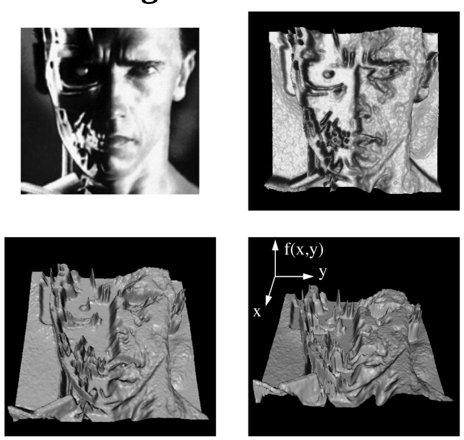
Image Processing

Definitions

- Many graphics techniques that operate only on images
- **Image processing**: operations that take images as input, produce images as output
- In its most general form, an **image** is a function *f* from R^2 to R
  - $f(x, y)$ gives the intensity of a channel at position (x, y)
  - defined over a rectangle, with a finite range:
    $f: [a,b] \times [c,d] \rightarrow [0,1]$
  - A color image is just three functions pasted together:
    - $\mathbf{f}(x, y) = (f_r(x, y), f_g(x, y), f_b(x, y))$

Images as Functions



What is a digital image?

- In computer graphics, we usually operate on **digital** (**discrete**) images:
  - **Sample** the space on a regular grid
  - **Quantize** each sample (round to nearest integer)
- If our samples are Δ apart, we can write this as:

$f[i, j] = \text{Quantize}\{ f(i\,\Delta, j\,\Delta) \}$

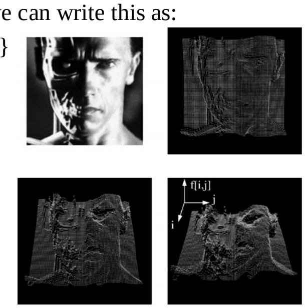


Image processing

- An **image processing** operation typically defines a new image *g* in terms of an existing image *f*.
- The simplest operations are those that transform each pixel in isolation. These pixel-to-pixel operations can be written:

$$g(x,y) = t(f(x,y))$$

- Example: threshold, RGB $\rightarrow$ grayscale
- Note: a typical choice for mapping to grayscale is to apply the YIQ television matrix and keep the Y.

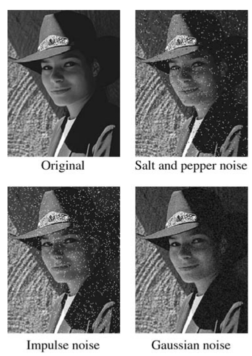
$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.528 & 0.311 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Pixel Movement

- Some operations preserve intensities, but move pixels around in the image

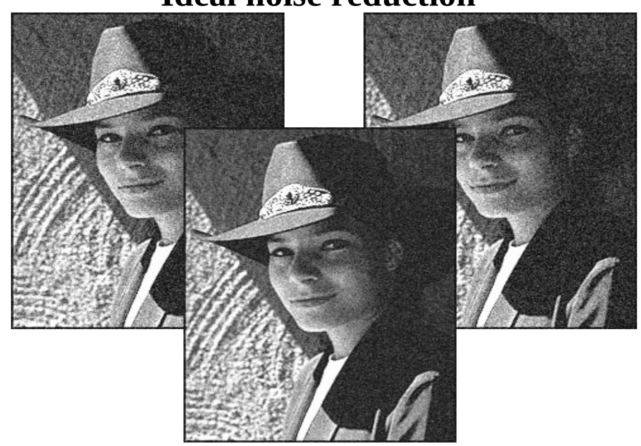
  $f'(x, y) = f(g(x,y), h(x,y))$
- Examples: many amusing warps of images

Noise



- Common types of noise:
  - **Salt and pepper noise**: contains random occurences of black and white pixels
  - **Impulse noise:** contains random occurences of white pixels
  - **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Ideal noise reduction



Ideal noise reduction

Noise Reduction

- How can we "smooth" away noise?

