# Paxos

# The Part-Time Parliament

- Parliament determines laws by passing sequence of numbered decrees
- Legislators can leave and enter the chamber at arbitrary times
- No centralized record of approved decrees–instead, each legislator carries a ledger

# Government 101

- No two ledgers contain contradictory information
- If a majority of legislators were in the Chamber and no one entered or left the Chamber for a sufficiently long time, then
  - any decree proposed by a legislator would eventually be passed
  - any passed decree would appear on the ledger of every legislator

# Back to the future

- A set of processes that can propose values
- Processes can crash and recover
- Processes have access to stable storage
- Asynchronous communication via messages
- Messages can be lost and duplicated, but not corrupted

# The Players

- Proposers
- Acceptors
- Learners

# Timeline

# The Game: Consensus

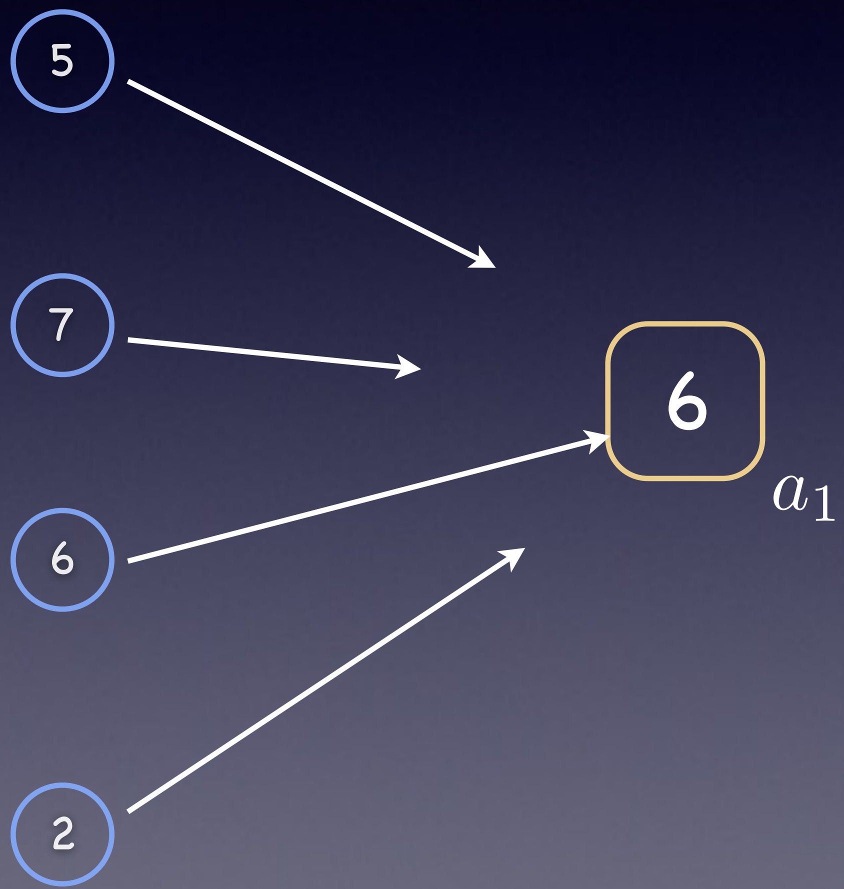
## SAFETY

- Only a value that has been proposed can be chosen
- Only a single value is chosen
- A process never learns that a value has been chosen unless it has been

