Subdivision surfaces

Brian Curless
CSE 557
Fall 2009

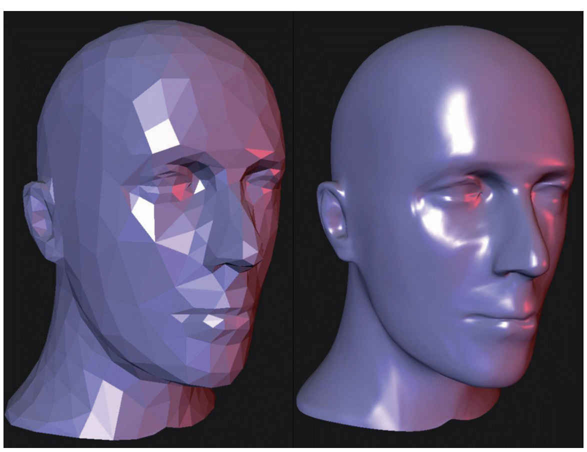
Reading

Recommended:

- Stollnitz, DeRose, and Salesin. *Wavelets for Computer Graphics: Theory and Applications,* 1996, section 10.2.
- DeRose, Kass, and Truong. Subdivision surfaces in character animation, SIGGRAPH '98, pp. 85-94.

Building complex models

We can extend the idea of subdivision from curves to surfaces…



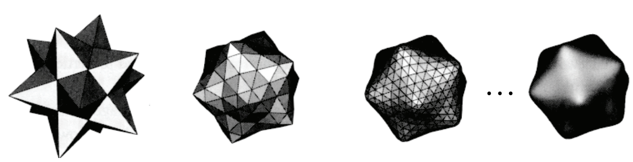
Subdivision surfaces

Chaikin's use of subdivision for curves inspired similar techniques for subdivision surfaces.

Iteratively refine a **control polyhedron** (or **control mesh**) to produce the limit surface

$$S = \lim_{j \to \infty} M^j$$

using splitting and averaging steps.

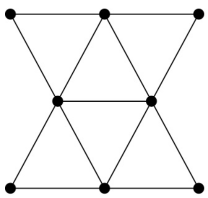


M^0 M^1 M^2 … M^∞

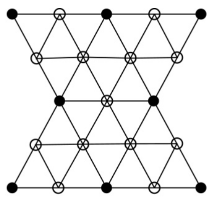
Triangular subdivision

There are a variety of ways to subdivide a poylgon mesh.

A common choice for triangle meshes is 4:1 subdivision – each triangular face is split into four subfaces:



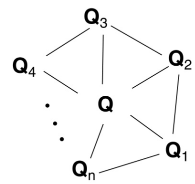
Original



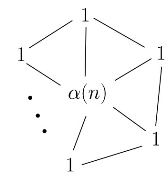
After splitting

Loop averaging step

Once again we can use **masks** for the averaging step:



Vertex neighorhood



Averaging mask
(before affine normalization)

$$\mathbf{Q} \leftarrow \frac{\alpha(n)\mathbf{Q} + \mathbf{Q}_1 + \cdots + \mathbf{Q}_n}{\alpha(n) + n}$$

where

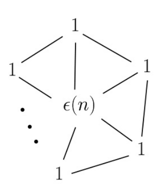
$$\alpha(n) = \frac{n(1-\beta(n))}{\beta(n)} \qquad \beta(n) = \frac{5}{4} - \frac{(3+2\cos(2\pi/n))^2}{32}$$

These values, due to Charles Loop, are carefully chosen to ensure smoothness – namely, tangent plane or normal continuity.

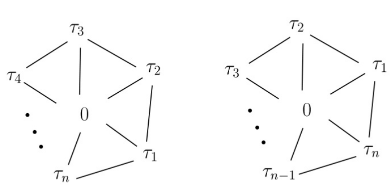
Note: tangent plane continuity is also know as G^1 continuity for surfaces.

Loop evaluation and tangent masks

As with subdivision curves, we can split and average a number of times and then push the points to their limit positions.



Evaluation mask
(before affine normalization)



Tangent masks

$$\mathbf{Q}^{\infty} = \frac{\varepsilon(n)\mathbf{Q} + \mathbf{Q}_1 + \cdots + \mathbf{Q}_n}{\varepsilon(n) + n}$$

$$\mathbf{T}_1^{\infty} = \tau_1(n)\mathbf{Q}_1 + \tau_2(n)\mathbf{Q}_2 + \cdots + \tau_n(n)\mathbf{Q}_n$$

$$\mathbf{T}_2^{\infty} = \tau_n(n)\mathbf{Q}_1 + \tau_1(n)\mathbf{Q}_2 + \cdots + \tau_{n-1}(n)\mathbf{Q}_n$$

where

$$\varepsilon(n) = \frac{3n}{\beta(n)} \qquad \tau_i(n) = \cos(2\pi i/n)$$

Note that the eigenvalues of the related subdivision matrix have the form: $\lambda_1 = 1 > \lambda_2 = \lambda_3 > \cdots \geq \lambda_n \geq 0$.

How do we compute the normal?

