## Announcements

- My office hours TODAY 3:30 pm - 4:30 pm CSE 666
- Poster Session - Pick one
- First poster session TODAY 4:30 pm - 7:30 pm CSE Atrium
- Second poster session December 12 4:30 pm - 7:30 pm CSE Atrium
- Support your peers and check out the posters!
- Poster description from website:
"We will hold a poster session in the Atrium of the Paul Allen Center. Each team will be given a stand to present a poster summarizing the project motivation, methodology, and results. The poster session will give you a chance to show off the hard work you put into your project, and to learn about the projects of your peers. We will provide poster boards that are $32 \times 40$ inches. Both one large poster or several pinned pages are OK (fonts should be easily readable from 5 feet away)."
- Course Evaluation: https://uw.iasystem.org/survey/200308 (or on MyUW)
- Other anonymous Google form course feedback: https://bit.ly/2rmdYAc
- Homework 3 Problem 5 "revisited".
- Optional. Can only increase your grade, but will not hurt it.


## $\equiv$ Spotify

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# NETFIX 

## $\equiv$ Spotify

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## Basics of Fair ML

You work at a bank that gives loans based on credit score.

You have historical data: $\left\{\left(x_{i}, y_{i}\right)\right\}_{i=1}^{n}$
credit score $x_{i} \in \mathbb{R}$
paid back loan $y_{i} \in\{0,1\}$
is paid bach
If the loan $\left(y_{i}=1\right)$ you receive $\$ 300$ in interest
If the loan defaults $\left(y_{i}=0\right)$ you lose $\$ 700$
For some threshold t

$$
\begin{aligned}
\text { Profit }= & 300 \cdot \mathbb{P}\left(x_{i}>t \mid y_{i}=1\right) \\
& -700 \cdot \mathbb{P}\left(x_{i}>t \mid y_{i}=0\right)
\end{aligned}
$$

## Basics of Fair ML

You work at a bank that gives loans based on credit score. Boss tells you "make sure it doesn't discriminate on race"

You have historical data: $\left\{\left(x_{i}, a_{i}, y_{i}\right)\right\}_{i=1}^{n}$
credit score $x_{i} \in \mathbb{R}$
paid back loan $y_{i} \in\{0,1\}$
$\underset{\text { race }}{ } a_{i} \in\{$ asian, white, hispanic, black $\}$
If the $\left(y_{i}=1\right)$ you receive $\$ 300$ in interest If the loan defaults $\left(y_{i}=0\right)$ you lose $\$ 700$

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race $a_{i} \in\{$ asian, white, hispanic, black $\}$

- Fairness through unawareness. Ignore $a_{i}$, everyone gets same threshold
- Pro: simple,
- Con: features are often

$$
\mathbb{P}\left(x_{i}>t \mid a_{i}=\square\right)=\mathbb{P}\left(x_{i}>t\right)
$$ proxy for protected group

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race $a_{i} \in\{$ asian, white, hispanic, black $\}$

- Demographic parity. proportion of loans to each group is the same
- Pro: sounds fair,
- Con: groups more likely

$$
\mathbb{P}\left(x_{i}>t_{\square} \mid a_{i}=\square\right)=\mathbb{P}\left(x_{i}>t_{\diamond} \mid a_{i}=\diamond\right)
$$ to pay back loans penalized

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paid back loan $y_{i} \in\{0,1\}$

$$
P(x>t \mid y=1)=\frac{\mathbb{P}(x>t, y=1)}{\mathbb{P}(y=1)}
$$

race $a_{i} \in\{$ asian, white, hispanic, black $\}$

- Equal opportunity. proportion of those who would pay back loans equal
- Pro: Bayes optimal if conditional distributions are the same, TPR=equal
- Con: needs one class to be "good", another "bad"

$$
\mathbb{P}\left(x_{i}>t_{\square} \mid y_{i}=1, a_{i}=\square\right)=\mathbb{P}\left(x_{i}>t_{\diamond} \mid y_{i}=1, a_{i}=\diamond\right)
$$

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race $a_{i} \in\{$ asian, white, hispanic, black $\}$
Per-group ROC curve



## Fairness, Accountability, and Transparency in Machine Learning www.fatml.org

## Trees

Machine Learning - CSE546
Kevin Jamieson
University of Washington
December 4, 2018