## LIVENESS

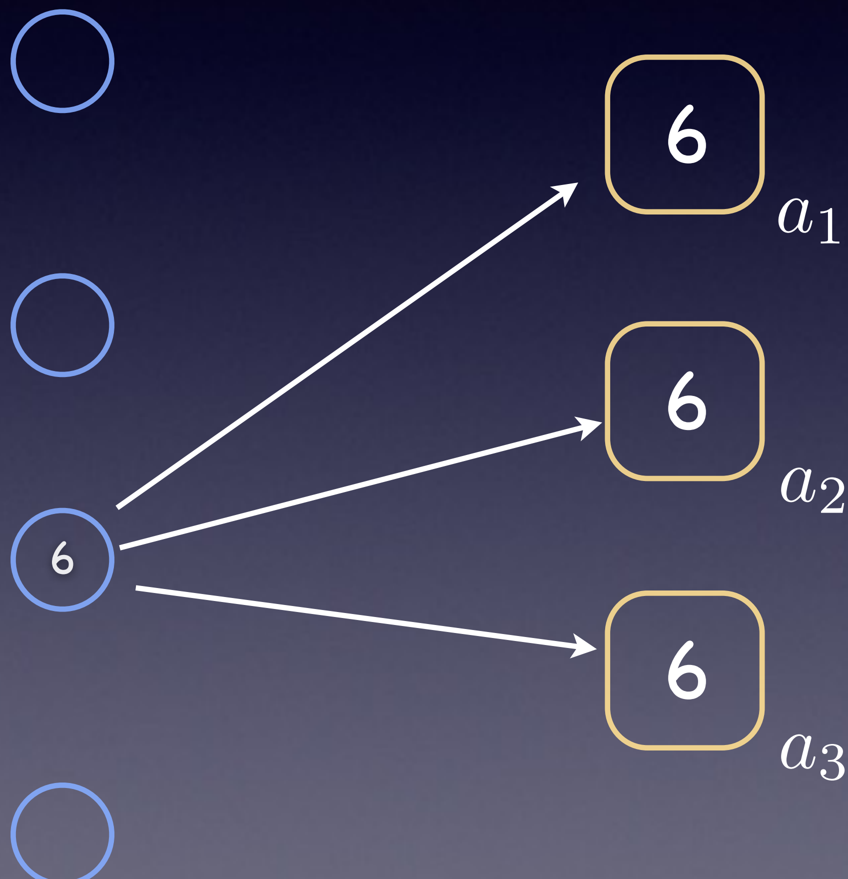
- Some proposed value is eventually chosen
- If a value is chosen, a process eventually learns it

# Choosing a value




Use a single acceptor

# What if the acceptor fails?




6 is chosen!

- Choose only when a "large enough" set of acceptors accepts
- Using a majority set guarantees that at most one value is chosen

