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Support Vector Machine

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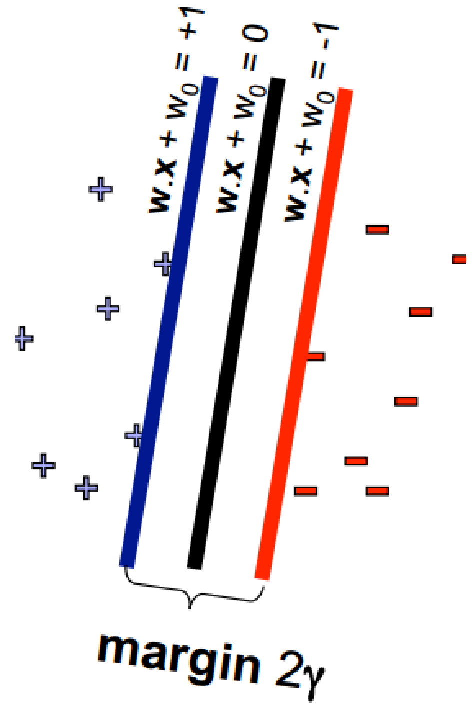
The Linear SVM Objective

- Maximizing the margin

$$argmax_{\mathbf{w},w_0}\gamma$$
$$\text{subject to}\quad \frac{1}{\|w\|}y^{(j)}(\mathbf{w}^T\mathbf{x}^{(j)}+w_0)\geq\gamma, j\in\{1,2,\cdots,N\}$$

Distance between x and hyper-plane, why?



- The objective that is usually used in

$$argmin\|w\|^2$$
$$\text{subject to}\quad y^{(j)}(\mathbf{w}^T\mathbf{x}^{(j)}+w_0)\geq 1, j\in\{1,2,\cdots,N\}$$

- This is the objective used when the data is linearly separable

Constraint Violation and Slack Variables

Original SVM

$$
\begin{aligned}
& argmin\|w\|^2 \\
\text{subject to} \quad & y^{(j)}(\mathbf{w}^T\mathbf{x}^{(j)} + w_0) \geq 1, j \in \{1, 2, \cdots, N\}
\end{aligned}
$$

The soft constraint version

$$
\begin{aligned}
& argmin\|w\|^2 + C\sum_{j=1}^{N} \xi^{(j)} \\
\text{subject to} \quad & y^{(j)}(\mathbf{w}^T\mathbf{x}^{(j)} + w_0) \geq 1 - \xi^{(j)},\ \xi^{(j)} \geq 0, j \in \{1, 2, \cdots, N\}
\end{aligned}
$$

Slack variable: how much violation instance j have on the constraint

- This allows the constraint to be violated for some (outlier) j
- We add a linear penalty to the violations of constraint

Soft Constraint and Hinge Loss

- The soft constraint version

$$argmin\|w\|^2 + C\sum_{j=1}^{N}\xi^{(j)}$$
$$\text{subject to} \quad y^{(j)}(\mathbf{w}^T\mathbf{x}^{(j)} + w_0) \geq 1 - \xi^{(j)}, \ \ \xi^{(j)} \geq 0, j \in \{1, 2, \cdots, N\}$$

- This means $\xi^{(j)} \geq 1 - y^{(j)}(\mathbf{w}^T\mathbf{x}^{(j)} + w_0)$ also note $\xi^{(j)} \geq 0$
- The equivalent form

$$argmin\|w\|^2 + C\sum_{j=1}^{N}\max\left(1 - y^{(j)}(\mathbf{w}^T\mathbf{x}^{(j)} + w_0), 0\right)$$

Hinge Loss

Soft Constraint and Hinge Loss(cont')

- Think of following new problem
- Assume we have set of pairs $\{(\mathbf{x}_1, \mathbf{z}_1), (\mathbf{x}_2, \mathbf{z}_2), \cdots (\mathbf{x}_N, \mathbf{z}_N)\}$
  - We know that for each pair, x is better than z
  - How can we learn the rank of the items from these pairs?
  - Objective will look like

$$
\begin{aligned}
& argmin \|w\|^2 + C \textstyle\sum_{j=1}^{N} \xi^{(j)} \\
\text{subject to} \quad & (\mathbf{w}^T \mathbf{x}^{(j)} + w_0) \geq (\mathbf{w}^T \mathbf{z}^{(j)} + w_0) + 1 - \xi^{(j)}, j \in \{1, 2, \cdots, N\}
\end{aligned}
$$

  - What is the corresponding hinge loss form?

SGD for Linear Model

- Think of how can you implement SGD for both logistic regression, linear regression and linear SVM
- General loss function

$$L(\mathbf{w}, w_0) = \frac{\lambda}{N}\|\mathbf{w}\|^2 + \frac{1}{N}\sum_{j=1}^{N} l(\hat{y}^{(j)}, y^{(j)}), \;\; \hat{y}^{(j)} = \mathbf{w}^T\mathbf{x}^{(j)} + w_0$$

- SGD update rule (derived using chain rule)

$$\mathbf{w}_i^{(t+1)} \leftarrow \mathbf{w}_i^{(t)} - \eta\left(2\frac{\lambda}{N}\mathbf{w}_i^{(t)} + \mathbf{x}_i^{(j)}\partial_{\hat{y}^{(j)}} l(\mathbf{w}^T\mathbf{x}^{(j)} + w_0, y^{(j)})\right)$$

  - SVM hinge loss

$$l(\hat{y}, y) = \max(1 - \hat{y}y, 0), \;\; \partial_{\hat{y}} l(\hat{y}, y) = \begin{cases} -y & \hat{y}y < 1 \\ 0 & \hat{y}y \geq 1 \end{cases}$$

  - Ridge regression, square loss

$$l(\hat{y}, y) = \frac{1}{2}(\hat{y} - y)^2, \quad \partial_{\hat{y}} l(\hat{y}, y) = \hat{y} - y$$

SGD for Linear Model (cont')

- Again, think of separation between model and objective function (loss and regularization)

- Think of this question: How can you implement a SGD solver for logistic/linear regression and linear SVM, with L1 or L2 regularization supported.
  - I would encourage you to try, and see how much code you can reuse
  - Same thing applies beyond linear models(e.g. Matrix Factorization, Neural Nets)

One thing you need to know about Kernel

- Many machine learning models accepts kernel as input instead of explicit feature mapping.

$$K(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}) = \phi^T(\mathbf{x}^{(i)})\phi(\mathbf{x}^{(j)})$$

Kernel **Feature mapping**

- When is kernel more helpful than explicit feature mapping?
  - Sometimes it is easier to specify inner product(distance) than explicit feature map
  - String kernels
  - Graph kernels
  - Image matching kernels

Midterm

- The grades has been posted
- When you have time, try to take a look at all the questions, including the one you did not manage to answer
- Try to learn from the questions☺