## Trees

$$
f(x)=\sum_{m=1}^{M} c_{m} I\left(x \in R_{m}\right)
$$

Build a binary tree, splitting along axes


## Learning decision trees

- Start from empty decision tree
- Split on next best attribute (feature)
$\square$ Use, for example, information gain to select attribute
$\square$ Split on $\arg \max _{i} I G\left(X_{i}\right)=\arg \max _{i} H(Y)-H\left(Y \mid X_{i}\right)$
- Recurse
- Prune


$$
f(x)=\sum_{m=1}^{M} c_{m} I\left(x \in R_{m}\right)
$$

## Trees

- Trees
- have low bias, high variance
- deal with categorial variables well
- intuitive, interpretable
- good software exists
- Some theoretical guarantees


# Random Forests 

Machine Learning - CSE546
Kevin Jamieson
University of Washington
December 4, 2018

## Random Forests

Tree methods have low bias but high variance.

One way to reduce variance is to construct a lot of "lightly correlated" trees and average them:
"Bagging:" Bootstrap aggregating


## Random Forrests

## Algorithm 15.1 Random Forest for Regression or Classification.

1. For $b=1$ to $B$ :
(a) Draw a bootstrap sample $\mathbf{Z}^{*}$ of size $N$ from the training data.
(b) Grow a random-forest tree $T_{b}$ to the bootstrapped data, by recursively repeating the following steps for each terminal node of the tree, until the minimum node size $n_{\min }$ is reached.
i. Select $m$ variables at random from the $p$ variables.
ii. Pick the best variable/split-point among the $m$.
iii. Split the node into two daughter nodes.
2. Output the ensemble of trees $\left\{T_{b}\right\}_{1}^{B}$.

To make a prediction point $x$ :
Regression: $\hat{f}_{\mathrm{rf}}^{B}(x)=\frac{1}{B} \sum_{b=1}^{B} T_{b}(x)$.
Classification: Let $\hat{C}_{b}(x)$ be the class prediction of the bth random-forest tree. Then $\hat{C}_{\mathrm{rf}}^{B}(x)=$ majority vote $\left\{\hat{C}_{b}(x)\right\}_{1}^{B}$ $\square$

## Random Forests

- Random Forests
- have low bias, low variance
- deal with categorial variables well
- not that intuitive or interpretable
- Notion of confidence estimates
- good software exists
- Some theoretical guarantees
- works well with default hyperparameters


## Boosting

## Machine Learning - CSE546 <br> Kevin Jamieson <br> University of Washington

December 4, 2018

## Boosting

- 1988 Kearns and Valiant: "Can weak learners be combined to create a strong learner?"

Weak learner definition (informal):
An algorithm $\mathcal{A}$ is a weak learner for a hypothesis class $\mathcal{H}$ that maps $\mathcal{X}$ to $\{-1,1\}$ if for all input distributions over $\mathcal{X}$ and $h \in \mathcal{H}$, we have that $\mathcal{A}$ correctly classifies $h$ with error at most $1 / 2-\gamma$

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- 1990 Robert Schapire: "Yup!"
- 1995 Schapire and Freund: "Practical for 0/1 loss" AdaBoost


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- 2001 Friedman: "Practical for arbitrary losses"
- 2014 Tianqi Chen: "Scale it up!" XGBoost


## Boosting and Additive Models

Machine Learning - CSE546
Kevin Jamieson
University of Washington
December 4, 2018

## Additive models

- Consider the first algorithm we used to get good classification for MNIST. Given: $\left\{\left(x_{i}, y_{i}\right)\right\}_{i=1}^{n} \quad x_{i} \in \mathbb{R}^{d}, y_{i} \in\{-1,1\}$
- Generate random functions: $\phi_{t}: \mathbb{R}^{d} \rightarrow \mathbb{R} \quad t=1, \ldots, p$
- Learn some weights: $\widehat{w}=\arg \min _{w} \sum_{i=1}^{n} \operatorname{Loss}\left(y_{i}, \sum_{t=1}^{p} w_{t} \phi_{t}\left(x_{i}\right)\right)$
- Classify new data: $f(x)=\operatorname{sign}\left(\sum_{t=1}^{p} \widehat{w}_{t} \phi_{t}(x)\right)$


