

Boosting (almost) by hand

Magic:  $\alpha_t = \frac{1}{2} \ln \left( \frac{1 - \epsilon_t}{\epsilon_t} \right)$

- Initialize weights to uniform dist:  $D_1(j) = 1/N$

- For  $t = 1 \dots T$

- Train weak learner  $h_t$  on distribution  $D_t$  over the data
- Choose weight  $\alpha_t$

- Update weights:

for each  $j$

$$D_{t+1}(j) = \frac{D_t(j) \exp(-\alpha_t y^j h_t(x^j))}{Z_t}$$

- Where  $Z_t$  is normalizer:

so weights  
add up to 1

$$Z_t = \sum_{j=1}^N D_t(j) \exp(-\alpha_t y^j h_t(x^j))$$

- Output final classifier:

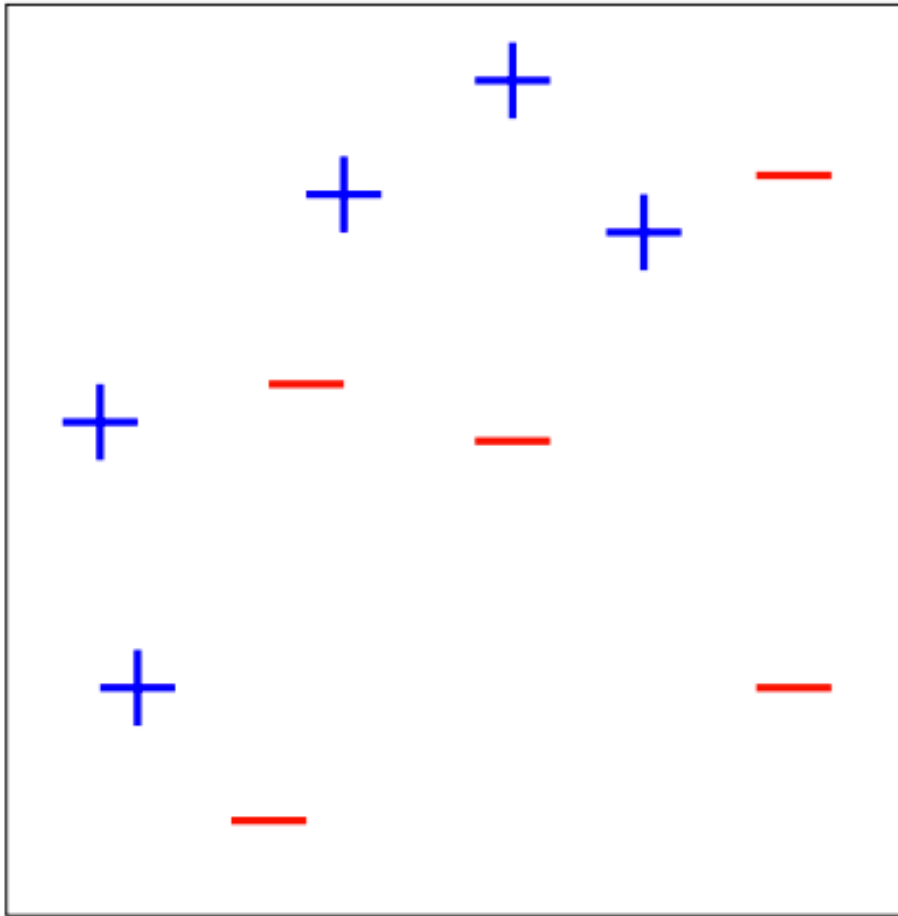
$$H(x) = \text{Sign} \left( \sum_{t=1}^T \alpha_t h_t(x) \right)$$