Convolution

- Convolution is a fancy way to combine two functions.
  - Think of f as an image and g as a "smear" operator
  - g determines a new intensity at each point in terms of intensities of a neighborhood of that point

$$\begin{aligned} h(x,y) &= f(x,y) * g(x,y) \\ &= \int_{-\infty}^{\infty} f(x',y')g(x-x',y-y')dx'dy' \end{aligned}$$

- The computation at each point (x,y) is like the computation of cone responses

Convolution

- One of the most common methods for filtering an image is called **convolution**.
- In 1D, convolution is defined as:

$$\begin{aligned} g(x) &= f(x) * h(x) \\ &= \int_{-\infty}^{\infty} f(x')h(x-x')dx' \\ &= \int_{-\infty}^{\infty} f(x')\tilde{h}(x'-x)dx' \end{aligned}$$

where $\tilde{h}(x) = h(-x)$.

- Example:

Convolution in 2D

- In two dimensions, convolution becomes:

$$g(x,y) = f(x,y) * h(x,y)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')h(x-x',y-y')dx'dy'$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')\tilde{h}(x'-x,y'-y)dx'dy'$$

where $\tilde{h}(x,y) = h(-x,-y)$.

- Similarly, discrete convolution in 2D becomes:

$$g[i,j] = f[i,j] * h[i,j]$$
$$= \sum_k \sum_l f[k,l]h[k-i,l-j]$$
$$= \sum_k \sum_l f[k,l]\tilde{h}[i-k,j-l]$$

where $\tilde{h}[i,j] = h[-i,-j]$.

Convolution Representation

- Since f and g are defined over finite regions, we can write them out in two-dimensional arrays:
- Note: *This is not matrix multiplication*!

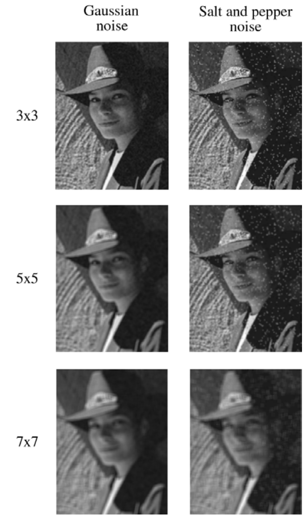
62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

$\times .2$	$\times 0$	$\times .2$
$\times 0$	$\times .2$	$\times 0$
$\times .2$	$\times 0$	$\times .2$

Mean Filters

- How can we represent our noise-reducing averaging filter as a convolution diagram?

Mean Filters



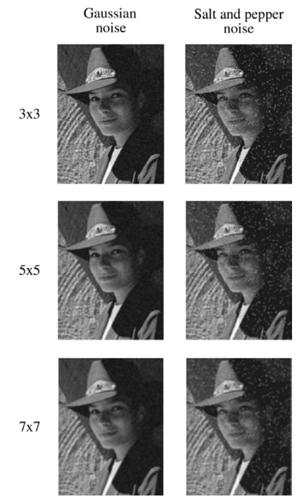
Gaussian Filters

- Gaussian filters weigh pixels based on their distance to the location of convolution.

$$g[i,j] = e^{-(i^2+j^2)/(2\sigma^2)}$$

- Blurring noise while preserving features of the image
- Smoothing the same in all directions
- More significance to neighboring pixels
- Width parameterized by σ
- Gaussian functions are separable
- Convolving with multiple Gaussian filters results in a single Gausian filter

