

15. Subdivision surfaces

1

Reading

Recommended:

- ♦ Stollnitz, DeRose, and Salesin. *Wavelets for Computer Graphics: Theory and Applications*, 1996, section 10.2.

2

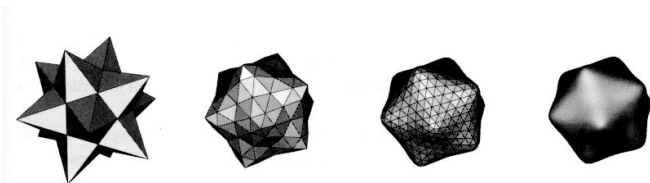
Subdivision surfaces

Chaikin's use of subdivision for curves inspired similar techniques for subdivision surfaces.

Iteratively refine a **control polyhedron** (or **control mesh**) to produce the limit surface

$$\sigma = \lim_{j \rightarrow \infty} M^j$$

using splitting and averaging steps.

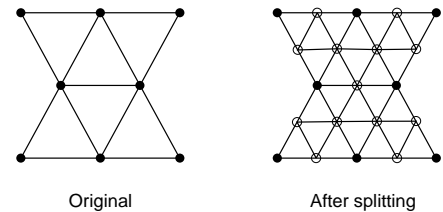


3

Triangular subdivision

There are a variety of ways to subdivide a polygon mesh.

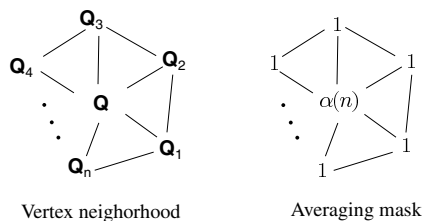
A common choice for triangle meshes is 4:1 subdivision – each triangular face is split into four subfaces:



4

Loop averaging step

Once again we can use **masks** for the averaging step:



$$\mathbf{Q} \leftarrow \frac{\alpha(n)\mathbf{Q} + \mathbf{Q}_1 + \dots + \mathbf{Q}_n}{\alpha(n) + n}$$

where

$$\alpha(n) = \frac{n(1-\beta(n))}{\beta(n)} \quad \beta(n) = \frac{5}{4} - \frac{(3+2\cos(2\pi/n))^2}{32}$$

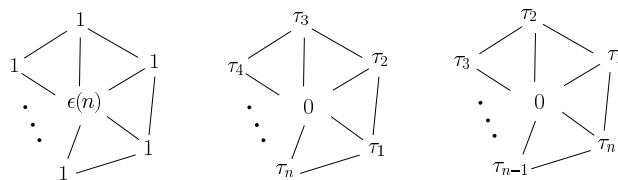
These values, due to Charles Loop, are carefully chosen to ensure smoothness – namely, tangent plane or normal continuity.

Note: tangent plane continuity is also known as G^1 continuity.

5

Loop evaluation and tangent masks

As with subdivision curves, we can split and average a number of times and then push the points to their limit positions.



Evaluation mask

Tangent masks

$$\mathbf{Q}^\infty = \frac{\epsilon(n)\mathbf{Q} + \mathbf{Q}_1 + \dots + \mathbf{Q}_n}{\epsilon(n) + n}$$

$$\mathbf{T}_1^\infty = \tau_1(n)\mathbf{Q}_1 + \tau_2(n)\mathbf{Q}_2 + \dots + \tau_n(n)\mathbf{Q}_n$$

$$\mathbf{T}_2^\infty = \tau_n(n)\mathbf{Q}_1 + \tau_1(n)\mathbf{Q}_2 + \dots + \tau_{n-1}(n)\mathbf{Q}_n$$

where

$$\epsilon(n) = \frac{3n}{\beta(n)} \quad \tau_i(n) = \cos(2\pi i/n)$$

How do we compute the normal?

6

Recipe for subdivision surfaces

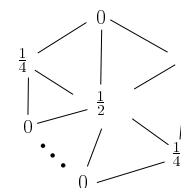
As with subdivision curves, we can now describe a recipe for creating and rendering subdivision surfaces:

- Subdivide (split+average) the control polyhedron a few times. Use the averaging mask.
- Compute two tangent vectors using the tangent masks.
- Compute the normal from the tangent vectors.
- Push the resulting points to the limit positions. Use the evaluation mask.
- Render!

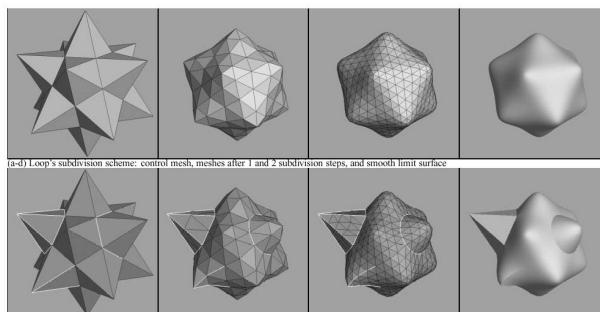
Adding creases without trim curves

In some cases, we want a particular feature such as a crease to be preserved. With NURBS surfaces, this required the use of trim curves.

For subdivision surfaces, we can just modify the subdivision mask:



This gives rise to G^0 continuous surfaces (i.e., having positional but not tangent plane continuity)



(a-d) Loop's subdivision scheme: control mesh, meshes after 1 and 2 subdivision steps, and smooth limit surface

(e-h) Our piecewise smooth subdivision scheme: tagged control mesh, meshes after 1 and 2 subdivision steps, and piecewise smooth limit surface

7

8

Creases without trim curves, cont.

Here's an example using Catmull-Clark surfaces (based on subdividing quadrilateral meshes):

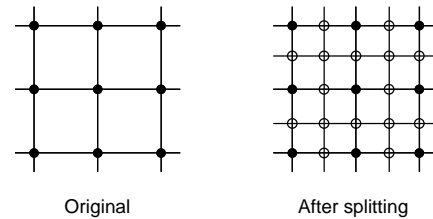


9

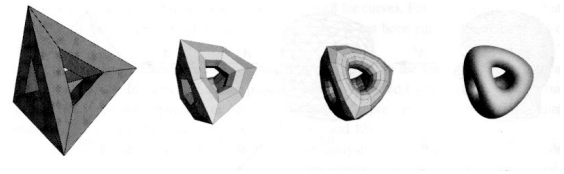
Face schemes

There are other kinds of subdivision techniques for surfaces besides 4:1 triangular subdivision.

For example, quadrilateral faces can be split into four subfaces:



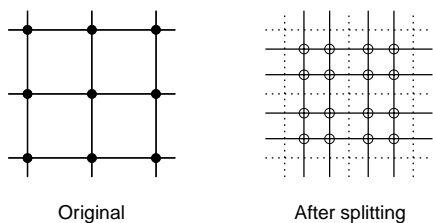
Catmull-Clark subdivision:



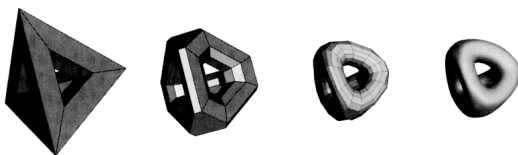
10

Vertex schemes

A vertex surrounded by n faces is split into n subvertices, one for each face:



Doo-Sabin subdivision:



11

Summary

What to take home:

- ♦ The meanings of all the **boldfaced** terms.
- ♦ How to construct and render Loop subdivision surfaces from the averaging masks, evaluation masks, and tangent masks.

12