← Focused on parts that have high weights

← Magic of taking derivative & setting to 0  
new weight ← old weight

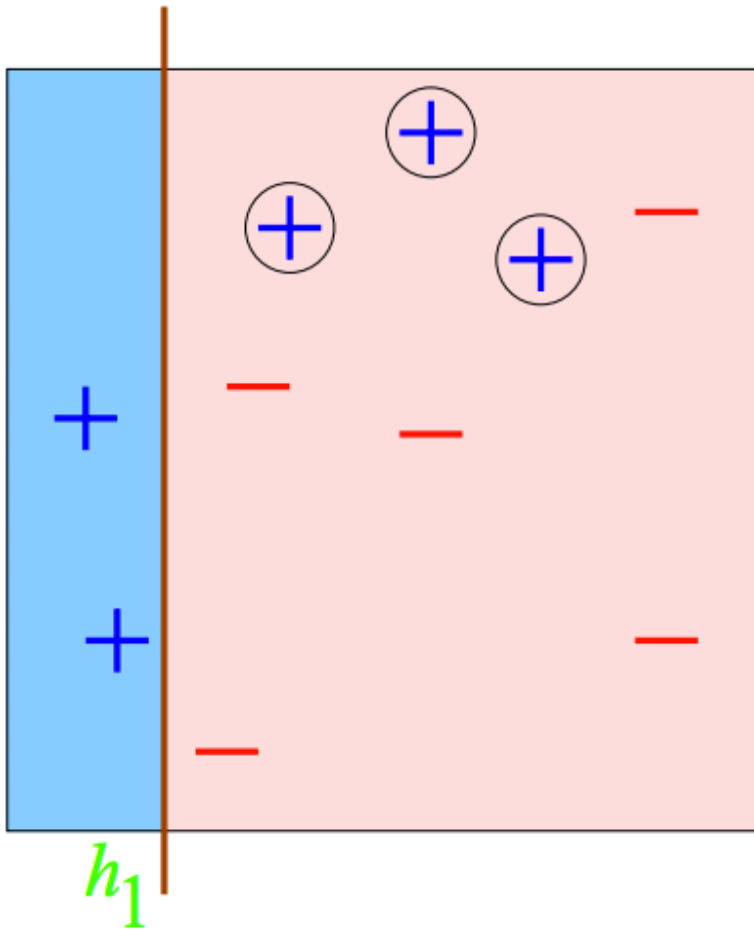
suppose  $\alpha_t > 0$   
if  $h_t$  correct on  $j$   
 $y^j h_t(x^j) > 0$   
 $\Rightarrow$  weight decrease  
if  $h_t$  incorrect on  $j$   
weight increases

(from Rob Schapire)



$N = 10$

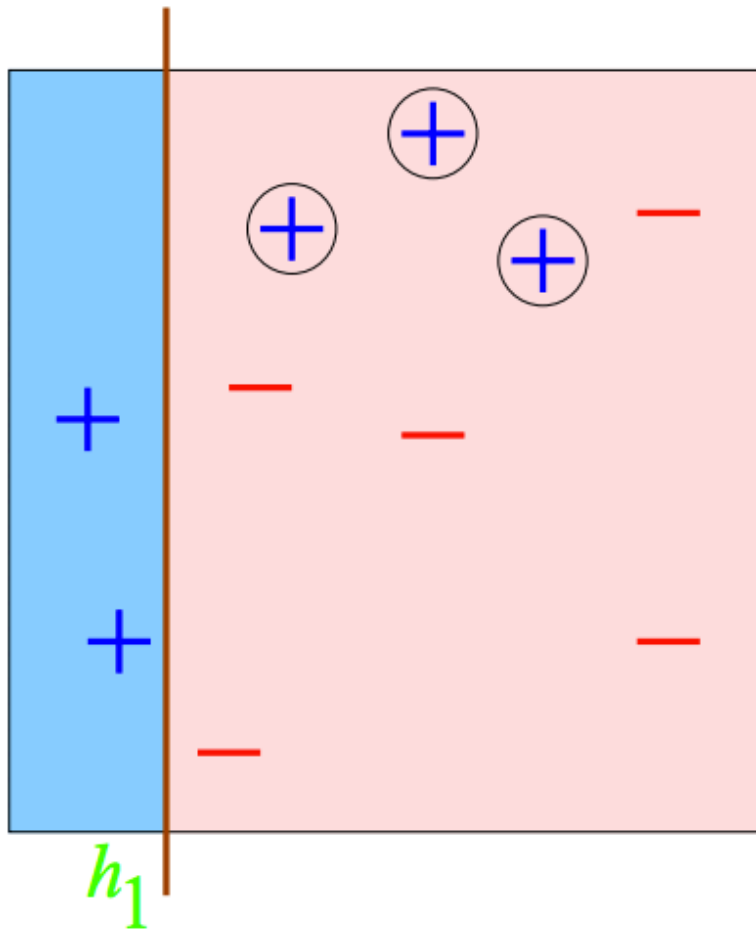
# Whats error train here?