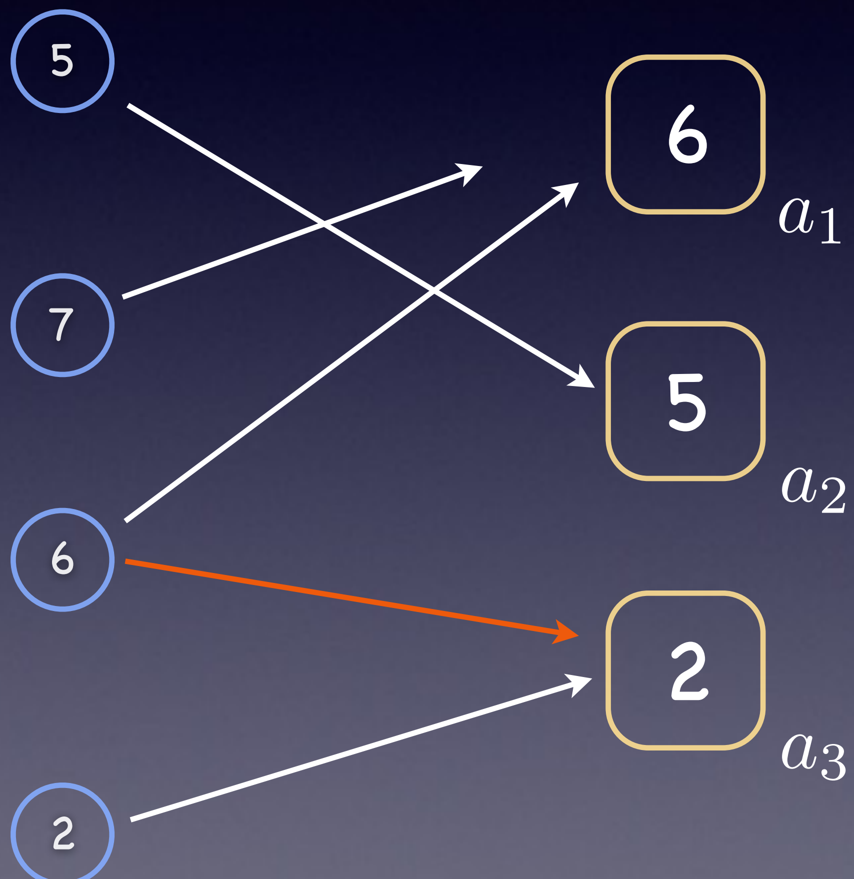
# Accepting a value

- Suppose only one value is proposed by a single proposer.
- That value should be chosen!
- First requirement:

  P1: An acceptor must accept the first proposal that it receives

- ...but what if we have multiple proposers, each proposing a different value?

# P1 + multiple proposers




No value is chosen!

# Handling multiple proposals

- Acceptors must accept more than one proposal
- To keep track of different proposals, assign a natural number to each proposal
  - A proposal is then a pair (*psn*, value)
  - Different proposals have different *psn*
  - A proposal is chosen when it has been accepted by a majority of acceptors
  - A value is chosen when a single proposal with that value has been chosen

# Choosing a unique value

P2. If a proposal with value $v$ is chosen, then every higher-numbered proposal that is chosen has value $v$

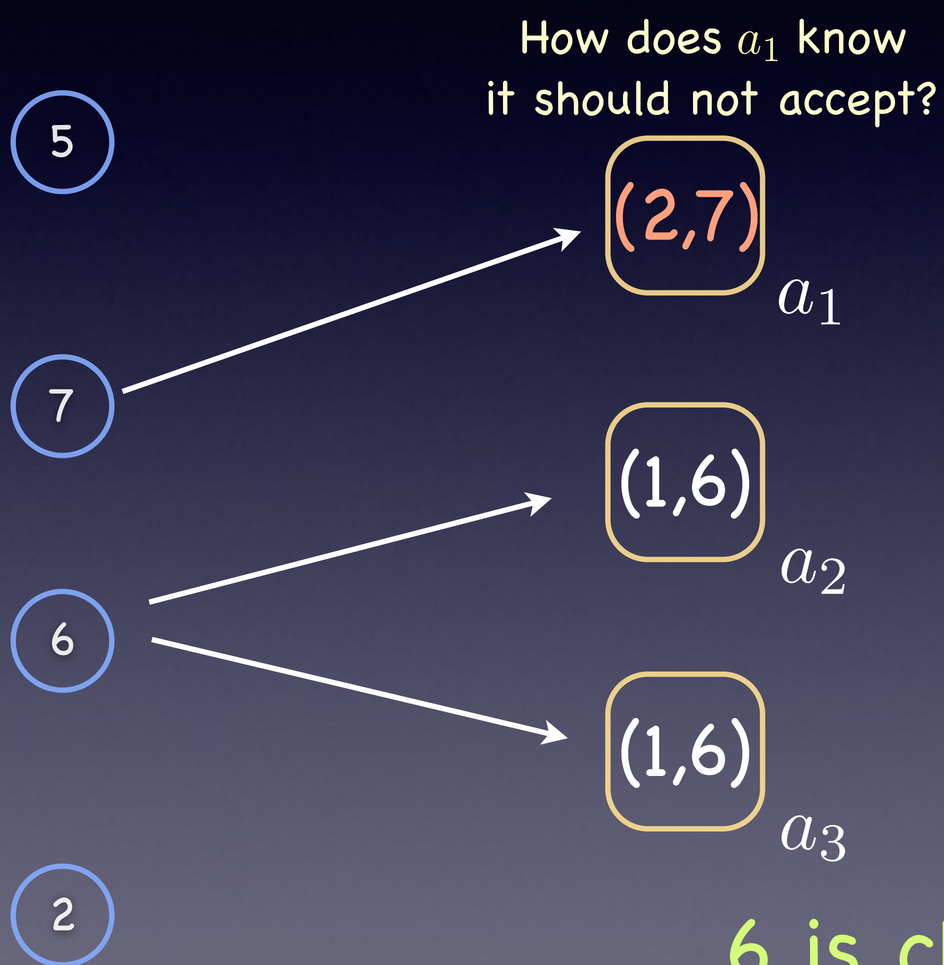
# It's up to the Acceptors!

