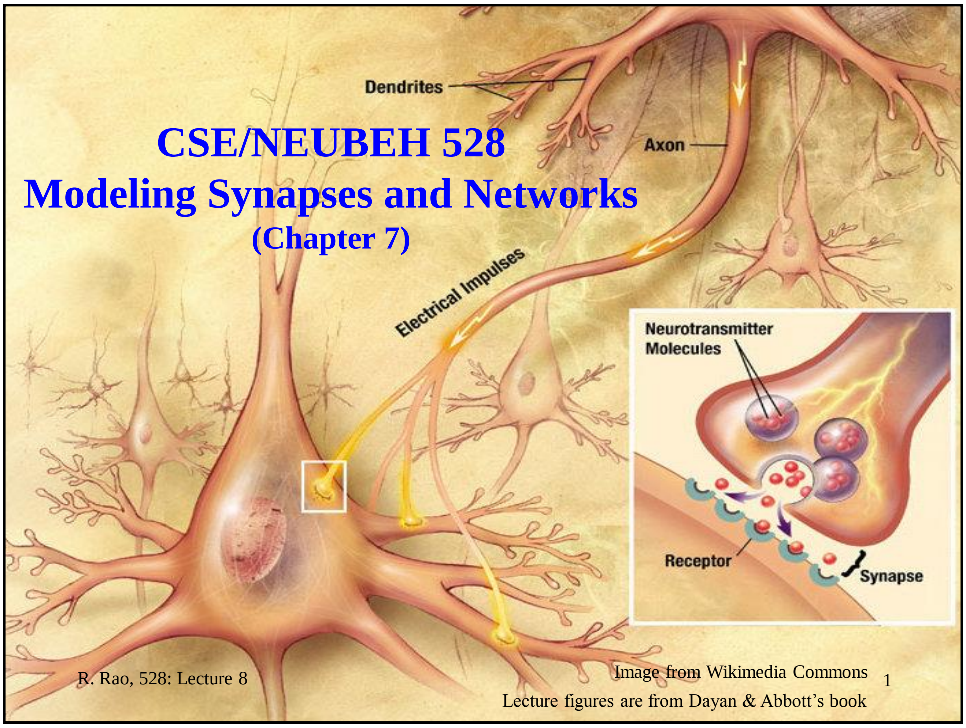

CSE/NEUBEH 528
Modeling Synapses and Networks
(Chapter 7)

Image from Wikimedia Commons

Lecture figures are from Dayan & Abbott's book

Course Summary (thus far)

- Neural Encoding
  - What makes a neuron fire? (STA, covariance analysis)
  - Poisson model of spiking
- Neural Decoding
  - Spike-train based decoding of stimulus
  - Stimulus Discrimination based on firing rate
  - Population decoding (Bayesian estimation)
- Single Neuron Models
  - RC circuit model of membrane
  - Integrate-and-fire model
  - Conductance-based Models

Today’s Agenda

- Computation in Networks of Neurons
  - Modeling synaptic inputs
  - From spiking to firing-rate based networks
  - Feedforward Networks
  - Multilayer Networks

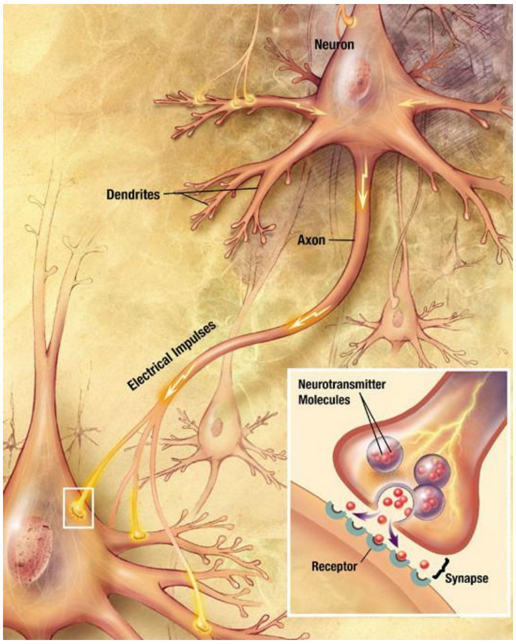

Image Source: Wikimedia Commons

How do neurons connect to form networks?

Using synapses!

