Paxos

The Part-Time Parliament

- Parliament determines laws by passing sequence of numbered decrees
- Legislators can leave and enter the chamber at arbitrary times
- No centralized record of approved decrees- instead, each legislator carries a ledger

Government 101

- No two ledgers contain contradictory information
- If a majority of legislators were in the Chamber and no one entered or left the Chamber for a sufficiently long time, then
  - any decree proposed by a legislator would eventually be passed
  - any passed decree would appear on the ledger of every legislator

Government 102

- Paxos legislature is non-partisan, progressive, and well-intentioned
- Legislators only care that something is agreed to, not what is agreed to
- We'll take care of Byzantine legislators later

Back to the future

- A set of processes that can propose values
- Processes can crash and recover
- Processes have access to stable storage
- Asynchronous communication via messages
- Messages can be lost and duplicated, but not corrupted

The Game: Consensus

SAFETY

- Only a value that has been proposed can be chosen
- Only a single value is chosen
- A process never learns that a value has been chosen unless it has been

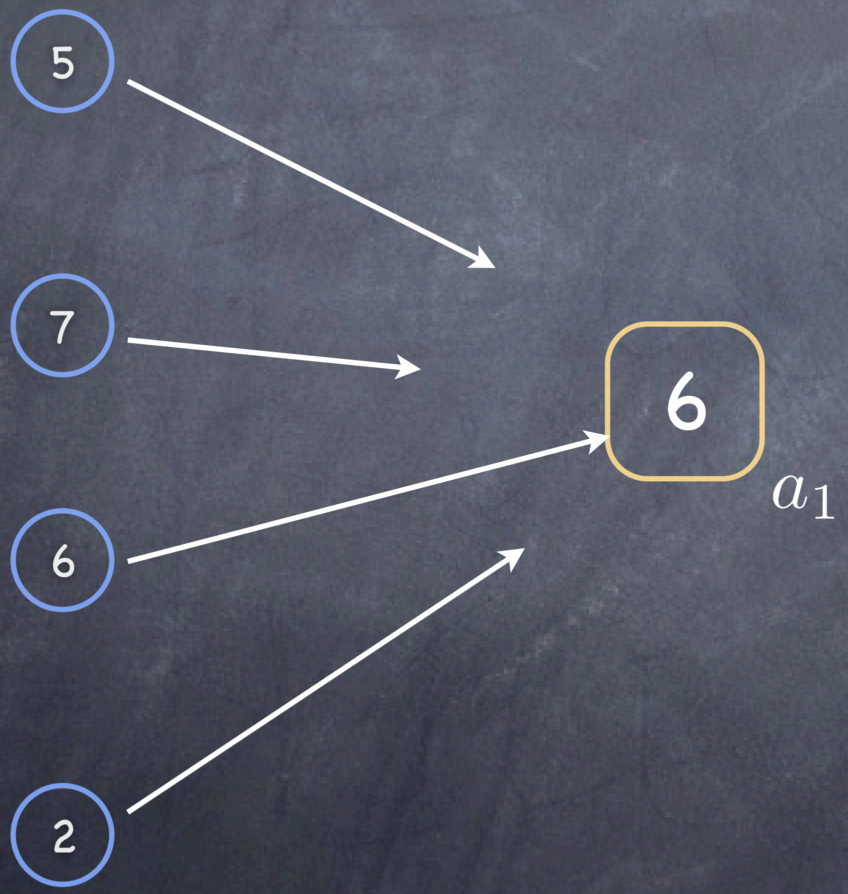
LIVENESS

- Some proposed value is eventually chosen
- If a value is chosen, a process eventually learns it

