Readings: K&F 12

Learning Problems in Real Applications & Approximate Inference

Lecture 14 – May 11, 2011
CSE 515, Statistical Methods, Spring 2011

Instructor: Su-In Lee
University of Washington, Seattle

Outline

- Learning problems in real applications: Robotics, AI, Natural Language Processing, Computational Biology, Computer Vision...
  - Robotic mapping
  - Part-of-speech tagging
  - Peptide identification in MSMS
  - Finding tumor-specific mutations
  - Collaborative filtering
    - Discovering user clusters
  - Computer vision
    - Learning spatial context: using stuff to find things
  - Machine learning
    - Structured prediction
- Particle-based approximate inference

PARTICLE-BASED APPROXIMATE INFERENCE

Inference Complexity Summary

- NP-Hard
  - Exact inference
  - Approximate inference
    - with relative error
    - with absolute error < 0.5 (given evidence)
- Hopeless?
  - No, we will see many network structures that have provably efficient algorithms and we will see cases when approximate inference works efficiently with high accuracy

Approximate Inference

- Particle-based methods
  - Create instances (particles) that represent part of the probability mass
    - Random sampling
    - Deterministically search for high probability assignments
- Global methods
  - Approximate the distribution in its entirety
    - Use exact inference on a simpler (but close) network (e.g. meanfield)
    - Perform inference in the original network but approximate some steps of the process (e.g., ignore certain computations or approximate some intermediate results)

37

Particle-Based Methods

- Particle definition
  - Full particles – complete assignments to all variables
  - Distributional particles – assignment to part of the variables
- Particle generation process
  - Generate particles deterministically
  - Generate particles by sampling

General framework

- Generate samples (particles) x[1],…,x[M] from P
- Estimate function by $E_P(f) \approx \frac{1}{M}\sum_{m=1}^{M} f(x[m])$

38

Particle-Based Methods Overview

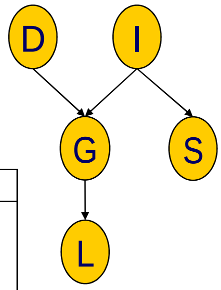
- Full particle methods
  - Sampling methods
    - ➡ Forward sampling
    - Importance sampling
    - Markov chain Monte Carlo
  - Deterministic particle generation
- Distributional particles

Forward Sampling

- Generate random samples from P(X)
  - Use the Bayesian network to generate samples
- Estimate function by $E_P(f) \approx \frac{1}{M}\sum_{m=1}^{M} f(x[m])$

Forward Sampling

d^0	d^1
0.4	0.6



i^0	i^1
0.7	0.3

I	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

D	*I*	g^0	g^1	g^2
d^0	i^0	0.3	0.4	0.3
d^0	i^1	0.05	0.25	0.7
d^1	i^0	0.9	0.08	0.02
d^1	i^1	0.5	0.3	0.2

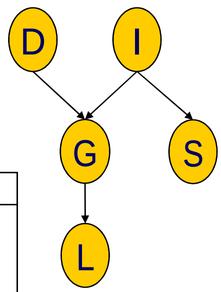
G	l^0	l^1
g^0	0.1	0.9
g^1	0.4	0.6
g^2	0.99	0.01

Samples

D	*I*	*G*	*S*	*L*

Forward Sampling

d^0	d^1
0.4	0.6



i^0	i^1
0.7	0.3

I	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

D	*I*	g^0	g^1	g^2
d^0	i^0	0.3	0.4	0.3
d^0	i^1	0.05	0.25	0.7
d^1	i^0	0.9	0.08	0.02
d^1	i^1	0.5	0.3	0.2

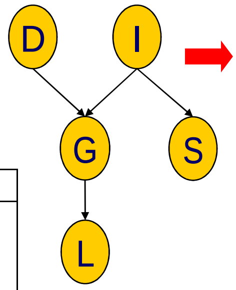
G	l^0	l^1
g^0	0.1	0.9
g^1	0.4	0.6
g^2	0.99	0.01