P2. If a proposal with value $v$ is chosen, then every higher-numbered proposal that is chosen has value $v$

We strengthen it to:

P2a. If a proposal with value $v$ is chosen, then every higher-numbered proposal accepted by any acceptor has value $v$

# What about P1?




6 is chosen!

- Do we still need P1?

  YES, to ensure that *some* proposal is accepted

- How well do P1 and P2a play together?

  Asynchrony is a problem...

# It's up to the Proposers!

Recall P2a:

P2a. If a proposal with value $v$ is chosen, then every higher-numbered proposal accepted by any acceptor has value $v$

We strengthen it to:

P2b. If a proposal with value $v$ is chosen, then every higher-numbered proposal issued by any proposer has value $v$

# What to propose

P2b: If a proposal with value $v$ is chosen, then every higher-numbered proposal issued by any proposer has value $v$

Suppose $p$ wants to issue a proposal numbered $n$.

- If $p$ can be certain that no proposal numbered $n' < n$ has been chosen then $p$ can propose any value!
  - If a proposal numbered $n' < n$ has been chosen, then it has been accepted by a majority set S
  - Any majority set S' must intersect S
  - If $p$ can find one S' in which no acceptors has accepted a proposal numbered $n' < n$, then no such proposal can have yet been chosen!
  - If no such S', a proposal numbered $n' < n$ may have been chosen...
  - *Then what?*

# What to propose

**P2b:** If a proposal with value $v$ is chosen, then every higher-numbered proposal issued by any proposer has value $v$

Suppose $p$ wants to issue a proposal numbered $n$.

- If $p$ can be certain that no proposal numbered $n' < n$ has been chosen then $p$ can propose any value!
- If not, $p$ should propose the chosen value. But how?
  - What about using induction...
  - Say proposal numbered $m$ with value $v$ is chosen: some majority-set C of acceptors has accepted it
  - Suppose every proposal issued with number $m...n\text{-}1$ has value $v$
  - *Consider proposal $n$ : since* every majority set S' intersects with C and every proposal accepted by any acceptor with sequence number in $m...n\text{-}1$ has value $v$, *then*
  - $p$ should propose the highest numbered proposal among all proposals, numbered less than $n$, accepted by some majority set S

