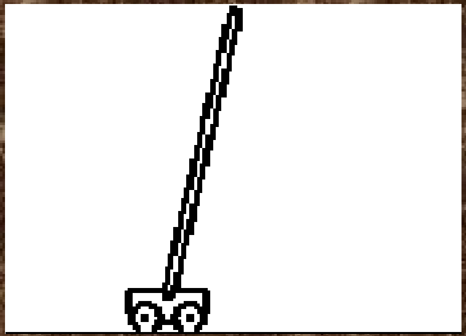
CSE/NB 528

Lecture 14: Reinforcement Learning

(Chapter 9)



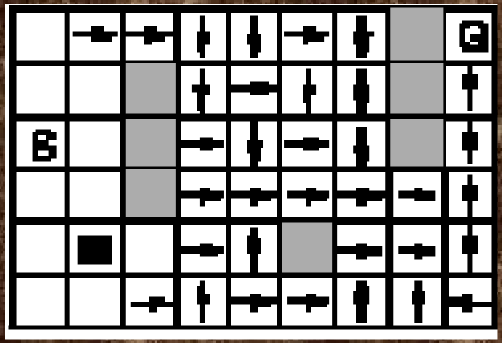


Image from http://clasdean.la.asu.edu/news/images/ubep2001/neuron3.jpg

Lecture figures are from Dayan & Abbott's book
http://people.brandeis.edu/~abbott/book/index.html

Today's Agenda

- Reinforcement Learning
  - What is reinforcement learning?
  - Classical conditioning
    - Learning to salivate (predicting reward)
  - Predicting Delayed Rewards
    - Temporal Difference Learning
  - Learning to Act
    - Q-learning
    - Actor-Critic Architecture

Some Supervised Learning Demos on the Web

- Function Approximation:

  http://neuron.eng.wayne.edu/bpFunctionApprox/bpFunctionApprox.html

- Pattern Recognition

  http://eecs.wsu.edu/~cook/ai/lectures/applets/hnn/JRec.html

- Image Compression

  http://neuron.eng.wayne.edu/bpImageCompression9PLUS/bp9PLUS.html

- Backpropagation for Control: Ball Balancing

  http://neuron.eng.wayne.edu/bpBallBalancing/ball5.html

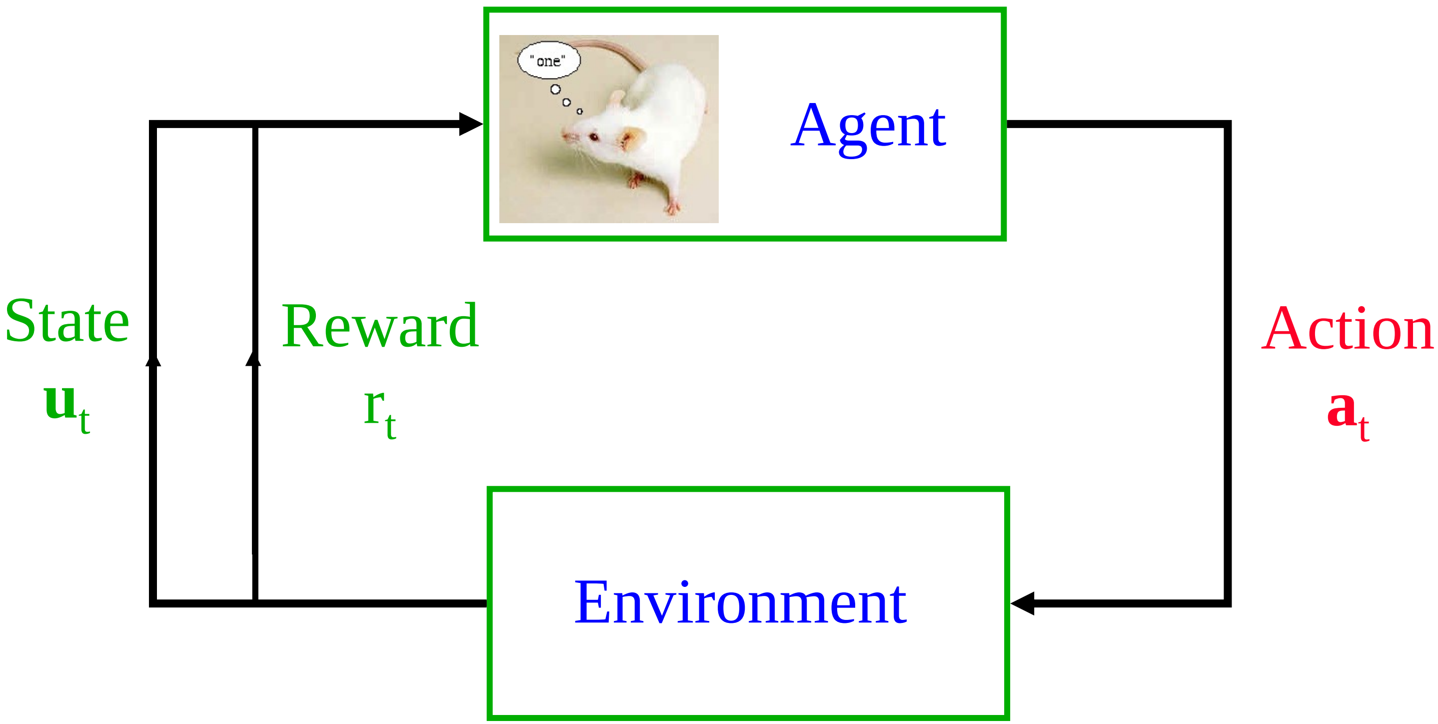
Humans don’t get exact supervisory signals (commands for muscles) for learning to talk, walk, ride a bicycle, play the piano, drive, etc.