Samples

D	*I*	*G*	*S*	*L*
d^1				

Forward Sampling

d^0	d^1
0.4	0.6



i^0	i^1
0.7	0.3

I	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

D	*I*	g^0	g^1	g^2
d^0	i^0	0.3	0.4	0.3
d^0	i^1	0.05	0.25	0.7
d^1	i^0	0.9	0.08	0.02
d^1	i^1	0.5	0.3	0.2

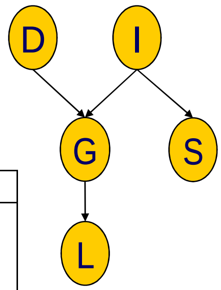
G	l^0	l^1
g^0	0.1	0.9
g^1	0.4	0.6
g^2	0.99	0.01

Samples

D	*I*	*G*	*S*	*L*
d^1	i^1			

Forward Sampling

d^0	d^1
0.4	0.6



i^0	i^1
0.7	0.3

I	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

D	*I*	g^0	g^1	g^2
d^0	i^0	0.3	0.4	0.3
d^0	i^1	0.05	0.25	0.7
d^1	i^0	0.9	0.08	0.02
d^1	i^1	0.5	0.3	0.2

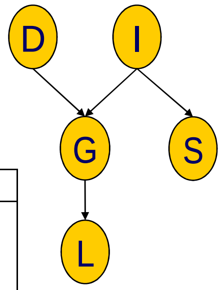
G	l^0	l^1
g^0	0.1	0.9
g^1	0.4	0.6
g^2	0.99	0.01

Samples

D	*I*	*G*	*S*	*L*
d^1	i^1	g^2		

Forward Sampling

d^0	d^1
0.4	0.6



i^0	i^1
0.7	0.3

I	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

D	*I*	g^0	g^1	g^2
d^0	i^0	0.3	0.4	0.3
d^0	i^1	0.05	0.25	0.7
d^1	i^0	0.9	0.08	0.02
d^1	i^1	0.5	0.3	0.2

G	l^0	l^1
g^0	0.1	0.9
g^1	0.4	0.6
g^2	0.99	0.01

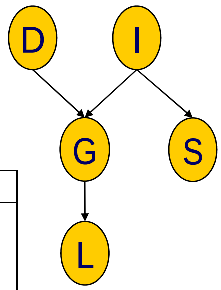
Samples

D	*I*	*G*	*S*	*L*
d^1	i^1	g^2	s^1	

45

Forward Sampling

d^0	d^1
0.4	0.6



i^0	i^1
0.7	0.3

I	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

D	*I*	g^0	g^1	g^2
d^0	i^0	0.3	0.4	0.3
d^0	i^1	0.05	0.25	0.7
d^1	i^0	0.9	0.08	0.02
d^1	i^1	0.5	0.3	0.2

G	l^0	l^1
g^0	0.1	0.9
g^1	0.4	0.6
g^2	0.99	0.01

Samples

D	*I*	*G*	*S*	*L*
d^1	i^1	g^2	s^1	l^0

46

Forward Sampling

- Let $X_1, \dots, X_n$ be a topological order of the variables
- For i = 1,…,n
  - Sample x_i from $P(X_i \mid pa_i)$
    - (Note: since $Pa_i \subseteq \{X_1,\dots,X_{i-1}\}$, we already assigned values to them)
- return $x_1, \dots, x_n$

- Estimate function by: $E_P(f) \approx \frac{1}{M}\sum_{m=1}^{M} f(x[m])$

- Estimate P(y) by: $P(y) \approx \frac{1}{M}\sum_{m=1}^{M} \mathbf{1}\{x[m](y) = y\}$

47

Forward Sampling

- Sampling cost
  - Per variable cost: $O(\log(\mathrm{Val}|X_i|))$
    - Sample uniformly in [0,1]
    - Find appropriate value of all $\mathrm{Val}|X_i|$ values that X_i can take
  - Per sample cost: $O(n\log(d))$ ($d = \max_i \mathrm{Val}|X_i|$)
  - Total cost: $O(Mn\log(d))$