The Players

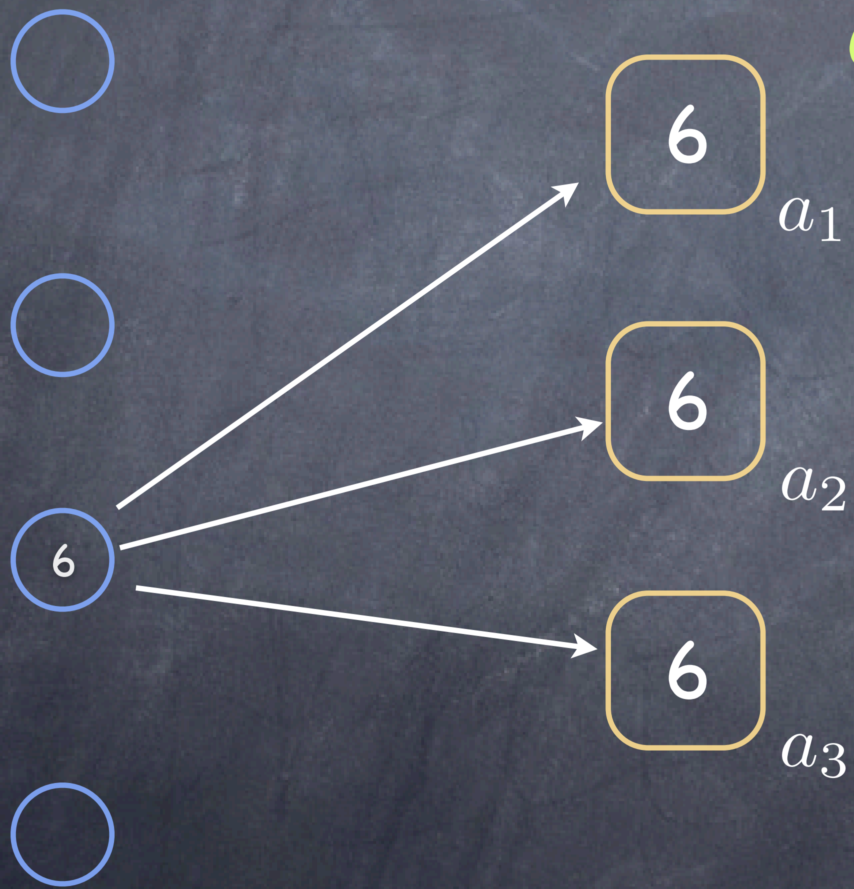
- Proposers
- Acceptors
- Learners

Choosing a value



Use a single acceptor

What if the acceptor fails?



6 is chosen!

- Choose only when a "large enough" set of acceptors accepts
- Using a majority set guarantees that at most one value is chosen

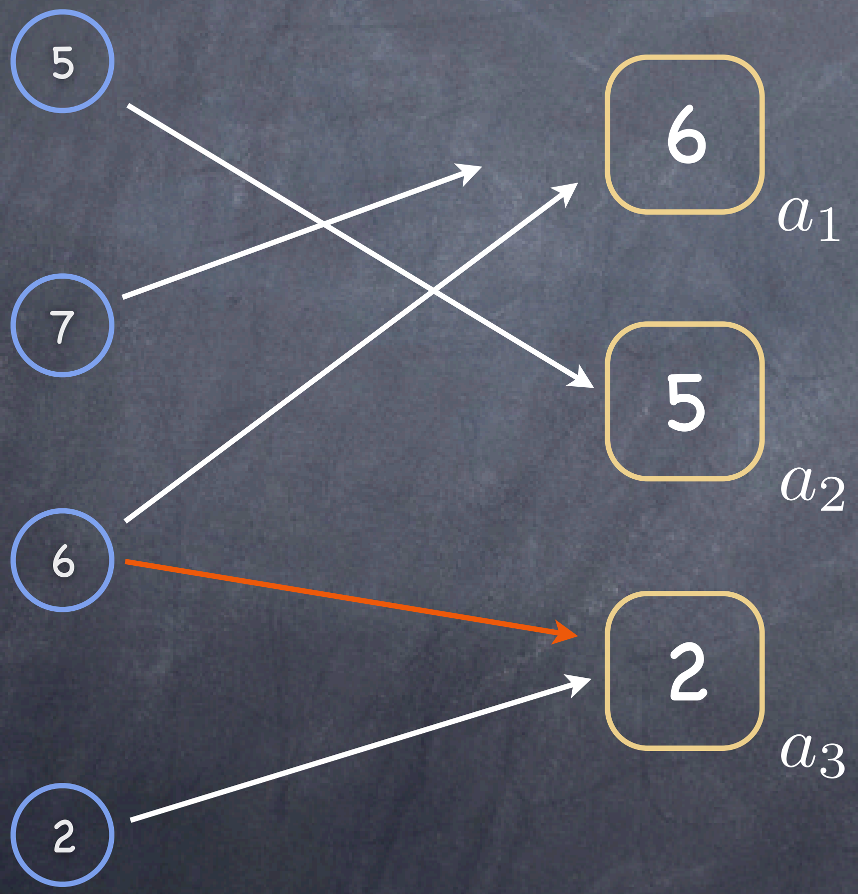
Accepting a value

- Suppose only one value is proposed by a single proposer.
- That value should be chosen!
- First requirement:

  P1: An acceptor must accept the first proposal that it receives

- ...but what if we have multiple proposers, each proposing a different value?