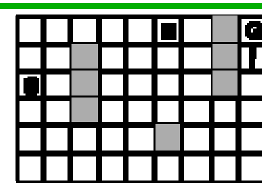
We learn by trial-and-error and by watching others

Might get “rewards and punishments” along the way

Enter…Reinforcement Learning

The Reinforcement Learning "Agent"



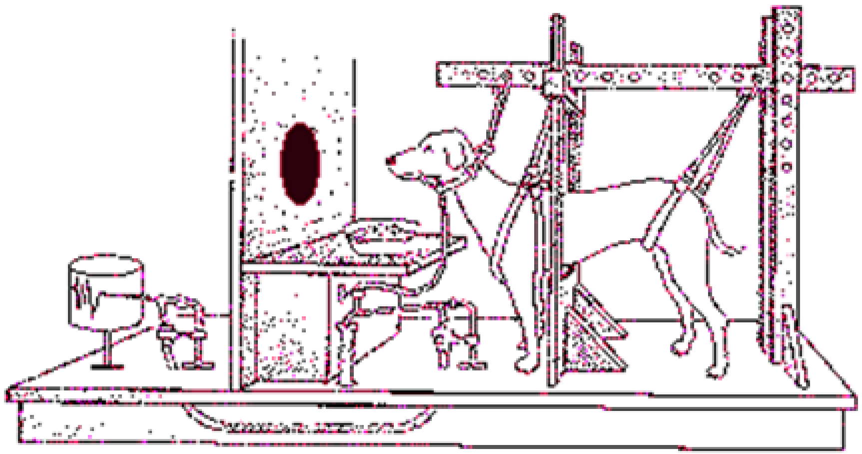


The Reinforcement Learning Framework

- **Unsupervised learning:** Learn the hidden causes of inputs
- **Supervised learning:** Learn a function based on training examples of (input, desired output) pairs
- **Reinforcement Learning:** Learn the best action for any given state so as to maximize total expected (future) reward
  - Learn by trial and error
  - Intermediate between unsupervised and supervised learning
    Instead of explicit teaching signal (or desired output), you get *rewards or punishments*
  - Inspired by classical conditioning experiments (remember Pavlov's hyper-salivating dog?)

Early Results: Pavlov and his Dog

- Classical (Pavlovian) conditioning experiments
- Training: Bell →Food
- After: Bell → Salivate
- Conditioned stimulus (bell) predicts future reward (food)



(http://employees.csbsju.edu/tcreed/pb/pdoganim.html)

Predicting Reward

- Stimulus u = 0 or 1
- Expected reward $v = wu$
- Delivered reward = r
- Learn w by minimizing $(r - v)^2$

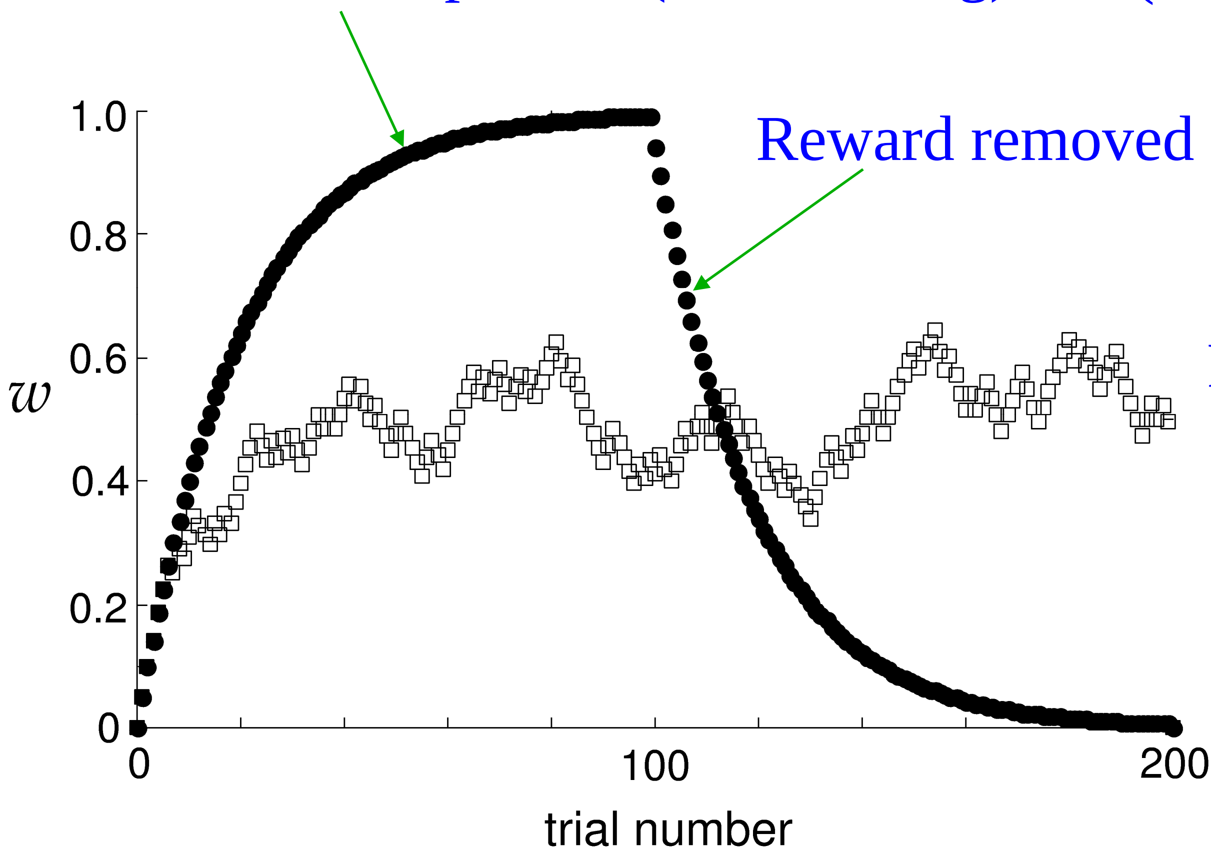
$$w \rightarrow w + \varepsilon(r - v)u$$