Synapses on an actual neuron

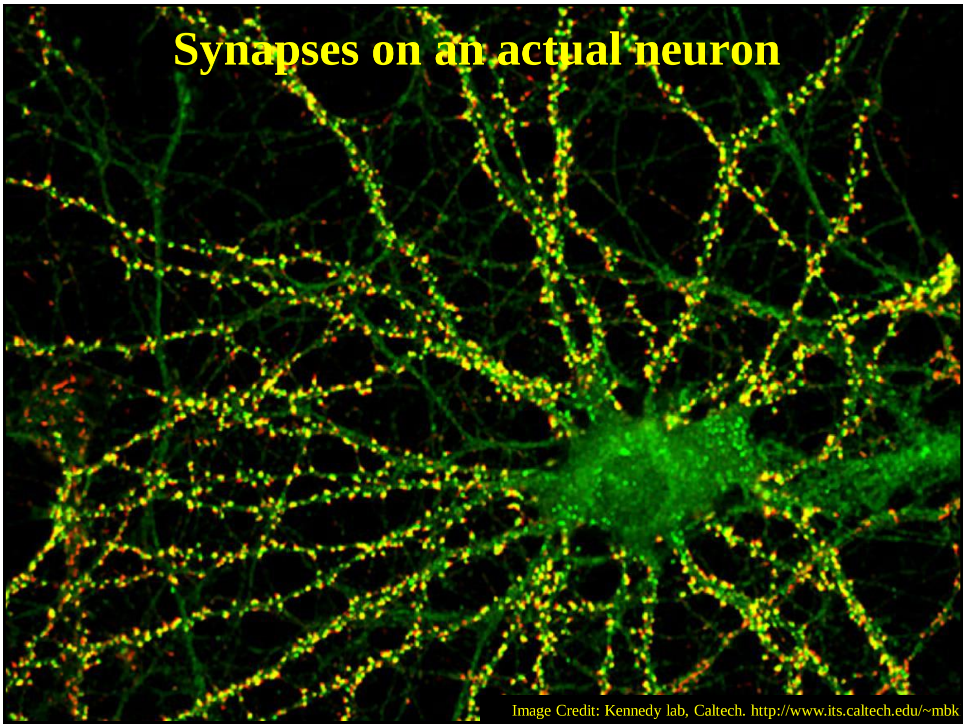

Image Credit: Kennedy lab, Caltech. http://www.its.caltech.edu/~mbk

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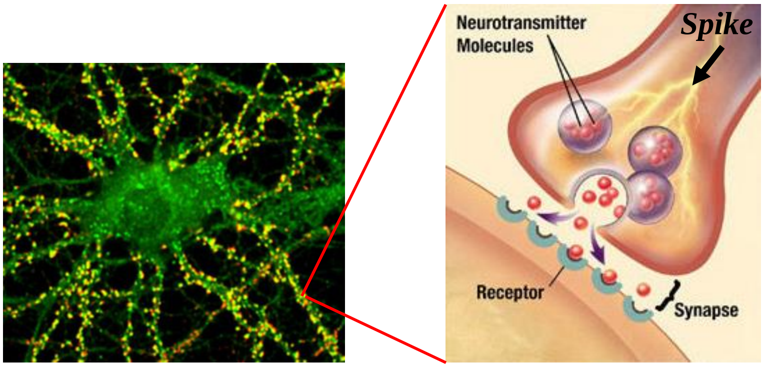
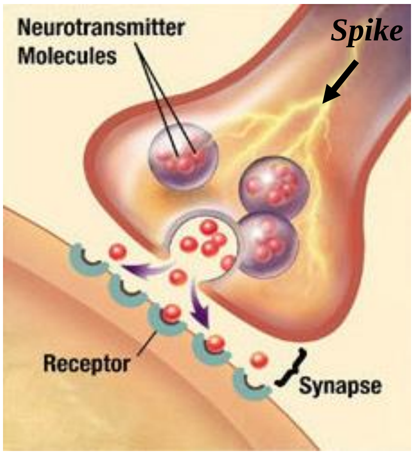
What do synapses do?