# Example

# It's up to an invariant!

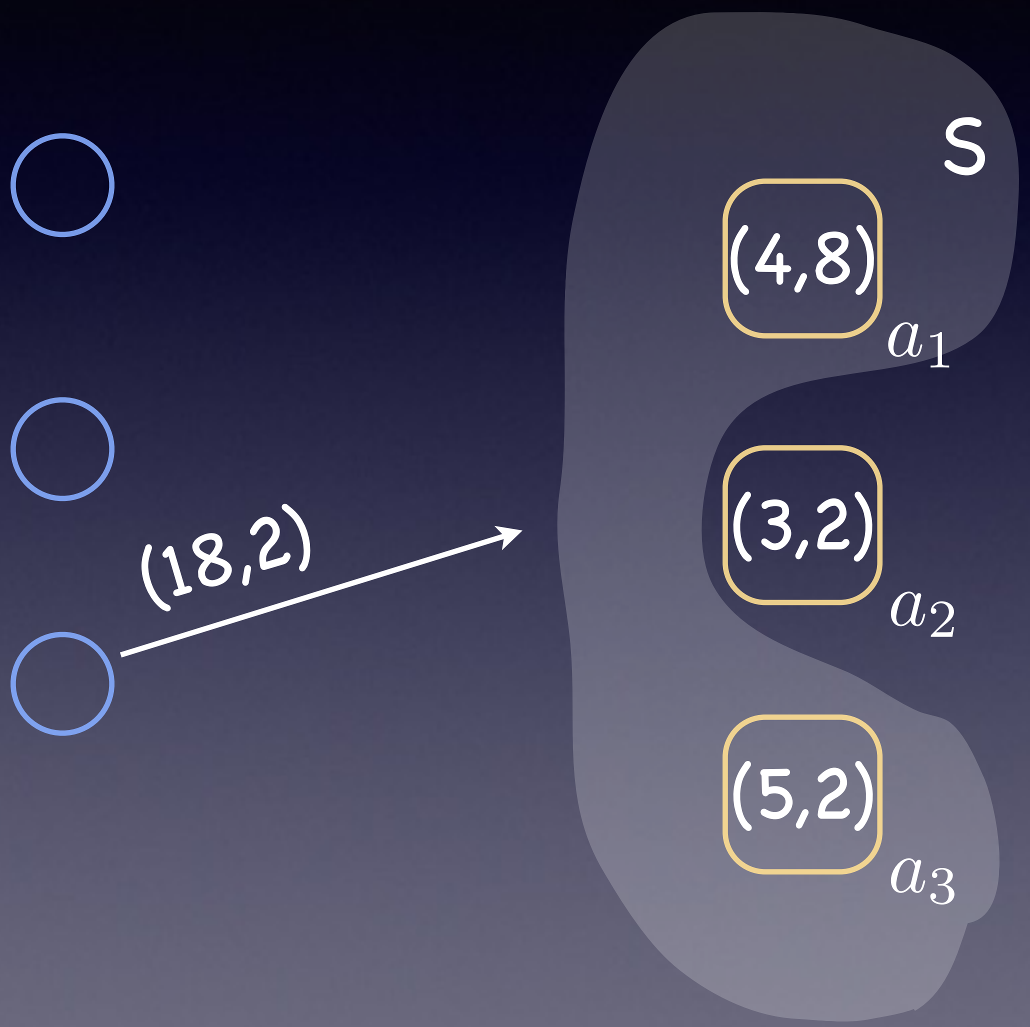
P2b: If a proposal with value $v$ is chosen, then every higher-numbered proposal issued by any proposer has value $v$

Achieved by enforcing the following invariant

P2c: For any $v$ and $n$, if a proposal with value $v$ and number $n$ is issued, then there is a set S consisting of a majority of acceptors such that either:

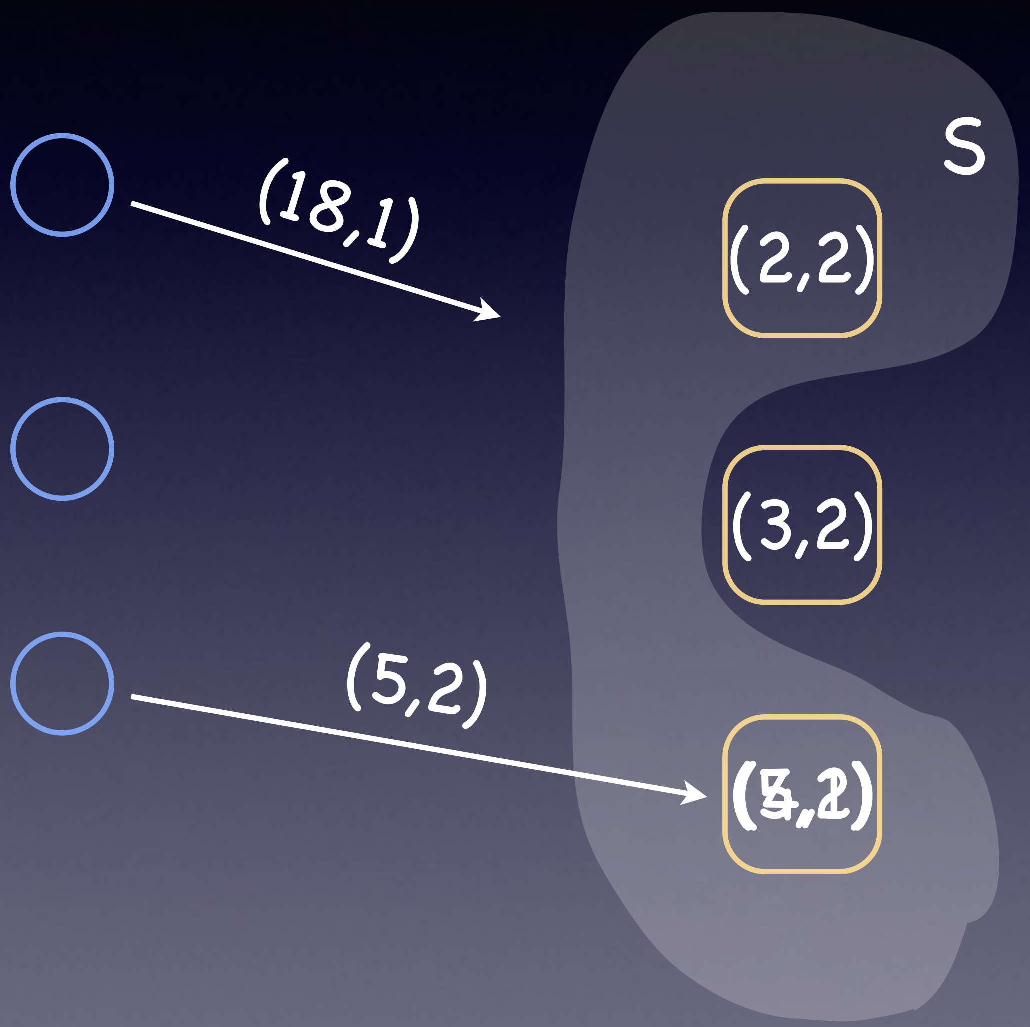
- no acceptor in S has accepted any proposal numbered less than $n$, or
- $v$ is the value of the highest-numbered proposal among all proposals numbered less than $n$ accepted by the acceptors in S

# P2c in action




- $v$ is the value of the highest-numbered proposal among all proposals numbered less than $n$ and accepted by the acceptors in S

# P2c in action




- $v$ is the value of the highest-numbered proposal among all proposals numbered less than $n$ and accepted by the acceptors in S

The invariant is violated

# Future telling?

- $p$ must learn the highest-numbered proposal with number less than $n$, if any, that has been or will be accepted by each acceptor in some majority of acceptors.
- Avoid predicting the future by extracting a promise from a majority of acceptors not to subsequently accept any proposals numbered less than $n$

# The proposer's protocol (I)

- A proposer chooses a new proposal number $n$ and sends a request to each member of some set of acceptors, asking it to respond with:

  a. A promise never again to accept a proposal numbered less than $n$, and
  b. The accepted proposal with highest number less than $n$ if any.

...call this a prepare request with number $n$

# The proposer's protocol (II)

- If the proposer receives a response from a majority of acceptors, then it can issue a proposal with number $n$ and value $v$, where $v$ is
  - the value of the highest-numbered proposal among the responses, or
  - is any value selected by the proposer if responders returned no proposals

A proposer issues a proposal by sending, to some set of acceptors, a request that the proposal be accepted.