(same as the delta rule; also called Rescorla-Wagner rule)

- Prediction error $\delta = (r - v)$
- For small ε and u = 1, $w \rightarrow w + \varepsilon(r - w)$
  - Average value of w = $\langle w \rangle \approx \langle r \rangle$

Predicting Reward during Conditioning

Reward r present (conditioning) (r = 1, ε = 0.5)



Reward removed ("extinction")

Reward presented 50% of the trials

Predicting Delayed Rewards

- In more realistic cases, reward is typically delivered at the end (when you know whether you succeeded or not)
- Time: $0 \le t \le$ T with stimulus $u(t)$ and reward $r(t)$ at each time step t (Note: $r(t)$ can be zero)
- Key Idea: Make the output $v(t)$ predict *total expected future reward* starting from time t

$$v(t) \approx \left\langle \sum_{\tau=0}^{T-t} r(t+\tau) \right\rangle$$

Learning to Predict Delayed Rewards

- Use a set of modifiable weights $w(t)$ and *predict based on all past stimuli* $u(t)$:

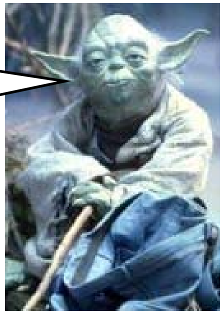
$$v(t) = \sum_{\tau=0}^{t} w(\tau) u(t-\tau)$$

- Would like to find $w(\tau)$ that minimize:

$$\left(\sum_{\tau=0}^{T-t} r(t+\tau) - v(t) \right)^2$$

(Can we minimize this using gradient descent and delta rule?)

Yes, BUT…not yet available are future rewards



Temporal Difference (TD) Learning

- **Key Idea**: Rewrite squared error to get rid of future terms:

$$\left(\sum_{\tau=0}^{T-t} r(t+\tau) - v(t)\right)^2 = \left(r(t) + \sum_{\tau=0}^{T-t-1} r(t+1+\tau) - v(t)\right)^2$$

$$\approx \left(r(t) + v(t+1) - v(t)\right)^2$$

- **Temporal Difference (TD) Learning**:

$$v(t) = \sum_{\tau=0}^{t} w(\tau) u(t-\tau)$$

For each time step t, do:

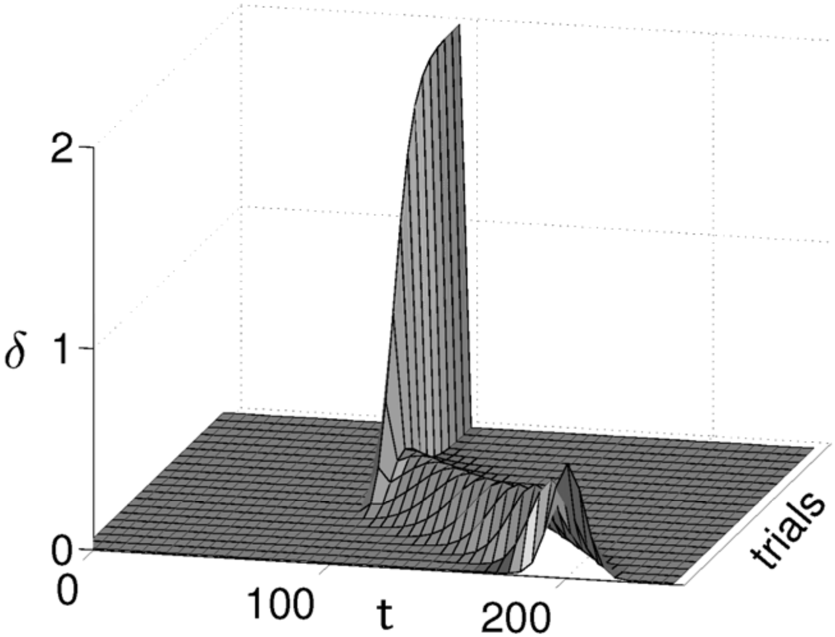
For all τ $(0 \le \tau \le$ t), do:

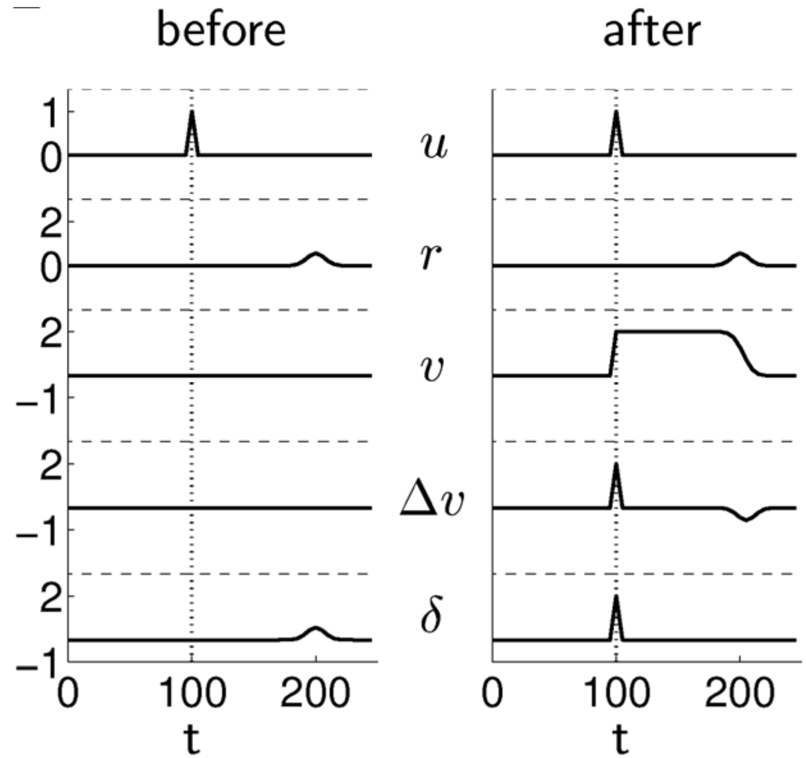
$$w(\tau) \to w(\tau) + \varepsilon \overbrace{[\underbrace{r(t) + v(t+1)}_{\text{Expected future reward}} - v(t)]}^{\delta} u(t-\tau)$$

Prediction

Predicting Delayed Reward: TD Learning

Stimulus at t = 100 and reward at t = 200



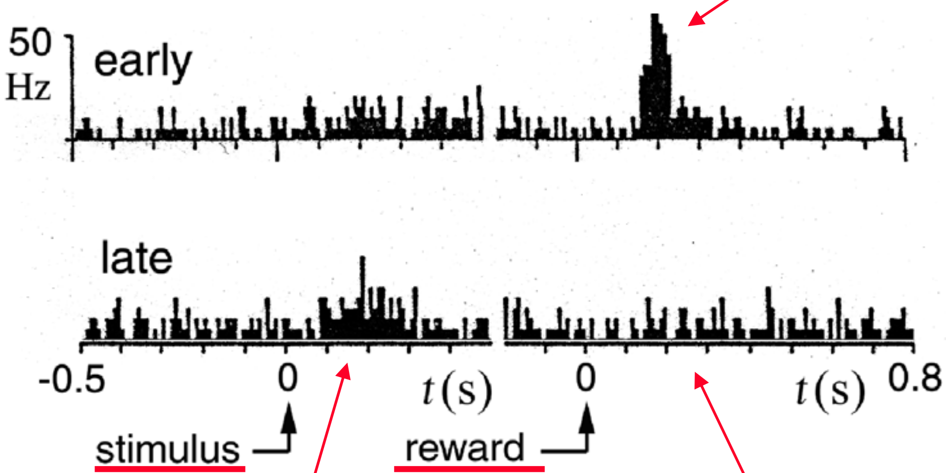


Prediction error δ for each time step (over many trials)

Reward Prediction Error Signal in Monkeys?

Dopaminergic cells in Ventral Tegmental Area (VTA)

Reward Prediction error? $[r(t)+v(t+1)-v(t)]$



Before Training

After Training

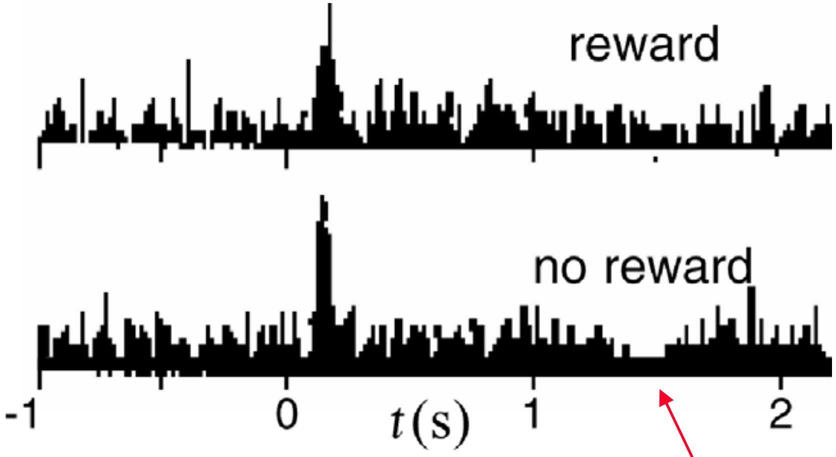
$[0+v(t+1)-v(t)]$

No error

$v(t)\approx r(t)+v(t+1)$

More Evidence for Prediction Error Signals

Dopaminergic cells in VTA



Negative error

$$r(t) = 0, v(t+1) = 0$$

$$[r(t) + v(t+1) - v(t)] = -v(t)$$

That’s great, but how does all that math help me get food in a maze?