- Number of samples needed
  - To get a relative error < ε, with probability $1-\delta$, we need

$$M \geq 3\frac{\ln(2/\delta)}{P(\mathbf{y})\varepsilon^2}$$

    - Note that number of samples grows inversely with P(y)
    - For small P(y) we need many samples, otherwise we report P(y)=0

48

Rejection Sampling

- In general we need to compute P(Y|e)
- We can do so with *rejection sampling*
  - Generate samples as in forward sampling
  - Reject samples in which $E \neq e$
  - Estimate function from accepted samples

- Problem: if evidence is unlikely (e.g., P(e)=0.001)) then we generate many rejected samples

49

Particle-Based Methods Overview

- Full particle methods
  - Sampling methods
    - Forward sampling
    - ➡ Importance sampling
    - Markov chain Monte Carlo
  - Deterministic particle generation

- Distributional particles

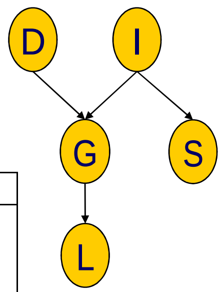
50

Likelihood Weighting

- Can we ensure that all of our samples satisfy E=e?
  - Solution: when sampling a variable $X \in E$, set $X=e$
    - Problem: we are trying to sample from the posterior $P(X|e)$ but our sampling process still samples from $P(X)$
    - Solution: weigh each sample by the joint probability of setting each variable to its evidence/observed value
    - In effect, we are sampling from $P(X,e)$ which we can normalize to then obtain $P(X|e)$ for a query of interest

Likelihood Weighting

d^0	d^1
0.4	0.6



i^0	i^1
0.7	0.3

I	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

D	*I*	g^0	g^1	g^2
d^0	i^0	0.3	0.4	0.3
d^0	i^1	0.05	0.25	0.7
d^1	i^0	0.9	0.08	0.02
d^1	i^1	0.5	0.3	0.2

G	l^0	l^1
g^0	0.1	0.9
g^1	0.4	0.6
g^2	0.99	0.01

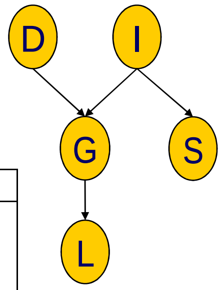
Samples

D	*I*	*G*	*S*	*L*	*W*

$E = \{S=s^1, G=g^2\}$

Likelihood Weighting

d^0	d^1
0.4	0.6



i^0	i^1
0.7	0.3

I	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

D	*I*	g^0	g^1	g^2
d^0	i^0	0.3	0.4	0.3
d^0	i^1	0.05	0.25	0.7
d^1	i^0	0.9	0.08	0.02
d^1	i^1	0.5	0.3	0.2

G	l^0	l^1
g^0	0.1	0.9
g^1	0.4	0.6
g^2	0.99	0.01

Samples

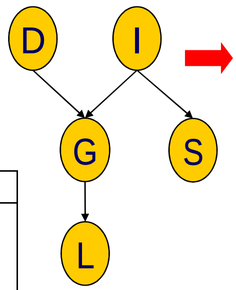
D	*I*	*G*	*S*	*L*	*W*
d^1					1

$E = \{S=s^1, G=g^2\}$

53

Likelihood Weighting

d^0	d^1
0.4	0.6



i^0	i^1
0.7	0.3

I	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

D	*I*	g^0	g^1	g^2
d^0	i^0	0.3	0.4	0.3
d^0	i^1	0.05	0.25	0.7
d^1	i^0	0.9	0.08	0.02
d^1	i^1	0.5	0.3	0.2