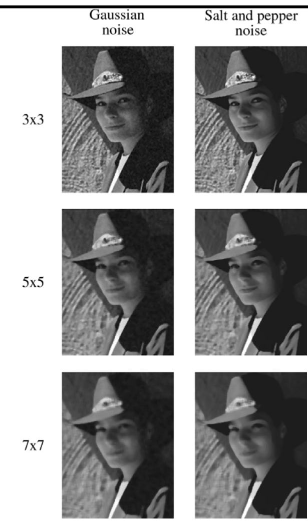
Gaussian Filters

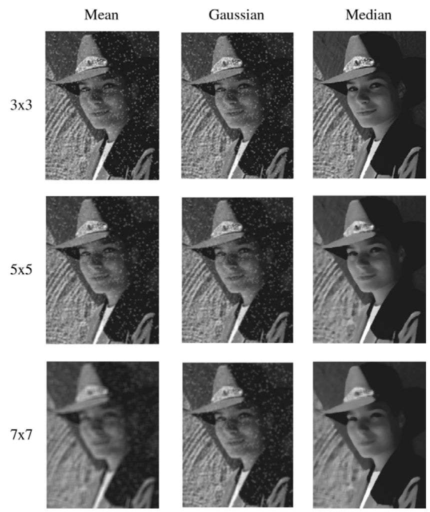


Median Filters

- A **Median Filter** operates over a *k*x*k* region by selecting the median intensity in the region.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?

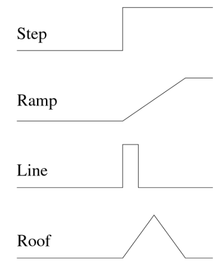
Median Filters





Edge Detection

- One of the most important uses of image processing is **edge detection**
  - Really easy for humans
  - Really difficult for computers
  - Fundamental in computer vision
  - Important in many graphics applications
- What defines an edge?



Gradient

- The **gradient** is the 2D equivalent of the derivative:

$$\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

- Properties of the gradient
  - It's a vector
  - Points in the direction of maximum increase of *f*
  - Magnitude is rate of increase
- How can we approximate the gradient in a discrete image?

Edge Detection Algorithms

- Edge detection algorithms typically proceed in three or four steps:
  - Filtering: cut down on noise
  - Enhancement: amplify the difference between edges and non-edges
  - Detection: use a threshold operation
  - Localization (optional): estimate geometry of edges beyond pixels

Edge Enhancement

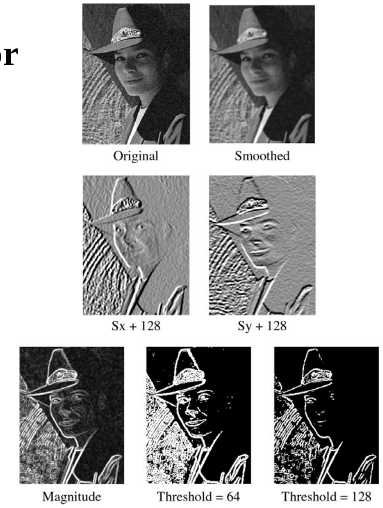
- A popular gradient magnitude computation is the **Sobel operator**:

$$s_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

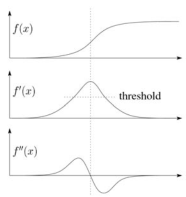
$$s_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

- We can then compute the magnitude of the vector (s_x, s_y)

Sobel Operator



Second derivative operators



- The Sobel operator can produce thick edges. Ideally, we're looking for infinitely thin boundaries.
- An alternative approach is to look for local extrema in the first derivative: places where the change in the gradient is highest.
- **Q**: A peak in the first derivative corresponds to what in the second derivative?

Localization with the Laplacian

- An equivalent measure of the second derivative in 2D is the **Laplacian**: $\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$
- Using the same arguments we used to compute the gradient filters, we can derive a Laplacian filter to be:

$$\Delta^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- Zero crossings of this filter correspond to positions of maximum gradient. These zero crossings can be used to localize edges.

Laplacian of Gaussian

- Combines
  - Gaussian smoothing
  - Second derivative enhancement (Laplacian)

$$LoG(x,y) = \left(\frac{x^2 + y^2 - 2\sigma^2}{\sigma^4}\right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Summary

- Formal definitions of image and image processing
- Kinds of image processing: pixel-to-pixel, pixel movement, convolution, others
- Types of noise and strategies for noise reduction
- Definition of convolution and how discrete convolution works
- The effects of mean, median and Gaussian filtering
- How edge detection is done
- Gradients and discrete approximations