P1 + multiple proposers



No value is chosen!

Handling multiple proposals

- Acceptors must accept more than one proposal
- To keep track of different proposals, assign a natural number to each proposal
  - A proposal is then a pair (*psn*, value)
  - Different proposals have different *psn*
  - A proposal is chosen when it has been accepted by a majority of acceptors
  - A value is chosen when a single proposal with that value has been chosen

Choosing a unique value

"Any acceptor can accept as many proposals as he wants, so long as they all propose the same value"

Leslie Lamport

P2. If a proposal with value v is chosen, then every higher-numbered proposal that is chosen has value v

It's up to the Acceptors!

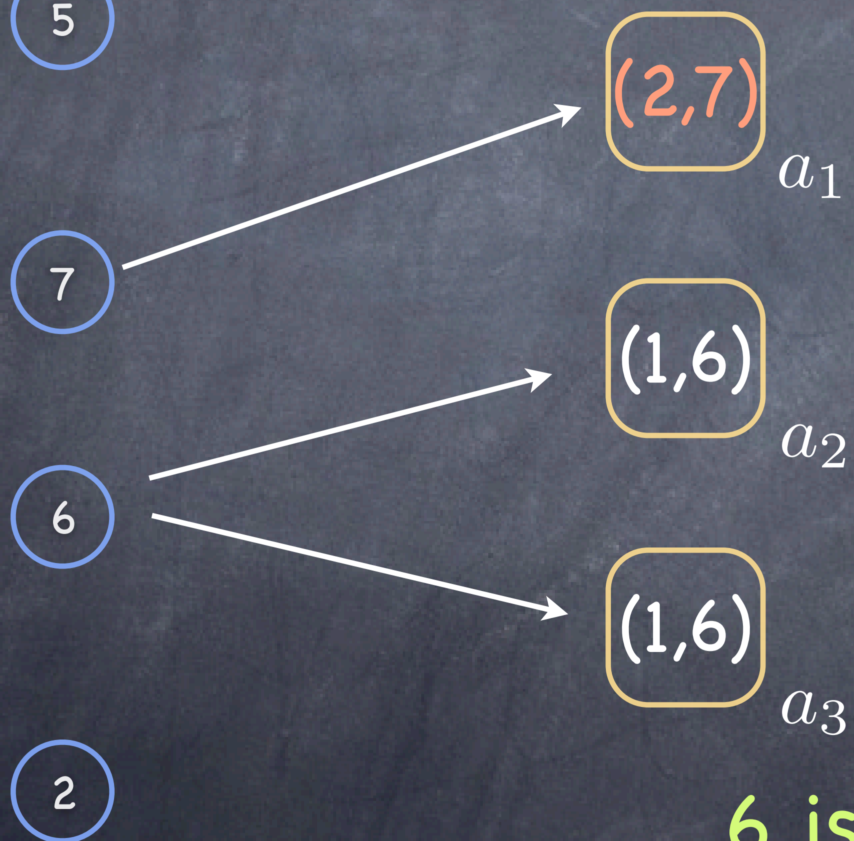
P2. If a proposal with value v is chosen, then every higher-numbered proposal that is chosen has value v

We strengthen it to:

P2a. If a proposal with value v is chosen, then every higher-numbered proposal accepted by any acceptor has value v

What about P1?

How does a_1 know
it should not accept?



6 is chosen!

- Do we still need P1?

  YES, to ensure that *some* proposal is accepted

- How well do P1 and P2a play together?

  Asynchrony is a problem...

It's up to the Proposers!

- Recall P2a:

  P2a. If a proposal with value v is chosen, then every higher-numbered proposal accepted by any acceptor has value v

We strengthen it to:

  P2b. If a proposal with value v is chosen, then every higher-numbered proposal issued by any proposer has value v

What to propose

P2b: If a proposal with value v is chosen, then every higher-numbered proposal issued by any proposer has value v

Suppose p wants to issue a proposal numbered n.

