

This Week's Plan


## Sequence Evolution

## Nothing in Biology Makes Sense Except in the Light of

 Evolution- Theodosius Dobzhansky, 1973
- Changes happen at random
- Deleterious/neutral/advantageous changes
unlikely/possibly/likely spread widely in a population
- Changes are less likely to be tolerated in positions involved in
many/close interactions, e.g.
- enzyme binding pocket
- protein/protein interaction surface
- ..


## BLAST:

Basic Local Alignment Search Tool Altschul, Gish, Miller, Myers, Lipman, J Mol Biol 1990

- The most widely used comp bio tool
- Which is better: long mediocre match or a few nearby, short, strong matches with the same total score?
- score-wise, exactly equivalent
- biologically, later may be more interesting, \& is common
- at least, if must miss some, rather miss the former
- BLAST is a heuristic emphasizing the later
- speed/sensitivity tradeoff: BLAST may miss former, but gains greatly in speed


## BLAST: What

- Input:
- a query sequence (say, 300 residues)
- a data base to search for other sequences similar to the query (say, $10^{6}-10^{9}$ residues)
- a score matrix $\sigma(r, s)$, giving cost of substituting $r$ for $s(\&$ perhaps gap costs)
- various score thresholds \& tuning parameters
- Output:
- "all" matches in data base above threshold
_ "E-value" of each


## BLAST: How

Idea: find parts of data base near a good match to some short subword of the query

- Break query into overlapping words $w_{i}$ of small fixed length (e.g. 3 aa or 11 nt)
- For each $w_{i}$, find (empirically, $\sim 50$ ) "neighboring" words $\mathrm{v}_{\mathrm{ij}}$ with score $\sigma\left(\mathrm{w}_{\mathrm{i}}, \mathrm{v}_{\mathrm{ij}}\right)>$ thresh $_{1}$
- Look up each $\mathrm{v}_{\mathrm{ij}}$ in database (via prebuilt index) -i.e., exact match to short, high-scoring word
- Extend each such "seed match" (bidirectional)
- Report those scoring > thresh ${ }_{2}$, calculate E-values