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An interpretation:
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$$
\widehat{w}, \widehat{\phi}_{1}, \ldots, \widehat{\phi}_{t}=\arg \min _{w, \phi_{1}, \ldots, \phi_{p}} \sum_{i=1}^{n} \operatorname{Loss}\left(y_{i}, \sum_{t=1}^{p} w_{t} \phi_{t}\left(x_{i}\right)\right)
$$

is in general computationally hard

## Forward Stagewise Additive models

$b(x, \gamma)$ is a function with parameters $\gamma \quad$ Examples: $b(x, \gamma)=\frac{1}{1+e^{-\gamma^{T} x}}$

```
Algorithm 10.2 Forward Stagewise Additive Modeling.
    \(b(x, \gamma)=\gamma_{1} \mathbf{1}\left\{x_{3} \leq \gamma_{2}\right\}\)
1. Initialize \(f_{0}(x)=0\).
2. For \(m=1\) to \(M\) :
```

(a) Compute

$$
\left(\beta_{m}, \gamma_{m}\right)=\arg \min _{\beta, \gamma} \sum_{i=1}^{N} L\left(y_{i}, f_{m-1}\left(x_{i}\right)+\beta b\left(x_{i} ; \gamma\right)\right) .
$$

(b) Set $f_{m}(x)=f_{m-1}(x)+\beta_{m} b\left(x ; \gamma_{m}\right)$.

Idea: greedily add one function at a time

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(b) Set $f_{m}(x)=f_{m-1}(x)+\beta_{m} b\left(x ; \gamma_{m}\right)$.

## Idea: greedily add one function at a time

AdaBoost: $b(x, \gamma)$ : classifiers to $\{-1,1\}$

$$
L(y, f(x))=\exp (-y f(x))
$$

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Idea: greedily add one function at a time
Boosted Regression Trees: $\quad L(y, f(x))=(y-f(x))^{2}$
$b(x, \gamma)$ : regression trees

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## Idea: greedily add one function at a time

Boosted Regression Trees: $\quad L(y, f(x))=(y-f(x))^{2}$

$$
\begin{aligned}
L\left(y_{i}, f_{m-1}\left(x_{i}\right)+\beta b\left(x_{i} ; \gamma\right)\right) & =\left(y_{i}-f_{m-1}\left(x_{i}\right)-\beta b\left(x_{i} ; \gamma\right)\right)^{2} \\
& =\left(r_{i m}-\beta b\left(x_{i} ; \gamma\right)\right)^{2}, \quad r_{i m}=y_{i}-f_{m-1}\left(x_{i}\right)
\end{aligned}
$$

## Forward Stagewise Additive models

$b(x, \gamma)$ is a function with parameters $\gamma \quad$ Examples: $b(x, \gamma)=\frac{1}{1+e^{-\gamma^{T} x}}$