G	l^0	l^1
g^0	0.1	0.9
g^1	0.4	0.6
g^2	0.99	0.01

Samples

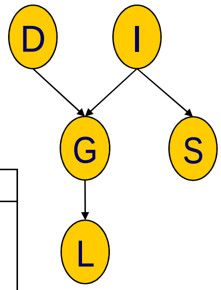
D	*I*	*G*	*S*	*L*	*W*
d^1	i^1				1

$E = \{S=s^1, G=g^2\}$

54

Likelihood Weighting

d^0	d^1
0.4	0.6



i^0	i^1
0.7	0.3

I	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

D	*I*	g^0	g^1	g^2
d^0	i^0	0.3	0.4	0.3
d^0	i^1	0.05	0.25	0.7
d^1	i^0	0.9	0.08	0.02
d^1	i^1	0.5	0.3	0.2

G	l^0	l^1
g^0	0.1	0.9
g^1	0.4	0.6
g^2	0.99	0.01

Samples

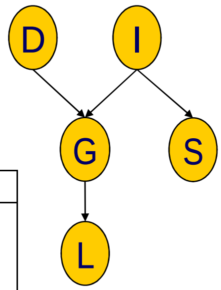
D	*I*	*G*	*S*	*L*	*W*
d^1	i^1	g^2			0.2

$E = \{S=s^1, G=g^2\}$

55

Likelihood Weighting

d^0	d^1
0.4	0.6



i^0	i^1
0.7	0.3

I	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

D	*I*	g^0	g^1	g^2
d^0	i^0	0.3	0.4	0.3
d^0	i^1	0.05	0.25	0.7
d^1	i^0	0.9	0.08	0.02
d^1	i^1	0.5	0.3	0.2

G	l^0	l^1
g^0	0.1	0.9
g^1	0.4	0.6
g^2	0.99	0.01

Samples

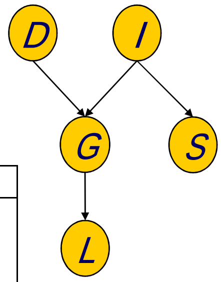
D	*I*	*G*	*S*	*L*	*W*
d^1	i^1	g^2	s^1		0.16

$E = \{S=s^1, G=g^2\}$

56

Likelihood Weighting

d^0	d^1
0.4	0.6



i^0	i^1
0.7	0.3

I	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

D	*I*	g^0	g^1	g^2
d^0	i^0	0.3	0.4	0.3
d^0	i^1	0.05	0.25	0.7
d^1	i^0	0.9	0.08	0.02
d^1	i^1	0.5	0.3	0.2

G	l^0	l^1
g^0	0.1	0.9
g^1	0.4	0.6
g^2	0.99	0.01

Samples

D	*I*	*G*	*S*	*L*	*W*
d^1	i^1	g^2	s^1	l^0	0.16

$E = \{S=s^1, G=g^2\}$

Likelihood Weighting

- Let $X_1, \ldots, X_n$ be a topological order of the variables
- For i = 1,...,n
  - If $X_i \notin E$
    - Sample x_i from $P(X_i \mid pa_i)$
  - If $X_i \in E$
    - Set $X_i = E[x_i]$
    - Set $w_i = w_i \cdot P(E[x_i] \mid pa_i)$
- return w_i and $x_1, \ldots, x_n$
- Estimate P(y|E) by: $P(y \mid e) \approx \dfrac{\sum_{m=1}^{M} w[m]\mathbf{1}\{x[m](y)=y\}}{\sum_{m=1}^{M} w[m]}$

Importance Sampling

- Generalization of likelihood weighting sampling

- Idea: to estimate a function relative to P, rather than sampling from the distribution P, sample from another distribution Q
  - P is called the target distribution
  - Q is called the proposal or the sampling distribution
  - Requirement from Q: $P(x) > 0 \rightarrow Q(x) > 0$
    - Q does not 'ignore' any non-zero probability events in P
    - In practice, performance depends on similarity between Q and P

Acknowledgement

- These lecture notes were generated based on the slides from Prof Eran Segal.