- If p can be certain that no proposal numbered $n' < n$ has been chosen then p can propose any value!
  - If a proposal numbered $n' < n$ has been chosen, then it has been accepted by a majority set S
  - Any majority set S′ must intersect S
  - If p can find one S′ in which no acceptors has accepted a proposal numbered $n' < n$, then no such proposal can have yet been chosen!
  - If no such S′, a proposal numbered $n' < n$ may have been chosen...
  - Then what?

What to propose

P2b: If a proposal with value v is chosen, then every higher-numbered proposal issued by any proposer has value v

Suppose p wants to issue a proposal numbered n.

- If p can be certain that no proposal numbered $n' < n$ has been chosen then p can propose any value!
- If not, p should propose the chosen value. But how?

What to propose

P2b: If a proposal with value v is chosen, then every higher-numbered proposal issued by any proposer has value v

Suppose p wants to issue a proposal numbered n.

- If p can be certain that no proposal numbered $n' < n$ has been chosen then p can propose any value!
- If not, p should propose the chosen value. But how?
  - What about using induction...
  - Say proposal numbered m with value v is chosen: some majority-set C of acceptors has accepted it
  - Suppose every proposal issued with number $m...n\text{-}1$ has value v
  - Consider proposal n : since every majority set S' intersects with C and ever proposal accepted by any acceptor with sequence number in $m...n\text{-}1$ has value v, *then*
  - p should propose the highest numbered proposal among all proposals, numbered less than n, accepted by some majority set S

It's up to an invariant!

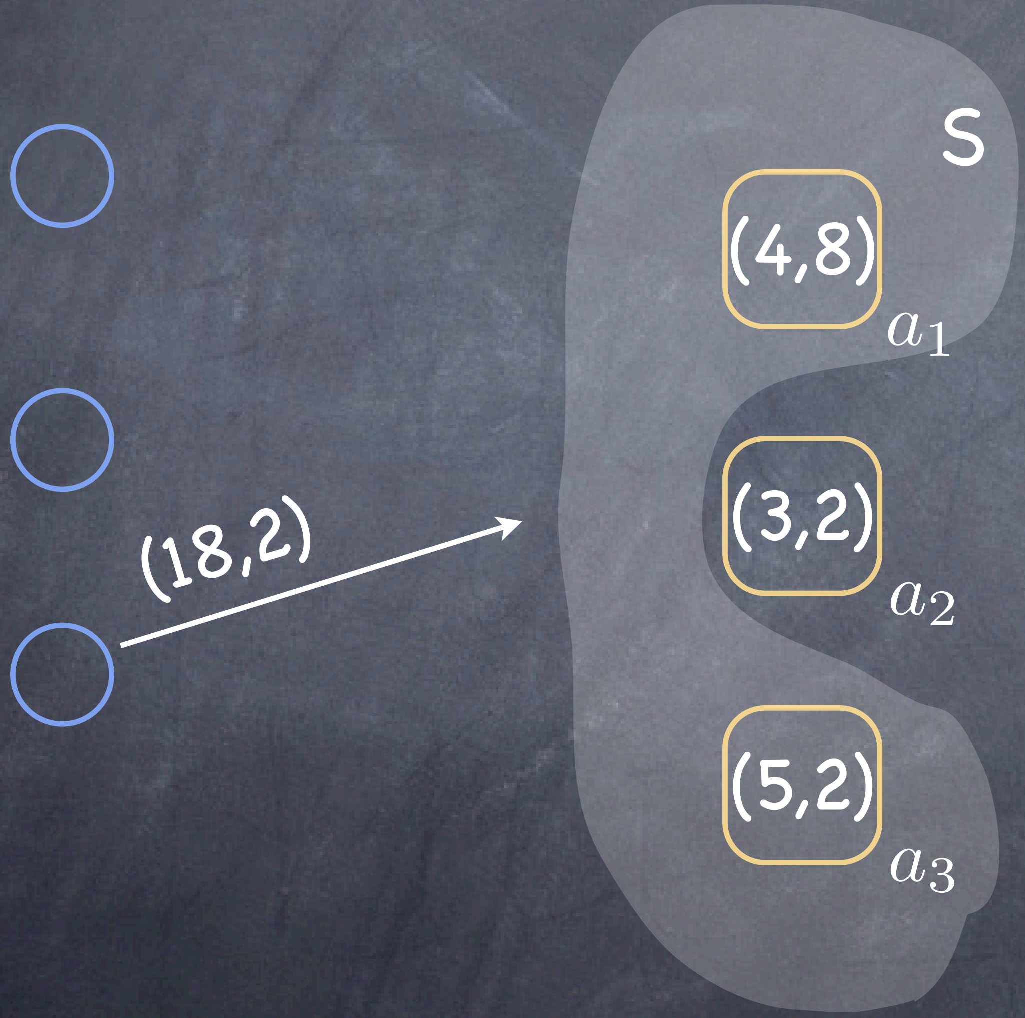
P2b: If a proposal with value v is chosen, then every higher-numbered proposal issued by any proposer has value v