Increase or decrease postsynaptic membrane potential

Image Source: Wikimedia Commons

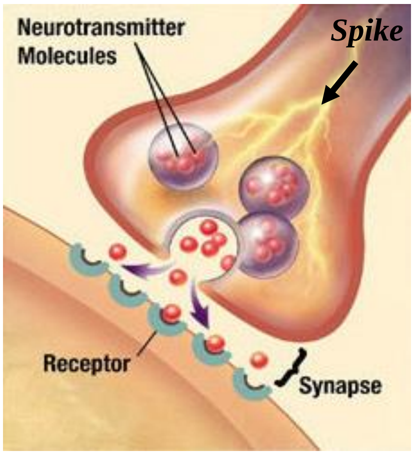
An Excitatory Synapse



Input spike → Neurotransmitter release (e.g., Glutamate) → Binds to ion channel receptors → Ion channels open → Na+ influx → Depolarization due to EPSP (excitatory postsynaptic potential)

Image Source: Wikimedia Commons

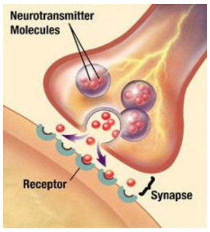
An Inhibitory Synapse

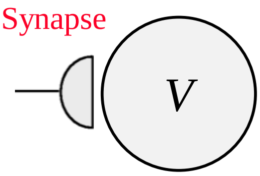


Input spike → Neurotransmitter release (e.g., GABA) → Binds to ion channel receptors → Ion channels open → Cl- influx→ Hyperpolarization due to IPSP (inhibitory postsynaptic potential)

Image Source: Wikimedia Commons

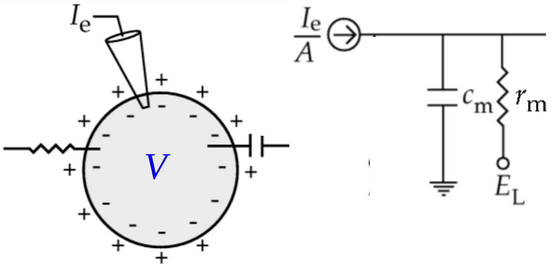
We want a *computational* model of the effects of a synapse on the membrane potential *V*





How do we do this?

Flashback Membrane Model



$c_m \approx 10\ \text{nF/mm}^2$

$r_m \approx 1\ \text{M}\Omega\ \text{mm}^2$

$C_m = c_m A$

$R_m = r_m / A$

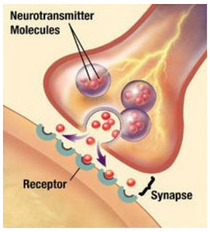
$c_m \frac{dV}{dt} = -\frac{(V-E_L)}{r_m} + \frac{I_e}{A}$, or equivalently:

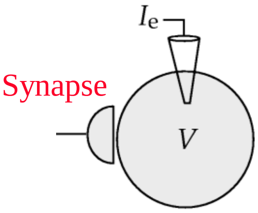
$$\boxed{\tau_m \frac{dV}{dt} = -(V-E_L) + I_e R_m}$$

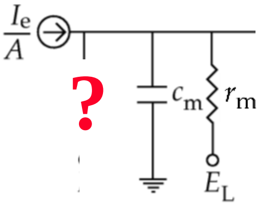
$\tau_m = r_m c_m = R_m C_m$ is the membrane time constant

Image Source: Dayan & Abbott textbook

How do we model the effects of a synapse on the membrane potential V ?



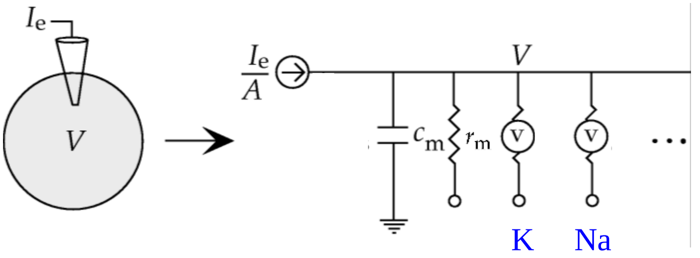




R. Rao, 528: Lecture 8

Hint!

Hodgkin-Huxley Model



$$\tau_m \frac{dV}{dt} = -i_m r_m + I_e R_m$$

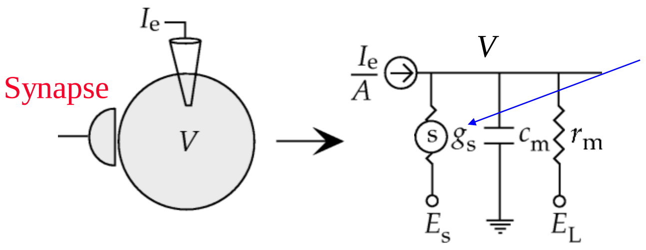
$$i_m = (1/r_m)(V - E_L) + g_{K,\max} n^4 (V - E_K) + g_{Na,\max} m^3 h (V - E_{Na})$$