$$D_1(j) = \frac{1}{N} = 0.1$$

$$\epsilon_t = \sum_{j=1}^N D_t(j) 1[\text{sign}(h_t(x^j)) \neq y^j]$$

# Whats error train here?



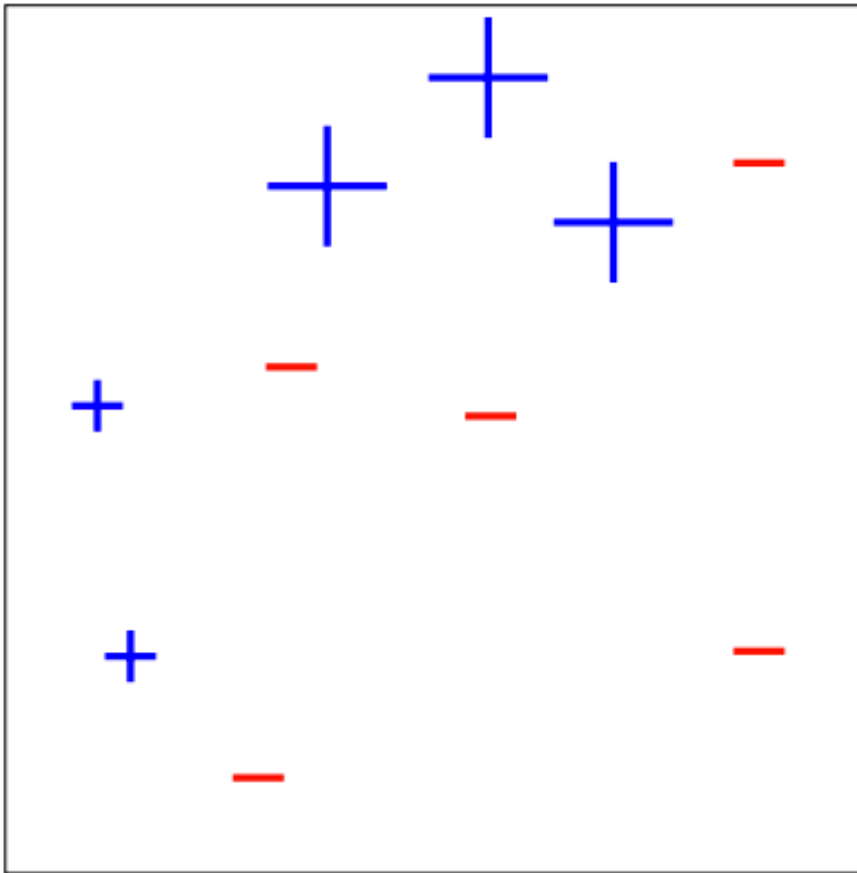
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$$\epsilon_1 = 0.3$$

$$\alpha_1 = \frac{1}{2} \ln\left(\frac{0.7}{0.3}\right) \approx 0.42$$

# New weights

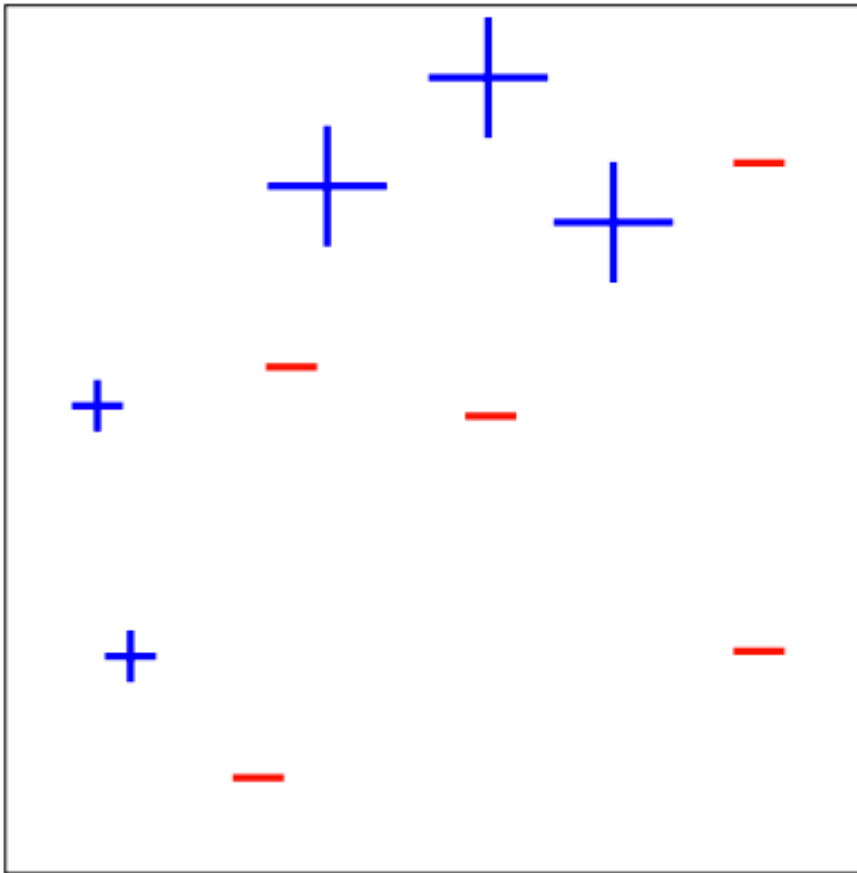


$$D_{t+1}(j) = \frac{D_t(j) \exp(-\alpha_t y^j h_t(x^j))}{Z_t}$$

When is  $y^j h_t(x^j) = 1$ ?