...call this an accept request.

# The acceptor's protocol

- An acceptor receives prepare and accept requests from proposers.
  - It can always respond to a prepare request
  - It can respond to an accept request, accepting the proposal, iff it has not promised not to, e.g.

P1a: An acceptor can accept a proposal numbered $n$ iff it has not responded to a prepare request having number greater than $n$

...which subsumes P1.

# Small optimizations

- If an acceptor receives a prepare request $r$ numbered $n$ when it has already responded to a prepare request for $n' > n$, then the acceptor can simply ignore $r$.

...so an acceptor needs only remember the highest numbered proposal it has accepted and the number of the highest-numbered prepare request to which it has responded.

# Choosing a value: Phase 1

- A proposer chooses a new *n* and sends <*prepare,n*> to a majority of acceptors
- If an acceptor a receives <*prepare,n'*>, where *n' > n* of any <*prepare,n*> to which it has responded, then it responds to <*prepare, n'* > with
  - a promise not to accept any more proposals numbered less than *n'*
  - the highest numbered proposal (if any) that it has accepted

# Choosing a value: Phase 2

- If the proposer receives a response to <*prepare,n*> from a majority of acceptors, then it sends to each <*accept,n,v*>, where *v* is either
  - the value of the highest numbered proposal among the responses
  - any value if the responses reported no proposals
- If an acceptor receives <*accept,n,v*>, it accepts the proposal unless it has in the meantime responded to <*prepare,n'*> , where $n' > n$

# Learning chosen values (I)

Once a value is chosen, learners should find out about it. Many strategies are possible:

i. Each acceptor informs each learner whenever it accepts a proposal.

ii. Acceptors inform a distinguished learner, who informs the other learners

iii. Something in between (a set of not-quite-as-distinguished learners)

# Questions

- What are the liveness properties of Paxos? Why is Paxos not considered live?

# Question

- What do you do when nodes fail?  How is Paxos robust to failures?

# Question

- Are there any advantages/disadvantages to having a designated leader?

# Implementing State Machine Replication

- Implement a sequence of separate instances of consensus, where the value chosen by the $i^{th}$ instance is the $i^{th}$ message in the sequence.
- Each server assumes all three roles in each instance of the algorithm.
- Assume that the set of servers is fixed

# The role of the leader

- In normal operation, elect a single server to be a leader. The leader acts as the distinguished proposer in all instances of the consensus algorithm.
  - Clients send commands to the leader, which decides where in the sequence each command should appear.
  - If the leader, for example, decides that a client command is the $k^{th}$ command, it tries to have the command chosen as the value in the $k^{th}$ instance of consensus.

# A new leader is elected...

- Since leader is a learner in all instances of consensus, it should know most of the commands that have already been chosen. For example, it might know commands 1-10, 13, and 15.
  - It executes phase 1 of instances 11, 12, and 14 and of all instances 16 and larger.
  - This might leave, say, 14 and 16 constrained and 11, 12 and all commands after 16 unconstrained.
  - leader then executes phase 2 of 14 and 16, thereby choosing the commands numbered 14 and 16

# Stop-gap measures

- All replicas can execute commands 1-10, but not 13-16 because 11 and 12 haven't yet been chosen.
- leader can either take the next two commands requested by clients to be commands 11 and 12, or can propose immediately that 11 and 12 be no-op commands.
- leader runs phase 2 of consensus for instance numbers 11 and 12.
- Once consensus is achieved, all replicas can execute all commands through 16.

# To infinity, and beyond

- leader can efficiently execute phase 1 for infinitely many instances of consensus! (e.g. command 16 and higher)
  - leader just sends a message with a sufficiently high proposal number for all instances
  - An acceptor replies non trivially only for instances for which it has already accepted a value

# Question

- What are the costs to using Paxos?  Is it practical?