Using Reward Predictions to Select Actions

- Suppose you have computed a "Value" for each action
- $Q(a)$ = value (predicted reward) for executing action a
  - Higher if action yields more reward, lower otherwise
- Can select actions probabilistically according to their value:

$$P(a) = \frac{\exp(\beta Q(a))}{\sum_{a'} \exp(\beta Q(a'))}$$

(High β selects actions with highest Q value. Low β selects more uniformly)

Simple Example: Bee Foraging

- Experiment: Bees select either yellow (y) or blue (b) flowers based on nectar reward

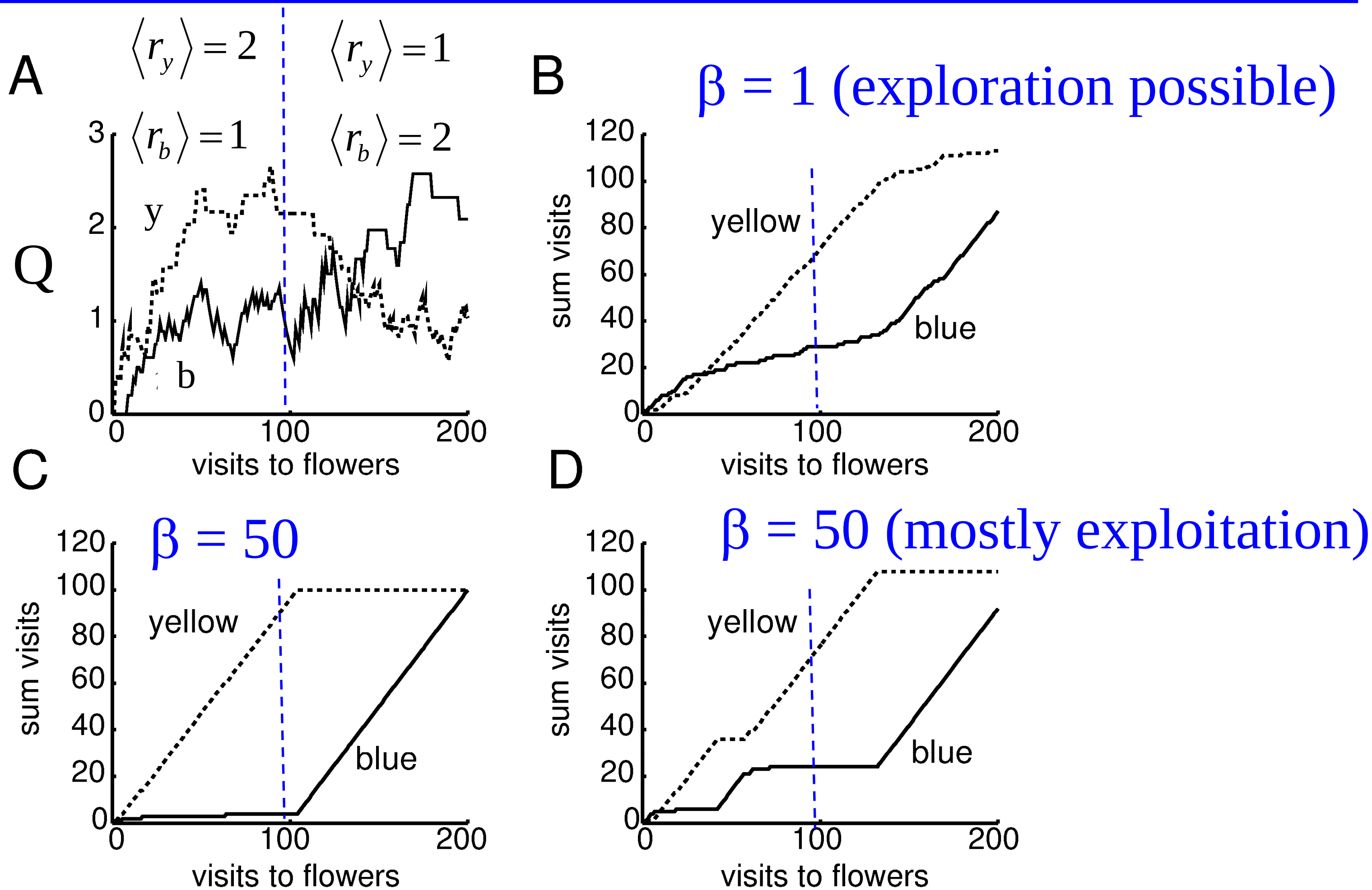
- Idea: Value of yellow/blue = average reward obtained so far

$$\left.\begin{aligned} Q(y) &\rightarrow Q(y) + \varepsilon(r_y - Q(y)) \\ Q(b) &\rightarrow Q(b) + \varepsilon(r_b - Q(b)) \end{aligned}\right\} \text{delta rule (running average)}$$

