CSE522 - Learning Theory - Homework Exercise 2

Assume that $\mathcal{Y}$ (the range of each hypothesis) is $\mathbb{R}$.

1. Let $\mathcal{H}$ and $\mathcal{H}'$ be hypothesis classes such that $\mathcal{H} \subseteq \mathcal{H}'$. Prove that $\mathcal{R}_m(\mathcal{H}) \leq \mathcal{R}_m(\mathcal{H}')$.
2. Let $\mathcal{H}$ be a hypothesis class. For any $\alpha \in \mathbb{R}$ define $\mathcal{H}_\alpha = \{\alpha h : h \in \mathcal{H}\}$. Prove that $\mathcal{R}_m(\mathcal{H}_\alpha) = |\alpha|\mathcal{R}_m(\mathcal{H})$.
3. Let $\mathcal{H}$ be a hypothesis class and let $\bar{\mathcal{H}}$ be the convex hull of $\mathcal{H}$ (namely, each function in $\bar{\mathcal{H}}$ is a convex combination of a finite set of functions from $\mathcal{H}$). Prove that $\mathcal{R}_m(\mathcal{H}) = \mathcal{R}_m(\bar{\mathcal{H}})$.
4. Let $\mathcal{H}_1, \ldots, \mathcal{H}_k$ be arbitrary hypothesis classes. Let $\sum_{i=1}^k \mathcal{H}_k$ denote the set $\{\sum_{i=1}^k h_i : h_i \in \mathcal{H}_i\}$. Prove that $\mathcal{R}_m(\sum_{i=1}^k \mathcal{K}_i) \leq \sum_{i=1}^k \mathcal{R}_m(\mathcal{H}_i)$.
5. Let $\mathcal{H}$ and $\mathcal{H}'$ be hypothesis classes. Either prove or give a counter example to $\mathcal{R}_m(\mathcal{H} \cup \mathcal{H}') \leq \mathcal{R}_m(\mathcal{H}) + \mathcal{R}_m(\mathcal{H}')$.
6. Let $\mathcal{H}$ be a hypothesis class. Either prove or give a counter example to $\mathcal{R}_m(\mathcal{H} \cup -\mathcal{H}) \leq 2\mathcal{R}_m(\mathcal{H})$.