Achieved by enforcing the following invariant

P2c: For any v and n, if a proposal with value v and number n is issued, then there is a set S consisting of a majority of acceptors such that either:

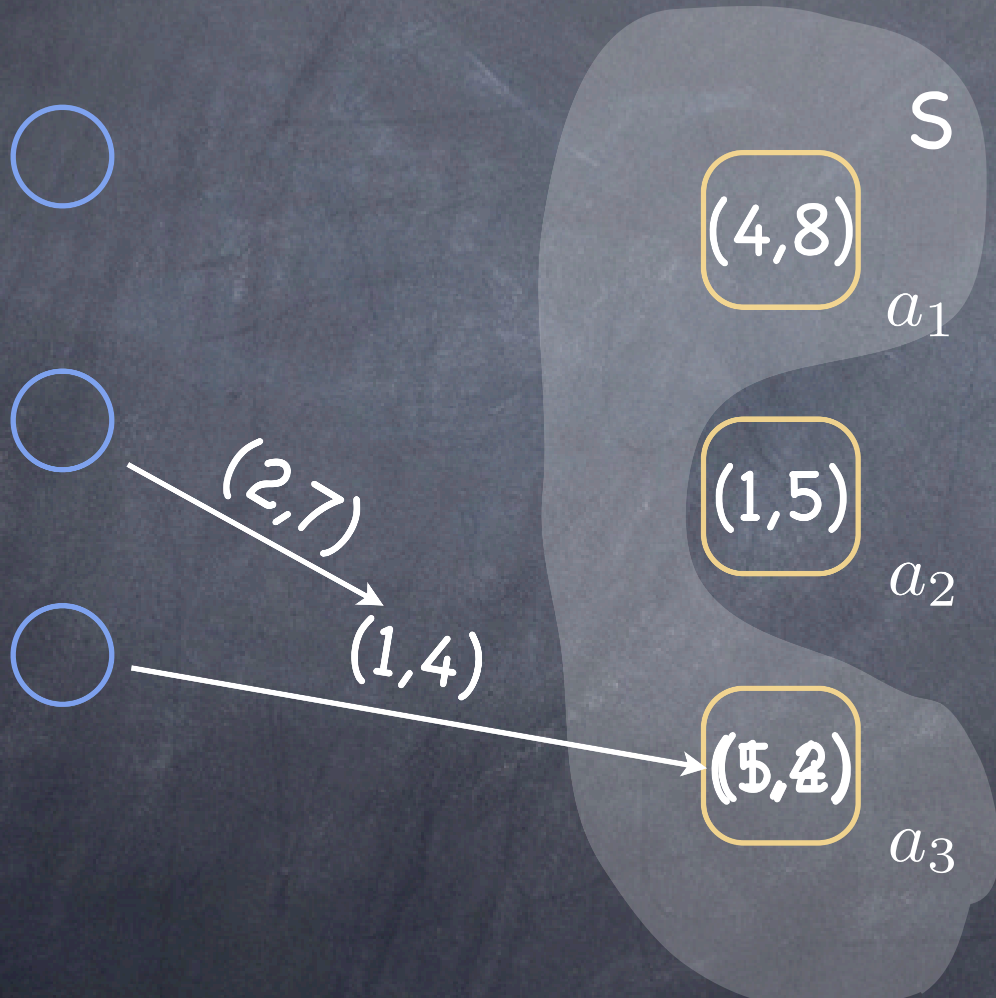
- no acceptor in S has accepted any proposal numbered less than n, or
- v is the value of the highest-numbered proposal among all proposals numbered less than n accepted by the acceptors in S

P2c in action



- v is the value of the highest-numbered proposal among all proposals numbered less than n and accepted by the acceptors in S

P2c in action



- No acceptor in S has accepted any proposal numbered less than n

The invariant is violated

Future telling?