When is  $y^j h_t(x^j) = -1$ ?

# New weights

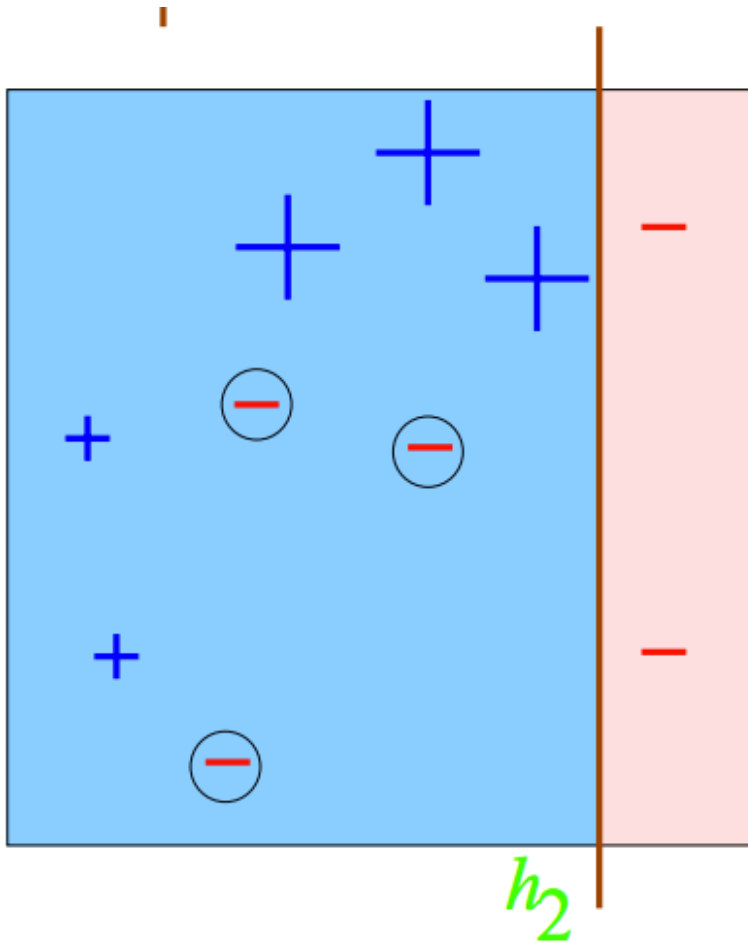


$$D_{t+1}(j) = \frac{D_t(j) \exp(-\alpha_t y^j h_t(x^j))}{Z_t}$$

$$D_2(\text{right}) = \frac{0.1 \exp(-0.42)}{Z_t} \approx 0.071$$

$$D_2(\text{wrong}) = \frac{0.1 \exp(0.42)}{Z_t} \approx 0.166$$

## Step 2



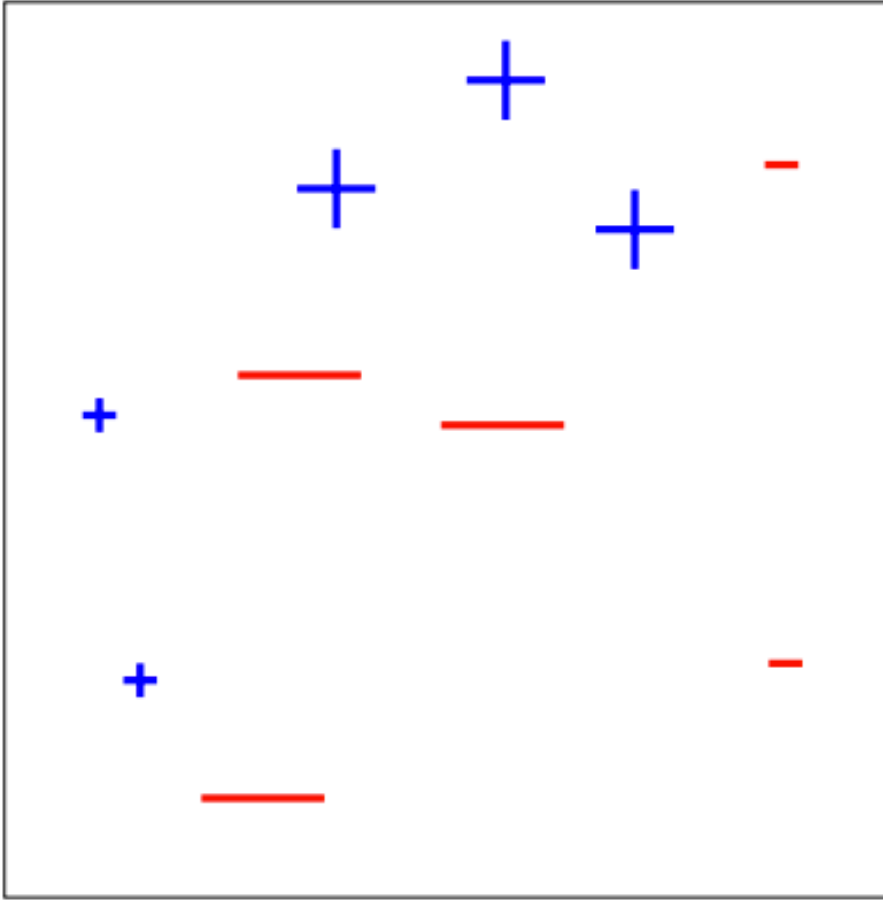
$$\epsilon_2 = 0.071 * 3 \approx 0.21$$