## BLAST: Example



## BLOSUM 62

|  |  | A R | R N | N |  |  |  |  | , | H |  |  | L K | K | M |  |  |  | T | W | Y |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 4 -1 | 1 -2 | -2 | - |  | -1 | -1 |  | -2 | -1 | -1 | - 1 | -1 | - | -2 |  |  |  | -3 | -2 |  |
| R |  | -1 | 50 | 0 -2 | -2 -3 | 3 |  | 0 | -2 | 0 | -3 | -2 | -2 | 2 -1 | 1 | -3 |  | -1 | -1 |  | -2 |  |
| N |  | -2 0 | 06 | 6 | -3 | - |  |  | 0 |  | -3 | -3 | - 0 | - 2 | -2 | -3 -2 |  |  |  | -4 | -2 |  |
| D |  | -2 -2 | 2 | 1 | - | 3 | 0 | 2 | -1 | -1 | - |  | -4 | -3 | - | -3 |  | 0 | -1 | -4 | -3 |  |
| c |  | - | 3 -3 | - | -3 |  | -3 | -4 | -3 | -3 | -1 |  | 1 | -1 | -2 | -2 |  | -1 | -1 | -2 | -2 |  |
|  |  | - |  |  | 0 -3 |  | 5 |  | -2 |  | -3 |  | - |  |  |  |  |  |  | -2 |  |  |
| E |  | -1 | 00 | 0 | -4 | - |  | 5 | -2 | 0 | -3 |  | - | -2 | -2 | -3-1 |  |  | -1 | -3 | -2 |  |
| G |  | $0-2$ | 2 | 0 | -1 -3 |  | -2 | -2 | 6 | -2 | -4 |  | 4 | 2 -3 | -3 | - |  | 0 | -2 | -2 | -3 |  |
| H |  | -2 |  |  | - |  |  | 0 | -2 |  | -3 |  | - -1 |  | -2 | - |  |  |  | -2 |  |  |
| 1 |  | - -3 | 3 -3 | 3 | -3 -1 |  | -3 | -3 | -4 | -3 | 4 |  | $2-3$ |  | 1 | 0 |  | -2 | -1 | -3 | -1 |  |
|  |  | -1 | 2 -3 | 3 | -4 -1 |  | -2 | -3 | -4 | -3 |  |  |  |  | 2 |  |  | -2 | -1 |  | -1 |  |
| K |  | -1 | 2 | 0 -1 | -1 |  |  |  | -2 | -1 |  |  | -2 |  | - | -3-1 |  |  | -1 |  |  |  |
| M |  | -1 -1 | 1 -2 | -2 | -3 -1 |  |  | -2 | -3 | -2 |  |  |  |  |  |  |  |  | -1 |  | -1 |  |
|  |  | -2 -3 | 3 -3 | - | -3 -2 |  | -3 | -3 | -3 | -1 |  |  |  |  |  |  |  |  |  |  |  |  |
| P |  | -1-2 | $2-2$ | 2 | -1-3 |  | -1 | -1 | -2 | -2 | -3 |  | -3 | -2 | -2 | - |  | -1 | -1 | -4 | -3 |  |
| S |  | 1 | 1 |  | $0-1$ |  |  |  | 0 | -1 | -2 |  | -2 |  | - | - |  | 4 |  | -3 | -2 |  |
|  |  | 1 | 1 | 0 -1 | - |  | -1 | -1 | -2 | - | -1 | -1 | -1 -1 | -1 | 1 -2 | -2 |  | 1 | 5 | -2 | -2 |  |
| w |  | -3 -3 | 3 -4 | - | -4 -2 |  | -2 | -3 | -2 | -2 | -3 | -2 | 2 -3 | -1 | 1 |  |  | -3 |  |  |  |  |
|  |  | -2 -2 | $2-2$ | -2 |  |  | -1 | -2 | -3 |  |  |  | - -2 | -1 | 1 |  |  | -2 |  |  | 7 |  |
|  |  | -3 | $3-3$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $-1$ |  |

## BLAST Refinements

- "Two hit heuristic" -- need 2 nearby, nonoverlapping, gapless hits before trying to extend either
- "Gapped BLAST" -- run heuristic version of SmithWaterman, bi-directional from hit, until score drops by fixed amount below max
- PSI-BLAST -- For proteins, iterated search, using "weight matrix" pattern from initial pass to find weaker matches in subsequent passes


## Significance of Alignments

- Is " 42 " a good score?
- Compared to what?
- Usual approach: compared to a specific "null model", such as "random sequences"


## Hypothesis Testing: A Very Simple Example

- Given: A coin, either fair $(p(H)=1 / 2)$ or biased $(p(H)=2 / 3)$
- Decide: which
- How? Flip it 5 times. Suppose outcome D = HHHTH
- Null Model/Null Hypothesis $M_{0}: p(H)=1 / 2$
- Alternative Model/Alt Hypothesis $M_{1}: p(H)=2 / 3$
- Likelihoods:
$-P\left(D \mid M_{0}\right)=(1 / 2)(1 / 2)(1 / 2)(1 / 2)(1 / 2)=1 / 32$
$-P\left(D \mid M_{1}\right)=(2 / 3)(2 / 3)(2 / 3)(1 / 3)(2 / 3)=16 / 243$
- Likelihood Ratio: $\frac{p\left(D \mid M_{1}\right)}{p\left(D \mid M_{0}\right)}=\frac{16 / 243}{1 / 32}=\frac{512}{243} \approx 2.1$
l.e., alt model is $\approx 2.1 \mathrm{x}$ more likely than null model, given data