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\]
```

$b(x, \gamma)=\gamma_{1} \mathbf{1}\left\{x_{3} \leq \gamma_{2}\right\}$
(b) Set $f_{m}(x)=f_{m-1}(x)+\beta_{m} b\left(x ; \gamma_{m}\right)$.

> Idea: greedily add one function at a time

Boosted Logistic Trees: $\quad L(y, f(x))=y \log (f(x))+(1-y) \log (1-f(x))$

$$
b(x, \gamma): \text { regression trees }
$$

Computationally hard to update

## Gradient Boosting

## Least squares, exponential loss easy. But what about cross entropy? Huber?

Algorithm 10.3 Gradient Tree Boosting Algorithm.

1. Initialize $f_{0}(x)=\arg \min _{\gamma} \sum_{i=1}^{N} L\left(y_{i}, \gamma\right)$.
2. For $m=1$ to $M$ :
(a) For $i=1,2, \ldots, N$ compute

$$
r_{i m}=-\left[\frac{\partial L\left(y_{i}, f\left(x_{i}\right)\right)}{\partial f\left(x_{i}\right)}\right]_{f=f_{m-1}}
$$

(b) Fit a regression tree to the targets $r_{i m}$ giving terminal regions $R_{j m}, j=1,2, \ldots, J_{m}$.
(c) For $j=1,2, \ldots, J_{m}$ compute

$$
\gamma_{j m}=\arg \min _{\gamma} \sum_{x_{i} \in R_{j m}} L\left(y_{i}, f_{m-1}\left(x_{i}\right)+\gamma\right)
$$

(d) Update $f_{m}(x)=f_{m-1}(x)+\sum_{j=1}^{J_{m}} \gamma_{j m} I\left(x \in R_{j m}\right)$.
3. Output $\hat{f}(x)=f_{M}(x)$.

LS fit regression tree to n-dimensional gradient, take a step in that direction

## Gradient Boosting

Least squares, 0/1 loss easy. But what about cross entropy? Huber?


AdaBoost uses 0/1 loss, all other trees are minimizing binomial deviance

## Additive models

- Boosting is popular at parties: Invented by theorists, heavily adopted by practitioners.


## Additive models

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- Computationally efficient with "weak" learners. But can also use trees! Boosting can scale.
- Kind of like sparsity?


## Additive models

- Boosting is popular at parties: Invented by theorists, heavily adopted by practitioners.
- Computationally efficient with "weak" learners. But can also use trees! Boosting can scale.
- Kind of like sparsity?
- Gradient boosting generalization with good software packages (e.g., XGBoost). Effective on Kaggle
- Robust to overfitting and can be dealt with with "shrinkage" and "sampling"


## Bagging versus Boosting

- Bagging averages many low-bias, lightly dependent classifiers to reduce the variance
- Boosting learns linear combination of high-bias, highly dependent classifiers to reduce error
- Empirically, boosting appears to outperform bagging


## Which algorithm do I use?

TABLE 10.1. Some characteristics of different learning methods. Key: $\boldsymbol{\Delta}=$ good, $\stackrel{\text { fair, and }}{\boldsymbol{\nabla}}=$ poor.

| Characteristic | Neural Nets | SVM | Trees | MARS | $\mathrm{k}-\mathrm{NN},$ <br> Kernels |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Natural handling of data of "mixed" type | $\nabla$ | $\nabla$ | A | A | $\nabla$ |
| Handling of missing values | $\nabla$ | $\nabla$ | A | A | A |
| Robustness to outliers in input space | $\nabla$ | $\nabla$ | A | $\nabla$ | A |
| Insensitive to monotone transformations of inputs | $\nabla$ | $\nabla$ | A | $\nabla$ | $\nabla$ |
| Computational scalability (large $N$ ) | $\nabla$ | $\nabla$ | A | A | $\nabla$ |
| Ability to deal with irrelevant inputs | $\nabla$ | $\nabla$ | A | A | $\nabla$ |
| Ability to extract linear combinations of features | A | A | $\nabla$ | $\nabla$ | $\checkmark$ |
| Interpretability | $\nabla$ | $\nabla$ | $\stackrel{\rightharpoonup}{*}$ | $\Delta$ | $\nabla$ |
| Predictive power | A | A | $\nabla$ | $\checkmark$ | A |

$X$-sample space $\mid P_{x y}$ distribution on $x \times y$
$y=\{0,13$
Given $f: X \rightarrow\{0,1\}$, sample $\left.\left\{x_{i}, y\right\}\right\}_{i=1}^{n}$
De five
Empirical Loss: $\hat{R}_{n}(f)=\frac{1}{n} \sum_{i=1}^{n} \mathbb{1}\left\{f\left(x_{i}\right)_{\left.i y_{j}\right\}}\right.$
The Loss: $R(f)=\underset{\rho_{x y}}{\mathbb{F}}[f(x) \neq y]$
Empirical Risk Minimization

$$
\hat{f}=\min _{f \in H} \hat{R}_{\hat{R}_{\text {logistic }}(\text { liner, tres, }} \hat{R}_{n}(f)
$$

$\rightarrow$ How well does $\hat{f}$ generalize?

PAC Learning
all functions $x \rightarrow 20,12$
A hypothesis class $H \leq y^{X}$ is
PAC-learnable if there exists an function $m_{H_{H}}:[0,1]^{2} \rightarrow \mathbb{N}$, and an algorithm $A$, such that: For every $\varepsilon, \delta \in(0,1)$ and every $P_{\underline{x x}}$, when Gunning $A$ on $m \geq m_{H}(\varepsilon, \delta)$ i.i.d examples from $P_{x y}$ the algorithm returns $h \in H$ sit. w/ probubility $>1-\delta$
$\sim R(h) \xrightarrow{R(h) \leq \min } \underset{h^{\prime} \in H}{ } R\left(h^{\prime}\right)+\varepsilon$ $h^{\prime} \in \mathcal{H}$ \& generalization