- p must learn the highest-numbered proposal with number less than n, if any, that has been or will be accepted by each acceptor in some majority of acceptors.
- Avoid predicting the future by extracting a promise from a majority of acceptors not to subsequently accept any proposals numbered less than n

The proposer's protocol (I)

- A proposer chooses a new proposal number n and sends a request to each member of some set of acceptors, asking it to respond with:

  a. A promise never again to accept a proposal numbered less than n, and
  b. The accepted proposal with highest number less than n if any.

...call this a prepare request with number n

The proposer's protocol (II)

- If the proposer receives a response from a majority of acceptors, then it can issue a proposal with number n and value v, where v is

  a. the value of the highest-numbered proposal among the responses, or
  b. is any value selected by the proposer if responders returned no proposals

A proposes issues a proposal by sending, to some set of acceptors, a request that the proposal be accepted.

...call this an accept request.

The acceptor's protocol

- An acceptor receives prepare and accept requests from proposers.
  - It can always respond to a prepare request
  - It can respond to an accept request, accepting the proposal, iff it has not promised not to, e.g.

P1a: An acceptor can accept a proposal numbered n iff it has not responded to a prepare request having number greater than n

...which subsumes P1.

Small optimizations

- If an acceptor receives a prepare request r numbered n when it has already responded to a prepare request for $n' > n$, then the acceptor can simply ignore r.

...so an acceptor needs only remember the highest numbered proposal it has accepted and the number of the highest-numbered prepare request to which it has responded.

Choosing a value: Phase 1

- A proposer chooses a new *n* and sends <*prepare,n*> to a majority of acceptors

- If an acceptor a receives <*prepare,n'*>, where $n' > n$ of any <*prepare,n*> to which it has responded, then it responds to <*prepare, n'* > with
  - a promise not to accept any more proposals numbered less than *n'*
  - the highest numbered proposal (if any) that it has accepted

Choosing a value: Phase 2

- If the proposer receives a response to <*prepare,n*> from a majority of acceptors, then it sends to each <*accept,n,v*>, where *v* is either
  - the value of the highest numbered proposal among the responses
  - any value if the responses reported no proposals
- If an acceptor receives <*accept,n,v*>, it accepts the proposal unless it has in the meantime responded to <*prepare,n'*> , where $n' > n$

Learning chosen values (I)

Once a value is chosen, learners should find out about it. Many strategies are possible:

i. Each acceptor informs each learner whenever it accepts a proposal.

ii. Acceptors inform a distinguished learner, who informs the other learners

iii. Something in between (a set of not-quite-as-distinguished learners)

Questions

- What are the liveness properties of Paxos?

Question

- What do you do when nodes fail?

Question

- Are there any advantages/disadvantages to having a designated leader?

Question

- What are the costs to using Paxos? Is it practical?

Implementing State Machine Replication

- Implement a sequence of separate instances of consensus, where the value chosen by the i^{th} instance is the i^{th} message in the sequence.
- Each server assumes all three roles in each instance of the algorithm.
- Assume that the set of servers is fixed

The role of the leader

- In normal operation, elect a single server to be a leader. The leader acts as the distinguished proposer in all instances of the consensus algorithm.
  - Clients send commands to the leader, which decides where in the sequence each command should appear.
  - If the leader, for example, decides that a client command is the k^{th} command, it tries to have the command chosen as the value in the k^{th} instance of consensus.

A new leader λ is elected...

- Since λ is a learner in all instances of consensus, it should know most of the commands that have already been chosen. For example, it might know commands 1–10, 13, and 15.
  - It executes phase 1 of instances 11, 12, and 14 and of all instances 16 and larger.
  - This might leave, say, 14 and 16 constrained and 11, 12 and all commands after 16 unconstrained.
  - λ then executes phase 2 of 14 and 16, thereby choosing the commands numbered 14 and 16

Stop-gap measures

- All replicas can execute commands 1-10, but not 13-16 because 11 and 12 haven't yet been chosen.
- λ can either take the next two commands requested by clients to be commands 11 and 12, or can propose immediately that 11 and 12 be no-op commands.
- λ runs phase 2 of consensus for instance numbers 11 and 12.
- Once consensus is achieved, all replicas can execute all commands through 16.

To infinity, and beyond

- λ can efficiently execute phase 1 for infinitely many instances of consensus! (e.g. command 16 and higher)
  - λ just sends a message with a sufficiently high proposal number for all instances
  - An acceptor replies non trivially only for instances for which it has already accepted a value