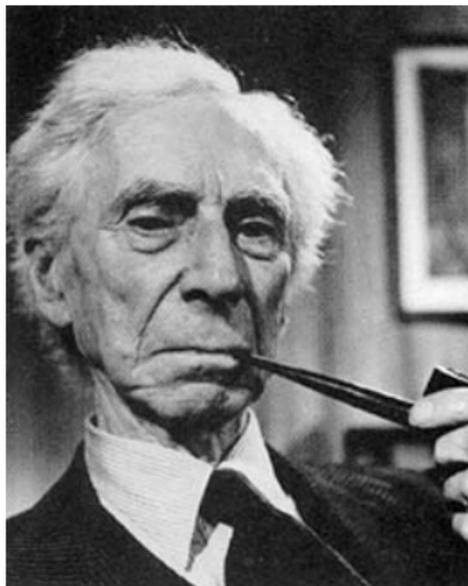
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And thus type theory was born ...

Adding constants

Enrich the Lambda Calculus with integer constants:

- Not strictly necessary, but makes types seem more natural

$$e ::= \lambda x.\, e \mid x \mid e\, e \mid c$$
$$v ::= \lambda x.\, e \mid c$$

No new operational-semantics rules since constants are values

We could add $+$ and other *primitives*

- Then we would need new rules (e.g., 3 small-step for $+$)
- Alternately, parameterize "programs" by primitives:
  $\lambda plus.\; \lambda times.\; ...\; e$
  - Like Pervasives in OCaml
  - A great way to keep language definitions small

Stuck

Key issue: can a program "get stuck" (reach a "bad" state)?

- ▶ Definition: e *is stuck* if e is not a value and there is no e' such that $e \rightarrow e'$

- ▶ Definition: e *can get stuck* if there exists an e' such that $e \rightarrow^* e'$ and e' is stuck
  - ▶ In a deterministic language, e "gets stuck"

Most people don't appreciate that stuckness depends on the operational semantics

- ▶ Inherent given the definitions above

Given our language, what are the set of stuck expressions?

- ▶ Note: Explicitly defining the stuck states is unusual

$$e \quad ::= \quad \lambda x.\, e \mid x \mid e\, e \mid c$$
$$v \quad ::= \quad \lambda x.\, e \mid c$$

$$\frac{}{(\lambda x.\, e_1)\, e_2 \to e_1[e_2/x]} \qquad \frac{e_1 \to e_1'}{e_1\, e_2 \to e_1'\, e_2}$$

(Hint: The full set is recursively defined.)

$$S ::=$$

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$$S ::= x \mid c\, v \mid S\, e \mid v\, S$$

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(Hint: The full set is recursively defined.)

$$S ::= x \mid c\ v \mid S\ e \mid v\ S$$

Note: Can have fewer stuck states if we add more rules

- Example: Javascript
- Example: $\dfrac{}{c\ v \rightarrow v}$
- In *unsafe* languages, stuck states can set the computer on fire

Soundness and Completeness

A *type system* is a judgment for classifying programs
- "accepts" a program if some complete derivation gives it a type, else "rejects"

A *sound* type system never accepts a program that can get stuck
- No false negatives

A *complete* type system never rejects a program that can't get stuck
- No false positives

It is typically *undecidable* whether a stuck state can be reachable
- Corollary: If we want an *algorithm* for deciding if a type system accepts a program, then the type system cannot be sound and complete
- We'll choose soundness, try to reduce false positives in practice

Wrong Attempt

$$\tau \ ::= \ \textbf{int} \mid \textbf{fn}$$

$$\boxed{\vdash e : \tau}$$

$$\frac{}{\vdash \lambda x. \ e : \textbf{fn}} \qquad \frac{}{\vdash c : \textbf{int}} \qquad \frac{\vdash e_1 : \textbf{fn} \qquad \vdash e_2 : \textbf{int}}{\vdash e_1 \ e_2 : \textbf{int}}$$

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1. NO: can get stuck, e.g., $(\lambda x. \ y) \ 3$