$$\alpha_2 = \frac{1}{2} \ln\left(\frac{0.79}{0.21}\right) \approx 0.65$$

Notice I still get 3 examples wrong, but they are worth less now.



# New weights



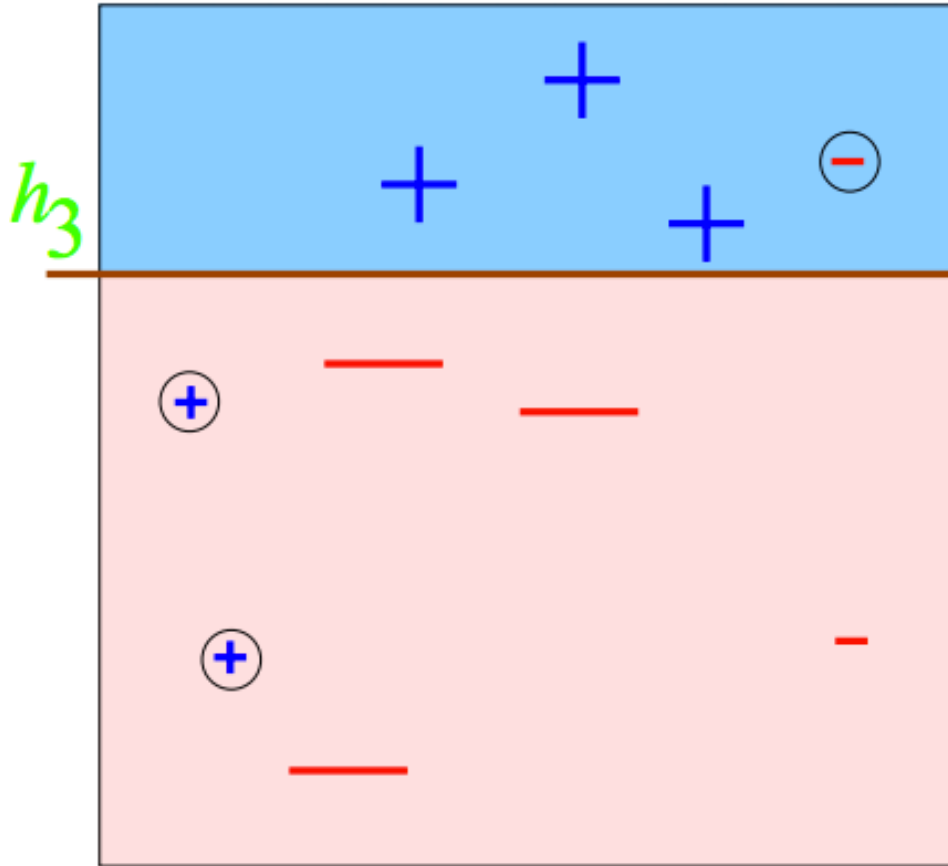
$$D_{t+1}(j) = \frac{D_t(j) \exp(-\alpha_t y^j h_t(x^j))}{Z_t}$$

$$D_3(\text{small} + \text{and} -) = \frac{0.071 \exp(-0.65)}{Z_t} \approx 0.045$$