Recipe for subdivision surfaces

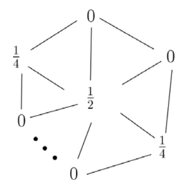
As with subdivision curves, we can now describe a recipe for creating and rendering subdivision surfaces:

- Subdivide (split+average) the control polyhedron a few times. Use the averaging mask.
- Compute two tangent vectors using the tangent masks.
- Compute the normal from the tangent vectors.
- Push the resulting points to the limit positions. Use the evaluation mask.
- Render!

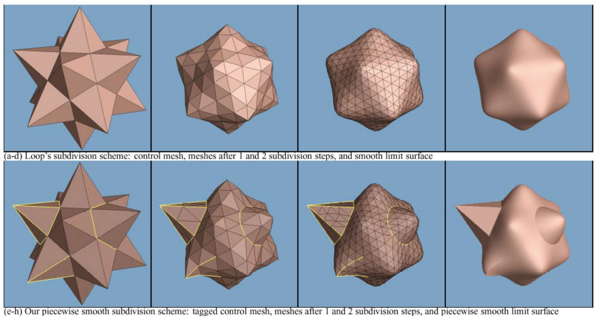
Adding creases without trim curves

For NURBS surfaces, adding sharp features like creases required the use of trim curves.

For subdivision surfaces, we can just modify the subdivision masks. E.g., we can mark some edges and vertices as “creases” and modify the subdivision mask for them (and their children):



This gives rise to G^0 continuous surfaces (i.e., having positional but not tangent plane continuity).



(a-d) Loop’s subdivision scheme: control mesh, meshes after 1 and 2 subdivision steps, and smooth limit surface

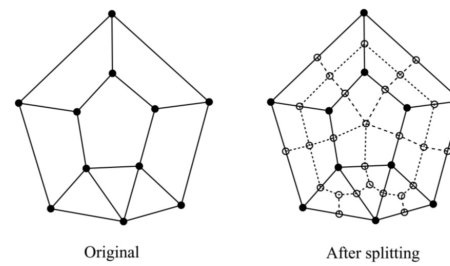
(e-h) Our piecewise smooth subdivision scheme: tagged control mesh, meshes after 1 and 2 subdivision steps, and piecewise smooth limit surface

[Hoppe, SIGGRAPH 1994]

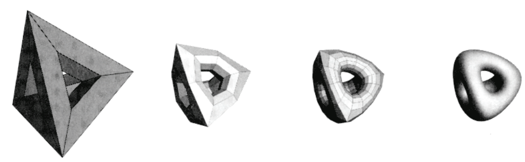
Face schemes

4:1 subdivision of triangles is sometimes called a **face scheme** for subdivision, as each face begets more faces.

An alternative face scheme starts with arbitrary polygon meshes and inserts vertices along edges and at face centroids:



Catmull-Clark subdivision:



Note: after the first subdivision, all polygons are quadilaterals in this scheme. Further, Catmull-Clark subdivision turns out to be a generalization of tensor product cubic B-spline surfaces!

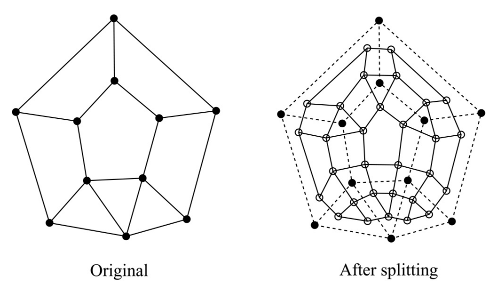
Creases without trim curves, cont.

Here’s an example using Catmull-Clark surfaces (based on subdividing quadrilateral meshes):

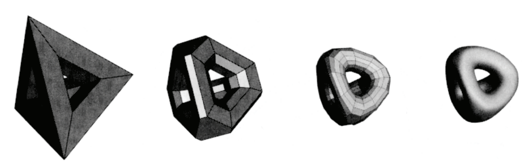
This particular example uses the hybrid technique of DeRose, et al., which applies sharp subdivision rules at some creases for a finite number of steps, and then switches to smooth subdivision, giving more gentle creases. This technique was used in Geri’s Game.

Vertex schemes

In a **vertex scheme**, each vertex begets more vertices. In particular, a vertex surrounded by *n* faces is split into *n* sub-vertices, one for each face:



Doo-Sabin subdivision:



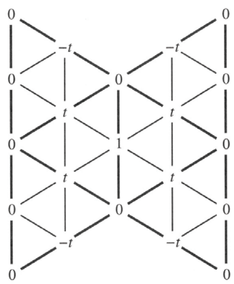
The number edges (faces) incident to a vertex is called its **valence**. Edges with only once incident face are on the **boundary**. After splitting in this subdivision scheme, all non-boundary vertices are of valence 4.

Interpolating subdivision surfaces

Interpolating schemes are defined by

- splitting
- averaging only new vertices

The following averaging mask is used in **butterfly subdivision**:



Setting t=0 gives the original polyhedron, and increasing small values of t makes the surface smoother, until t=1/8 when the surface is provably G^1.