## p -values

- the $p$-value of such a test is the probability, assuming that the null model is true, of seeing data as extreme or more extreme that what you actually observed
- e.g., we observed 4 heads; p-value is prob of seeing 4 or 5 heads in 5 tosses of a fair coin
- Why interesting? It measures probability that we would be making a mistake in rejecting null.
- Usual scientific convention is to reject null only if $p$-value is < 0.05 ; sometimes demand $p \ll 0.05$
- can analytically find $p$-value for simple problems like coins; often turn to simulation/permutation tests for more complex situations; as below


## Hypothesis Testing, II

- Log of likelihood ratio is equivalent, often more convenient
- add logs instead of multiplying..
- "Likelihood Ratio Tests": reject null if LLR > threshold - LLR > 0 disfavors null, but higher threshold gives stronger evidence against
- Neyman-Pearson Theorem: For a given error rate, LRT is as good a test as any.


## A Likelihood Ratio Test for Alignment

- Defn: two proteins are homologous if they are alike because of shared ancestry; similarity by descent
- suppose among proteins overall, residue $x$ occurs with frequency $p_{x}$
- then in a random alignment of 2 random proteins, you would expect to find $x$ aligned to $y$ with prob $p_{x} p_{y}$
- suppose among homologs, $x \& y$ align with prob $p_{x y}$
- are seqs $X \& Y$ homologous? Which is
more likely, that the alignment reflects chance or homology? Use a likelihood ratio test.


## Non-ad hoc Alignment Scores

- Take alignments of homologs and look at frequency of $x-y$ alignments vs freq of $x, y$ overall
- Issues
- biased samples
- evolutionary distance
- BLOSUM approach
- large collection of trusted alignments (the BLOCKS DB)
- subsetted by similarity, e.g BLOSUM62 => 62\% identity

$$
\frac{1}{\lambda} \log _{2} \frac{p_{x y}}{p_{x} p_{y}}
$$

## ad hoc Alignment Scores?

- Make up any scoring matrix you like
- Somewhat surprisingly, under pretty general assumptions**, it is equivalent to the scores constructed as above from some set of probabilities $\mathrm{p}_{\mathrm{xy}}$, so you might as well understand what they are
**e.g., average scores should be negative, but you probably want that anyway, otherwise local alignments turn into global ones, and some score must be $>0$, else best match is empty

BLOSUM 62


## Overall Alignment Significance, I A Theoretical Approach: EVD

- If $X_{i}$ is a random variable drawn from, say, a normal distribution with mean 0 and std. dev. 1, what can you say about distribution of $y=\max \left\{X_{i} \mid 1 \leq i \leq N\right\}$ ?
- Answer: it's approximately an Extreme Value Distribution (EVD)

$$
\begin{equation*}
P(y \leq z) \cong \exp \left(-K N e^{-\lambda z}\right) \tag{*}
\end{equation*}
$$

- For ungapped local alignment of seqs $x, y, N \sim|x| *|y|$ $\lambda, K$ depend on scores, etc., or can be estimated by curve-fitting random scores to (*). (cf. reading)


## EVD Pro/Con

- Pro:
- gives p -values for alignment scores
- Con:
- It's only approximate
- parameter estimation
- theory may not apply. E.g., it is NOT known to hold for gapped alignments (although empirically it seems to work pretty well).


## Generating Random Permutations

```
for (i= n-1; i>0; i--){
    j = random(0..i);
    swap X[i]<-> X[j];
}
```



Overall Alignment Significance, II Empirical (via randomization)

- generate N random sequences (say $\mathrm{N}=10^{3}-10^{6}$ )
- align x to each \& score
- if $k$ of them have better score than alignment of $x$ to $y$, then the (empirical) probability of a chance alignment as good as observed x : y alignment is $\mathrm{k} / \mathrm{N}$
- How to generate "random" sequences?
- Alignment scores often sensitive to sequence composition
- so uniform $1 / 20$ or $1 / 4$ is a bad idea
- even background $p_{i}$ can be dangerous
- Better idea: permute y N times