E_L = -54 mV, E_K = -77 mV, E_{Na} = +50 mV

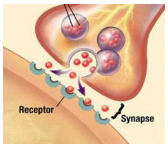
R. Rao, 528: Lecture 8

Image Source: Dayan & Abbott textbook

Modeling Synaptic Inputs



Synaptic conductance



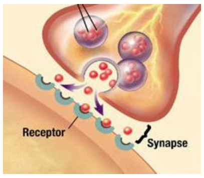
$$\tau_m \frac{dV}{dt} = -\big((V-E_L) + r_m g_s (V-E_s)\big) + I_e R_m$$

$$g_s = g_{s,\max} P_{rel} P_s$$

← Probability of postsynaptic channel opening (= fraction of channels opened)

Probability of transmitter release given an input spike

Basic Synapse Model



- Assume $P_{rel} = 1$
- Model the effect of a single spike input on P_s
- Kinetic Model of postsynaptic channels:

fraction of channels opened

$$\textbf{Closed} \underset{\beta_s}{\overset{\alpha_s}{\rightleftarrows}} \textbf{Open}$$

$$\frac{dP_s}{dt} = \alpha_s (1 - P_s) - \beta_s P_s$$

Opening rate

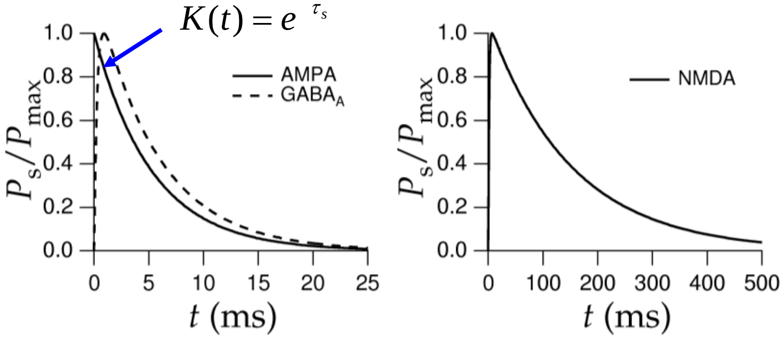
Closing rate

Fraction of channels closed

Fraction of channels open

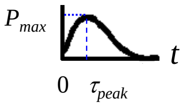
What does P_s look like over time given a spike?

$$K(t) = e^{-\frac{t}{\tau_s}}$$



Exponential function $K(t)$ gives reasonable fit for some synapses
Others can be fit using “Alpha” function:

$$K(t) = t \cdot e^{-\frac{t}{\tau_{peak}}}$$



Linear Filter Model of a Synapse

Synapse b

Input Spike Train ρ_b(t)

$\rho_b(t) = \Sigma_i \delta(t-t_i)$ (t_i are the input spike times, δ = delta function)

Filter for synapse b = $K(t)$

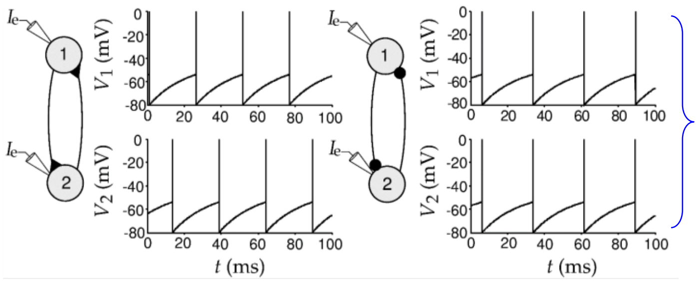
Synaptic conductance at b:

$$g_b(t) = g_{b,\max} \sum_{t_i<t} K(t - t_i)$$

$$= g_{b,\max} \int_{-\infty}^{t} K(t-\tau)\rho_b(\tau)d\tau$$

Example: Network of Integrate-and-Fire Neurons

Excitatory synapses (E_b = 0 mV) Inhibitory synapses (E_b = -80 mV)



Synchrony!

Each neuron: $$\tau_m \frac{dV}{dt} = -(V - E_L) - r_m g_b(t)(V - E_b) + I_e R_m$$

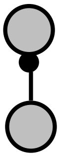
Synapses : Alpha function model $E_L = -70\,\text{mV}$ $V_{thresh} = -54\,\text{mV}$

$\tau_{peak} = 10\,\text{ms}$

Modeling Networks of Neurons

- Option 1: Use *spiking* neurons
  - *Advantages*: Model computation and learning based on:
    - Spike Timing
    - Spike Correlations/Synchrony between neurons
  - *Disadvantages*: Computationally expensive
- Option 2: Use neurons with *firing-rate outputs (real valued outputs)*
  - *Advantages*: Greater efficiency, scales well to large networks
  - *Disadvantages*: Ignores spike timing issues
- Question: How are these two approaches related?