$$D_3(\text{medium} +) = \frac{0.166 \exp(-0.65)}{Z_t} \approx 0.1$$

$$D_3(\text{large} -) = \frac{0.166 \exp(-0.65)}{Z_t} \approx 0.17$$

# Step 3



$$\epsilon_3 = 0.045 * 3 \approx 0.14$$

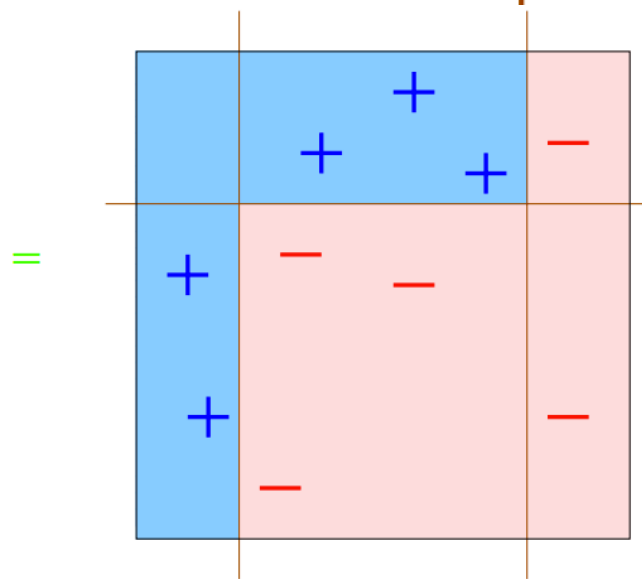
$$\alpha_3 = \frac{1}{2} \ln\left(\frac{0.86}{0.14}\right) \approx 0.92$$

Notice I still get 3 examples wrong.

- Output final classifier:

$$H(x) = \text{Sign} \left( \sum_{t=1}^T \alpha_t h_t(x) \right)$$

$$= \text{sign} \left( 0.42 \left[ \begin{array}{|c|} \hline \text{blue} \\ \hline \text{red} \end{array} \right] + 0.65 \left[ \begin{array}{|c|} \hline \text{blue} \\ \hline \text{red} \end{array} \right] + 0.92 \left[ \begin{array}{|c|} \hline \text{blue} \\ \hline \text{red} \end{array} \right] \right)$$



# Evaluation Metrics

- 0-1 error on test set:  $\sum_{j=1}^N 1[\text{sign}(h_t(x^j)) \neq y^j]$
- $1 - (0\text{-}1 \text{ error})/N = \text{accuracy}$
- Accuracy is just % of test samples I get right.

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- His classifier just always predicts 'not millionaire'.



# Confusion matrix

		Gold standard	
		X	Y
Your Result	X	true positive tp	false positive fp type I error
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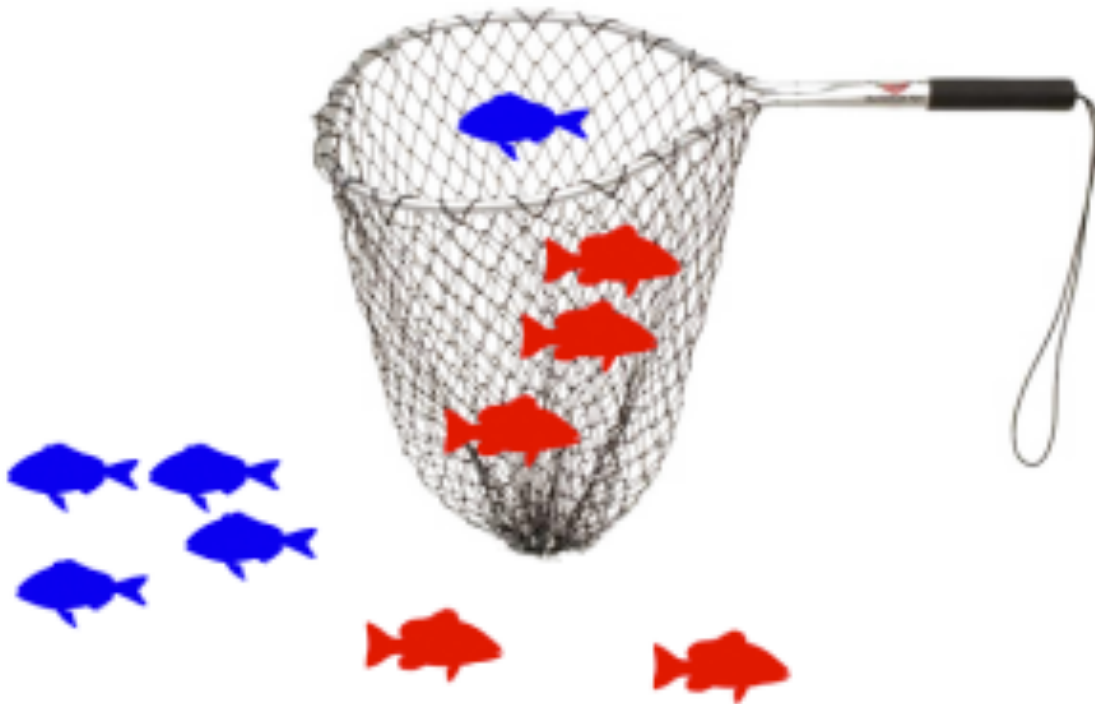
$$accuracy = \frac{tp + tn}{tp + tn + fp + fn}$$