$$P(y) = \frac{\exp(\beta Q(y))}{\exp(\beta Q(y)) + \exp(\beta Q(b))}$$

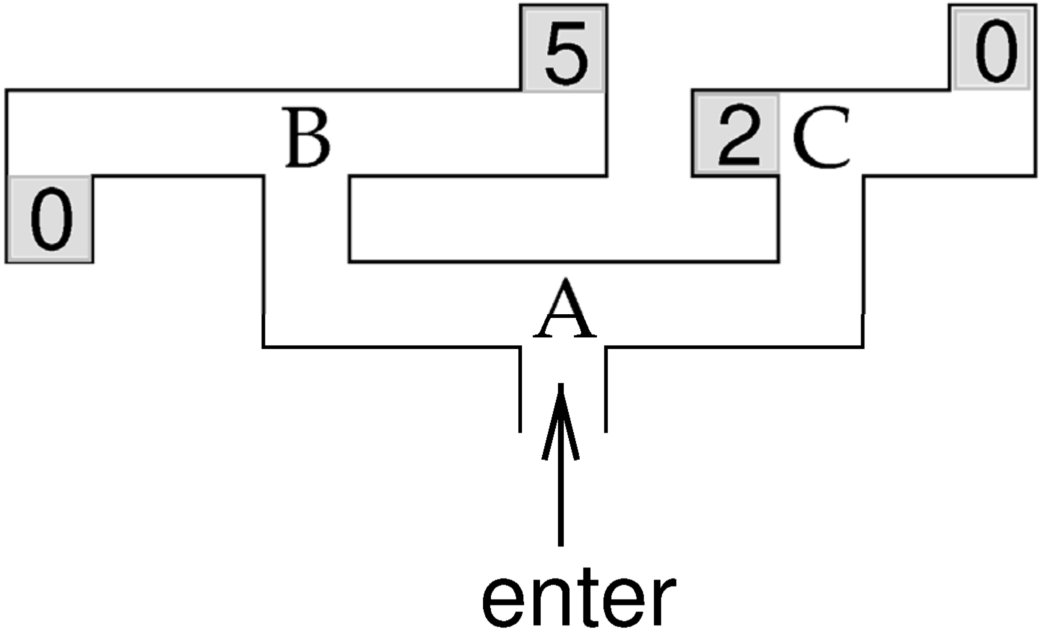
$$P(b) = 1 - P(y)$$

Simulating Bees




Forget bees, how do I get
to the food in the maze?

Selecting Actions when Reward is Delayed

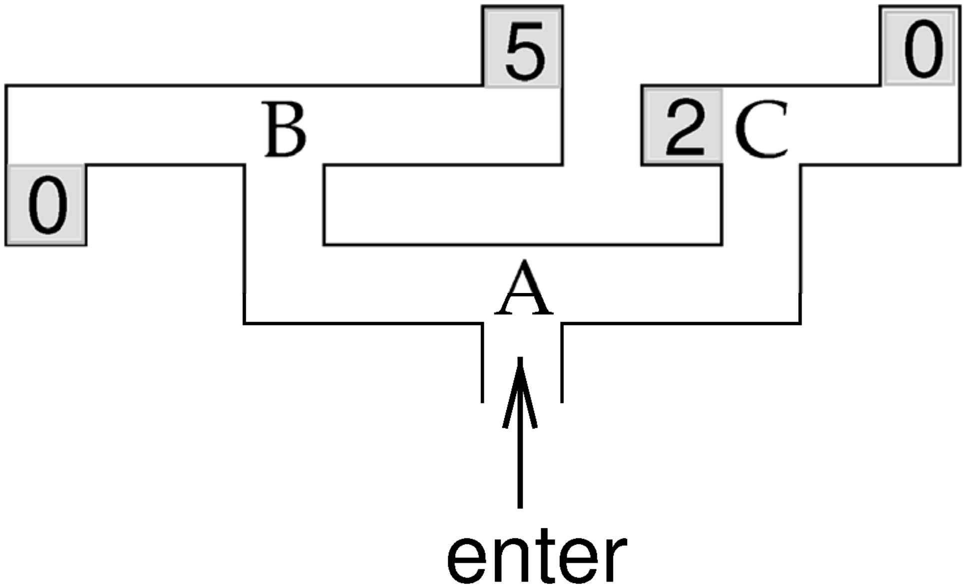


States: A, B, or C

Possible actions at any state: Left (L) or Right (R)

If you randomly choose to go L or R (random “policy”), what is the *value v of each state*?

Policy Evaluation



For random policy:

$$v(B) = \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 5 = 2.5$$

$$v(C) = \frac{1}{2}\cdot 2 + \frac{1}{2}\cdot 0 = 1$$

$$v(A) = \frac{1}{2}\cdot v(B) + \frac{1}{2}\cdot v(C) = 1.75$$

(Location, action) ⇨ new location

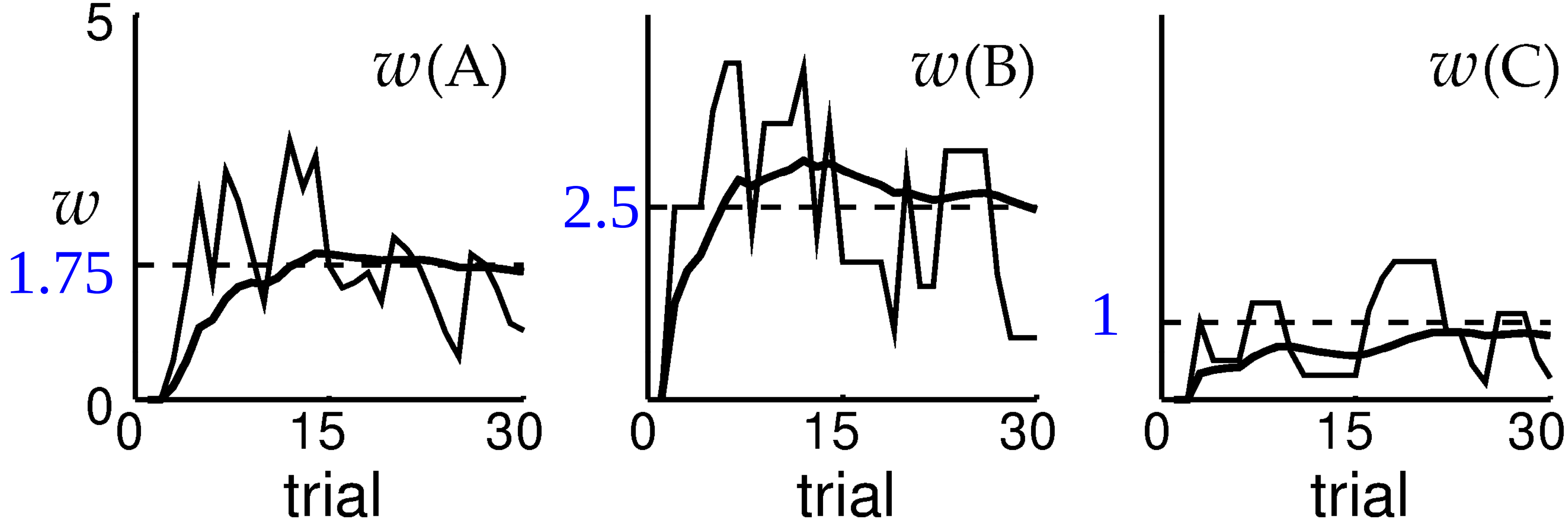
(u,a) ⇨ u'

Let $v(u) = w(u)$

Can learn this using TD learning:

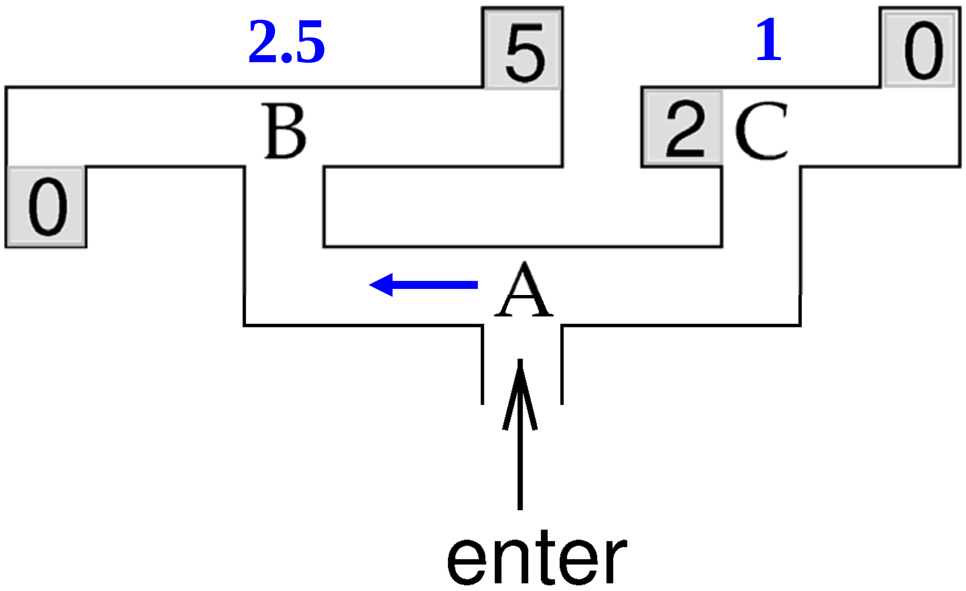
$$w(u) \rightarrow w(u) + \varepsilon\,[r_a(u) + v(u') - v(u)]$$

Maze Value Learning for Random Policy



Once I know the values, I can pick the action that leads to the higher valued state!

Selecting Actions based on Values



Values act as surrogate immediate rewards → Locally optimal choice leads to globally optimal policy (for Markov environments)

Related to *Dynamic Programming* in CS (see appendix in text)

Q learning

- A simple method for action selection based on action values (or *Q* values) $Q(x,a)$ where *x* is a state and *a* is an action

1. Let *u* be the current state. Select an action *a* according to:

$$P(a) = \frac{\exp(\beta Q(u,a))}{\sum_{a'} \exp(\beta Q(u,a'))}$$

2. Execute *a* and record new state u' and reward *r*. Update Q:

$$Q(u,a) \rightarrow Q(u,a) + \varepsilon(r + \underline{\max_{a'}} Q(u',a') - Q(u,a))$$

3. Repeat until an end state is reached

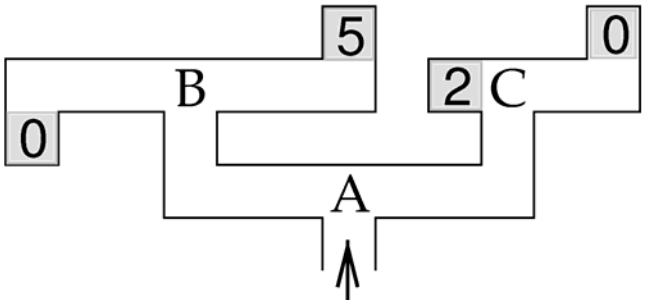
Another Variant: Actor-Critic Learning

- Two separate components: Actor (maintains policy) and Critic (maintains value of each state)