2. NO: too restrictive, e.g., $(\lambda x. \ x \ 3) \ (\lambda y. \ y)$

3. NO: types not preserved, e.g., $(\lambda x. \ \lambda y. \ y) \ 3$

Getting it right

1. Need to type-check function bodies, which have free variables
2. Need to classify functions using argument and result types

For (1): $\Gamma ::= \cdot \mid \Gamma, x : \tau$ and $\Gamma \vdash e : \tau$

- Require whole program to type-check under empty context $\cdot$

For (2): $\tau ::= \textbf{int} \mid \tau \rightarrow \tau$

- An infinite number of types:
  $\textbf{int} \rightarrow \textbf{int}$, $(\textbf{int} \rightarrow \textbf{int}) \rightarrow \textbf{int}$, $\textbf{int} \rightarrow (\textbf{int} \rightarrow \textbf{int})$, ...

Concrete syntax note: $\rightarrow$ is right-associative, so
$\tau_1 \rightarrow \tau_2 \rightarrow \tau_3$ is $\tau_1 \rightarrow (\tau_2 \rightarrow \tau_3)$

STLC Type System

$$\tau ::= \text{int} \mid \tau \rightarrow \tau$$
$$\Gamma ::= \cdot \mid \Gamma, x{:}\tau$$

$$\boxed{\Gamma \vdash e : \tau}$$

$$\overline{\Gamma \vdash c : \text{int}} \qquad\qquad \overline{\Gamma \vdash x : \Gamma(x)}$$

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x.\, e : \tau_1 \rightarrow \tau_2} \qquad \frac{\Gamma \vdash e_1 : \tau_2 \rightarrow \tau_1 \qquad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1\ e_2 : \tau_1}$$

The *function-introduction* rule is the interesting one...

A closer look

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x.\ e : \tau_1 \to \tau_2}$$

Where did τ_1 come from?

- Our rule "inferred" or "guessed" it
- To be syntax directed, change $\lambda x.\ e$ to $\lambda x : \tau.\ e$ and use that τ

Can think of "adding x" as shadowing or requiring $x \notin \mathbf{Dom}(\Gamma)$

- Systematic renaming (α-conversion) ensures $x \notin \mathbf{Dom}(\Gamma)$ is not a problem

A closer look

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x.\ e : \tau_1 \to \tau_2}$$

Is our type system too restrictive?

- That's a matter of opinion
- But it does reject programs that don't get stuck

Example: $(\lambda x.\ (x\ (\lambda y.\ y))\ (x\ 3))\ \lambda z.\ z$

- Does not get stuck: Evaluates to 3
- Does not type-check:
    - There is no τ_1, τ_2 such that $x : \tau_1 \vdash (x\ (\lambda y.\ y))\ (x\ 3) : \tau_2$
      because you have to pick *one* type for x

Always restrictive

Whether or not a program "gets stuck" is undecidable:

- If e has no constants or free variables, then e $(3\ 4)$ or e x
  gets stuck if and only if e terminates (cf. the halting problem)

Old conclusion: "Strong types for weak minds"

- Need a back door (unchecked casts)

Modern conclusion: Unsafe constructs almost never worth the risk

- Make "false positives" (rejecting safe program) rare enough
  - Have compile-time resources for "fancy" type systems
- Make workarounds for false positives convenient enough

How does STLC measure up?

So far, STLC is sound:

- As language dictators, we decided $c\ v$ and undefined variables were "bad" meaning neither values nor reducible
- Our type system is a conservative checker that an expression will never get stuck

But STLC is far too restrictive:

- In practice, just too often that it prevents safe and natural code reuse
- More fundamentally, it's not even Turing-complete
  - Turns out all (well-typed) programs terminate
  - A good-to-know and useful property, but inappropriate for a general-purpose PL
  - That's okay: We will add more constructs and typing rules

Type Soundness

We will take a *syntactic* (operational) approach to soundness/safety

- The popular way since the early 1990s

Theorem (Type Safety): If $\cdot \vdash e : \tau$ then e diverges or $e \rightarrow^n v$ for an n and v such that $\cdot \vdash v : \tau$

- That is, if $\cdot \vdash e : \tau$, then e cannot get stuck