$$precision = \frac{tp}{tp + fp}$$

$$error = \frac{fp + fn}{tp + tn + fp + fn}$$

$$recall = \frac{tp}{tp + fn}$$

# Examples:

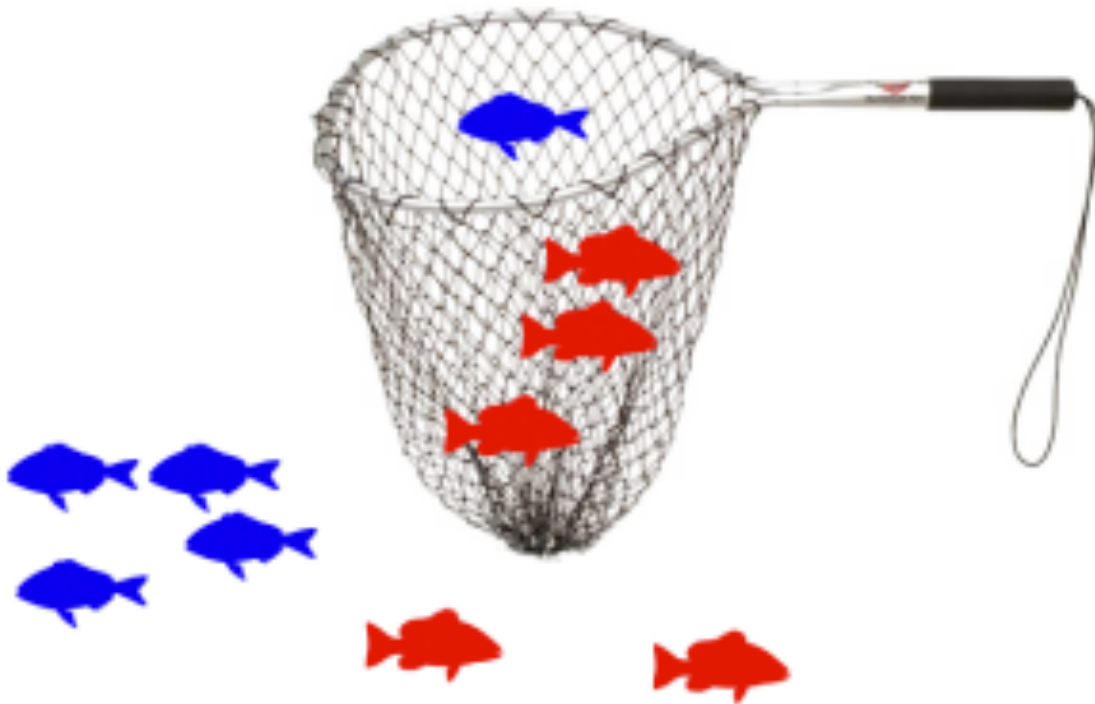


Let's say the net is trying to pick only blue fish. What's the precision and the recall?

I got this figure from

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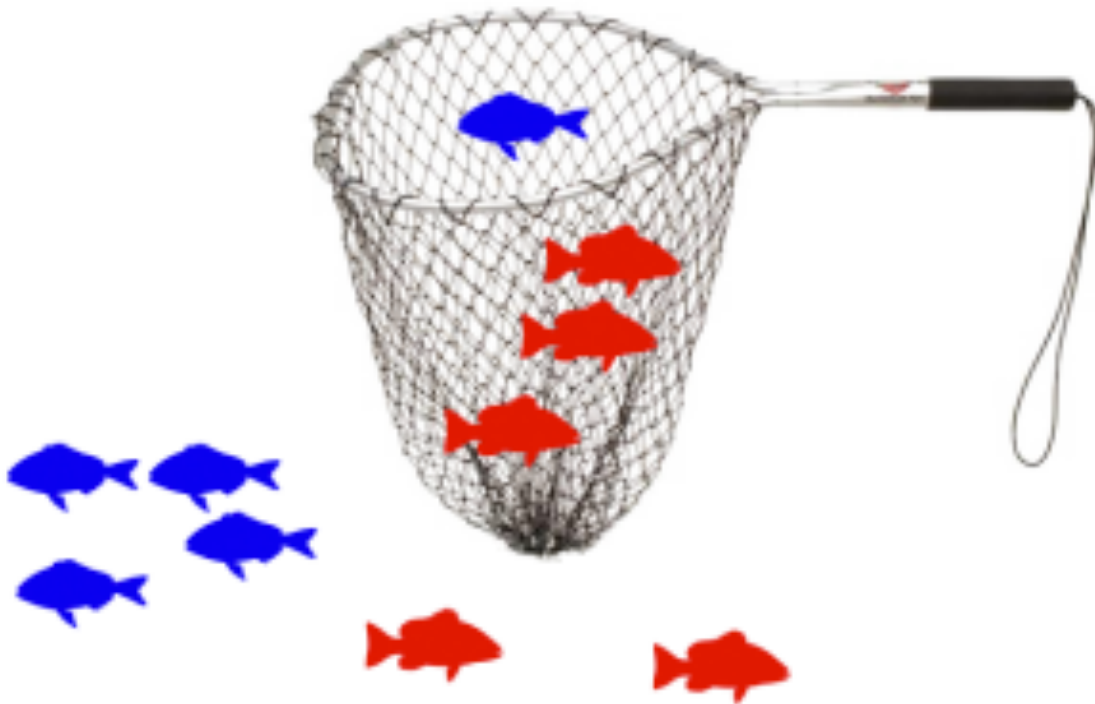
$$P = 1/4$$