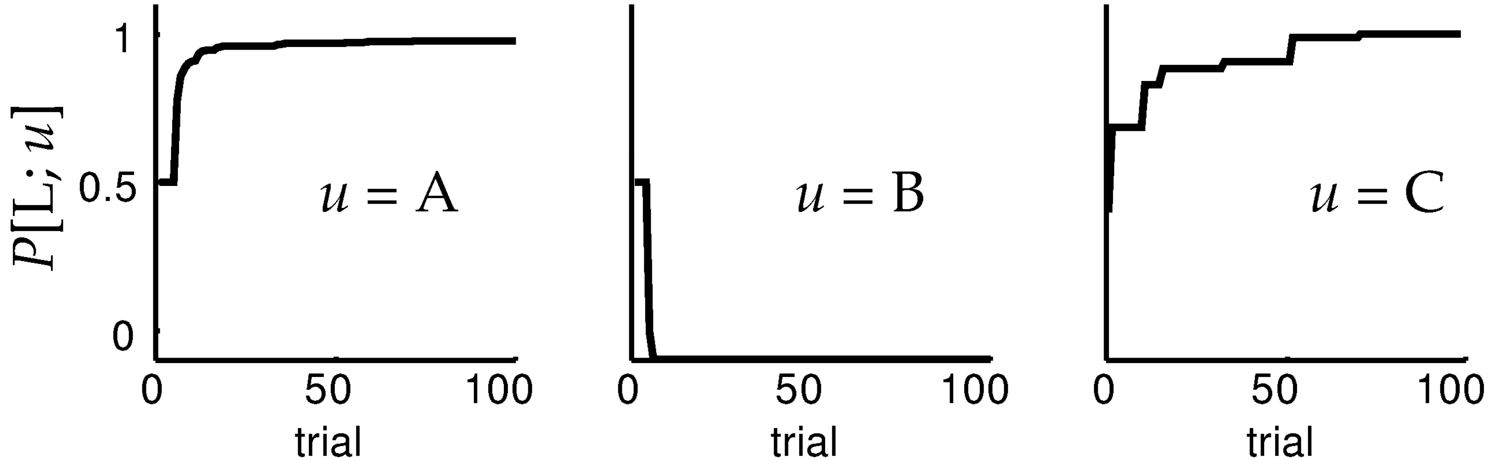
1. Critic Learning ("Policy Evaluation"):
   Value of state $u = v(u) = w(u)$
   $$w(u) \to w(u) + \varepsilon[r_a(u) + v(u') - v(u)]$$ (same as TD rule)
2. Actor Learning ("Policy Improvement"):
   $$P(a;u) = \frac{\exp(\beta Q_a(u))}{\sum_b \exp(\beta Q_b(u))}$$ Use this to select an action *a* in *u*

   For all *a*':
   $$Q_{a'}(u) \to Q_{a'}(u) + \varepsilon[r_a(u) + v(u') - v(u)](\delta_{aa'} - P(a';u))$$
3. Interleave 1 and 2

Actor-Critic Learning in the Maze Task

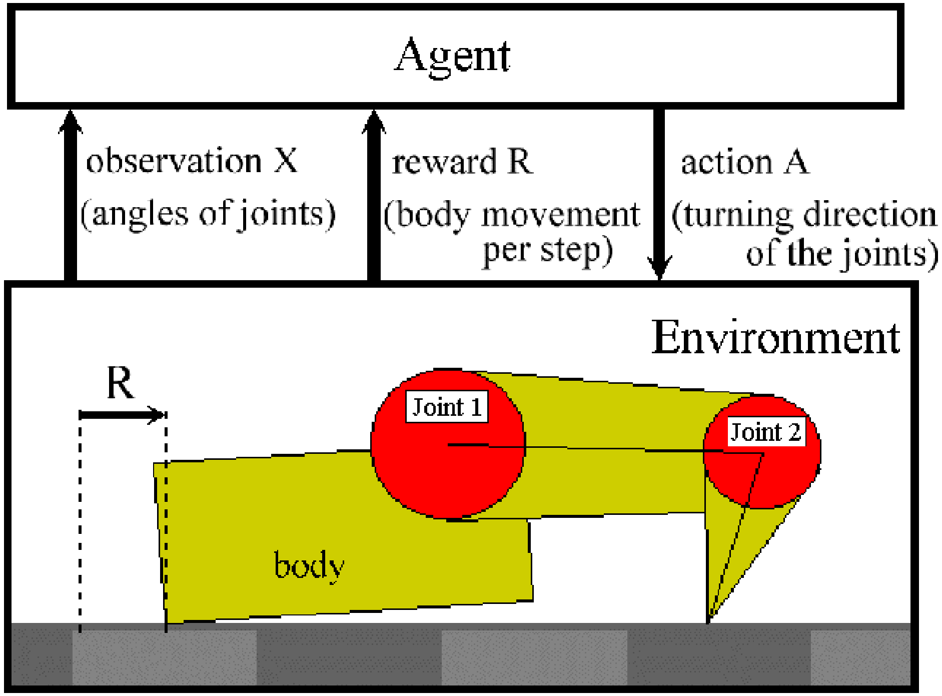


Probability of going Left at a location



Demo of Reinforcement Learning in a Robot

(from http://sysplan.nams.kyushu-u.ac.jp/gen/papers/JavaDemoML97/robodemo.html)



Things to do:

Work on mini-project

Next class:
Course Summary