## Permutation Pro/Con

- Pro:
- Gives empirical p-values for alignments with characteristics like sequence of interest, e.g. residue frequencies
- Con:
- Can be inaccurate if your method of generating random sequences is unrepresentative
- E.g., probably better to preserve di-, tri-residue statistics and/or other higher-order characteristics, but increasingly hard to know exactly what to model \& how
- Slow
- Especially if you want to assess low-probability p-values


## p-values \& multiple testing

- Above give " $p$-values": probability of a score more extreme than Abserved if the target sequence were random
- must be careful whether $p$-value means wrt comparison to one other random protein, or best of a database of $n$ random proteins
- E.g., suppose p-value for $x: y$ match is $10^{-3}$, then you'd expect to see a score that good only one time in a thousand among nonhomologous sequences
- Sounds good
- What if you found y by picking best match among $10^{4}$ proteins?
- Sounds not so good


## E-values

- "p-value": probability of a score more extreme than observed in a given random target data base
- E-value: expected number of matches that good or better in a random data base of the given size \& composition
- Related: $P=1-\exp (-E)$
- $\mathrm{E}=5$ <--> P = . 993
- $E=10$ <--> $P=.99995$
- both equally valid; E-value is perhaps a more intuitively interpretable quantity, \& perhaps makes role of data base size more explicit


## Issues

- What if the model is wrong?
- E.g., are adjacent positions really independent?


## Summary

- BLAST is a highly successful search/alignment heuristic. It looks for alignments anchored by short, strong, ungapped "seed" alignments
- Assessing statistical significance of alignment scores is crucial to practical applications
- score matrices derived from "likelihood ratio" test of trusted alignments vs random "null" model
- for gapless alignments, Extreme Value Distribution (EVD) is theoretically justified for overall significance of alignment scores; empirically seems ok for gapped alignments, too
- permutation tests are a simple (but brute force) alternative

| Weekly Bio(tech) Interlude <br> 2 Nobel Prizes: <br> PCR: Kary Mullis, 1993 <br> DNA Sequencing: Frederick Sanger, 1980 |
| :---: |



PCR

- Ingredients:
- many copies of deoxy nucleotide triphosphates
- many copies of two primer sequences (~20 nt each)
- readily synthesized
- many copies of Taq polymerase (Thermus aquaticus),
- readily available commercialy
- as little as 1 strand of template DNA
- a programmable "thermal cycler"
- Amplification: million to billion fold
- Range: up to 2 k bp routinely; 50k with other enzymes \& care
- Very widely used; forensics, archeology, cloning, sequencing, ...


## DNA Forensics

- E.g. FBI "CODIS" (combined DNA indexing system) data base
- pick 13 short, variable regions of human genome
- amplify each from, e.g., small spot of dried blood
- measure product lengths (next slides)
- PCR is important in that sample size is reduced from grams of tissue to a few cells



## DNA Sequencing

- Like one-cycle, one-primer PCR
- Suppose $0.1 \%$ of A’s:
- are di-deoxy adenosine's; backbone can't extend
- carry a green florescent dye
- Separate by capillary gel electrophoresis
- If frags of length $42,49,50,55 \ldots$ glow green, those positions are A's
- Ditto C's (blue), G's (yellow), T's (red)



## DNA Sequencing

- Highly automated
- Typically can "read" about 600 nt in one run
- "Whole Genome Shotgun" approach:
- cut genome randomly into ~ G / $600 \times 10$ fragments
- sequence each
- reassemble by computer

- Complications: repeated region, missed regions, sequencing errors, chimeric DNA fragments, ..
- But overall accuracy $\sim 10^{-4}$, if careful


## Summary

- PCR allows simple in vitro amplification of minute quantities of DNA (having pre-specified boundaries)
- Sanger sequencing uses
- a PCR-like setup with modified chemistry to generate varying length prefixes of a DNA template with the last nucleotide of each color-coded
- gel electrophoresis to separate DNA by size, giving sequence
- Sequencing random overlapping fragments allows genome sequencing