$$R = 1/5$$

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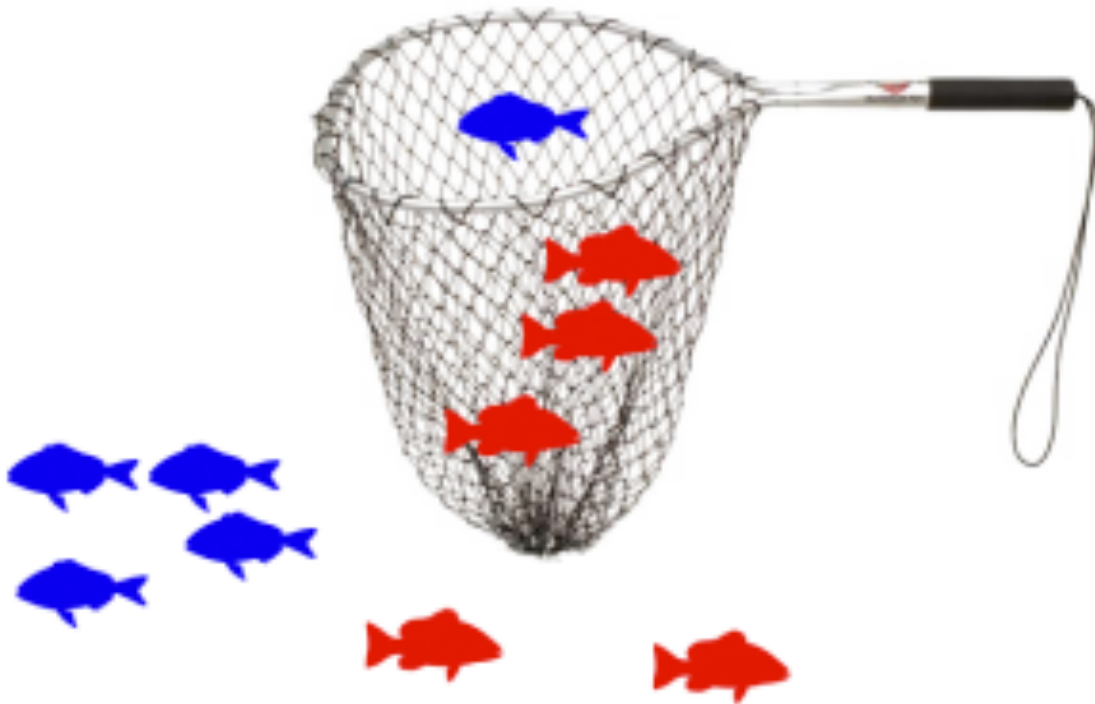
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# Examples:



Let's say the net is trying to pick only red fish. What's the precision and the recall?

$$P = 3/4$$

$$R = 3/5$$

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# A single metric?

- $F1 = \frac{2 * \text{precision} * \text{recall}}{\text{precision} + \text{recall}}$
- Fbeta
- AUC
- ...

# Which one is better?

- Our millionaire identification scenario?





# Which one is better?

- Spam classification



# Which one is better?

- Medical classifier:  $Y = (\text{operate}, \text{don't operate})$



# Which one is better?

- Search engine: query = legal



"Your Honor, I'd like to cite the precedent of 'Alien vs. Predator' ...."

# Which one is better?

- Search engine: query = “Husky football”
- By the way: why does google show more than 1 page?



# Which one is better?

- It depends on the task
- Is there imbalance?
- Are the misclassification costs the same?
- ...
- ...
- Think about evaluation when doing your projects!