Recall: Linear Filter Model of a Synapse



Synapse b

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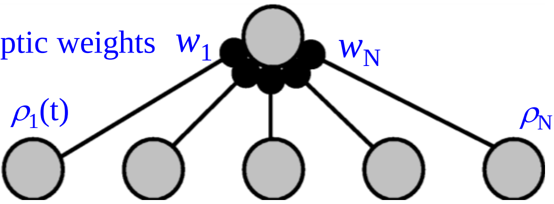
$$= g_{b,\max} \int_{-\infty}^{t} K(t-\tau)\rho_b(\tau)\,d\tau$$

From a Single Synapse to Multiple Synapses



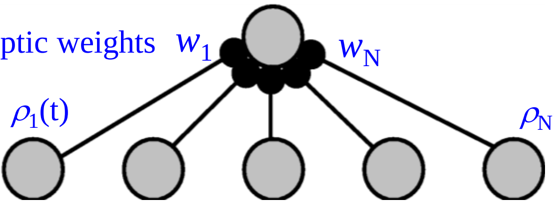
Synaptic weights w_1 w_N

Spike trains $\rho_1(t)$ $\rho_N(t)$

Total synaptic current $I_s(t) = \sum_{b=1}^{N} I_b(t)$

$$I_s(t) = \sum_{b=1}^{N} w_b \int_{-\infty}^{t} K(t-\tau)\rho_b(\tau)\,d\tau$$

From Spiking to Firing Rate Model



Synaptic weights w_1 w_N

Spike trains $\rho_1(t)$ $\rho_N(t)$

Firing rate $u_1(t)$ $u_N(t)$

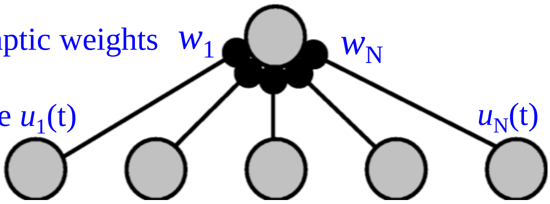
Total synaptic current

$$I_s(t) = \sum_{b=1}^{N} w_b \int_{-\infty}^{t} K(t-\tau)\rho_b(\tau)d\tau \quad \textbf{Spike train } \boldsymbol{\rho_b}\textbf{(t)}$$

$$\Big\downarrow$$

$$\approx \sum_{b=1}^{N} w_b \int_{-\infty}^{t} K(t-\tau)u_b(\tau)d\tau \quad \textbf{Firing rate } \boldsymbol{u_b}\textbf{(t)}$$

Simplifying the Input Current Equation



Synaptic weights w_1 w_N

Weight vector **w**

Firing rate $u_1(t)$ $u_N(t)$

Input vector **u**

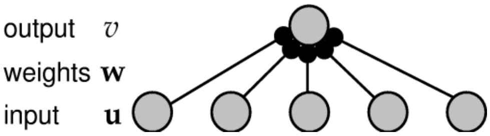
Suppose synaptic filter *K* is exponential: $K(t) = \frac{1}{\tau_s} e^{-\frac{t}{\tau_s}}$

Differentiating $I_s(t) = \sum_b w_b \int_{-\infty}^{t} K(t-\tau)u_b(\tau)d\tau$ w.r.t. time *t*,

we get $\tau_s \frac{dI_s}{dt} = -I_s + \sum_b w_b u_b$

$$= -I_s + \mathbf{w} \cdot \mathbf{u}$$

General Firing-Rate-Based Network Model



F is the “activation function”

Output firing rate changes like this: $$\tau_r \frac{dv}{dt} = -v + F(I_s(t))$$

Input current changes like this: $$\tau_s \frac{dI_s}{dt} = -I_s + \mathbf{w} \cdot \mathbf{u}$$

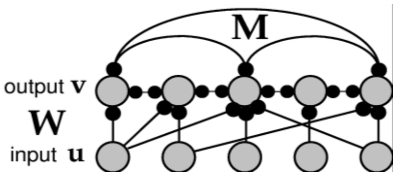
What happens when:

$\tau_s << \tau_r$?

$\tau_r << \tau_s$?

Static input?

Next Class: Networks



- To Do:
  - Homework 3
  - Finalize a final project topic and partner(s)
    - Email Raj, Adrienne and Rich your topic and partners, or ask to be assigned to a team