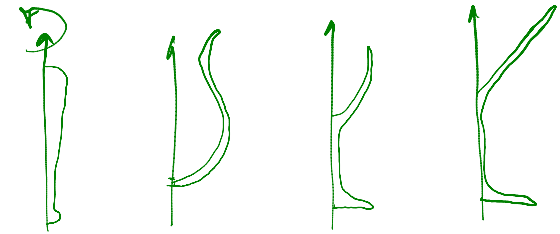


Surfaces of Revolution

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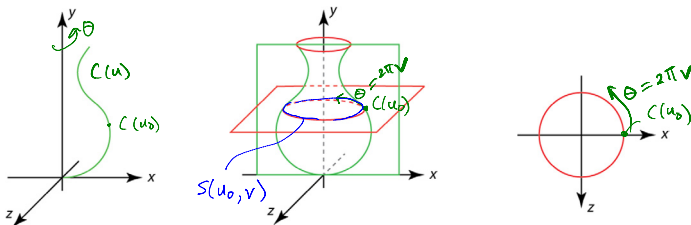
Surfaces of revolution



Idea: rotate a 2D **profile curve** around an axis.

What kinds of shapes can you model this way?

Constructing surfaces of revolution



Given: A curve $C(u)$ in the xy -plane:

$$C(u) = \begin{bmatrix} c_x(u) \\ c_y(u) \\ 0 \\ 1 \end{bmatrix}$$

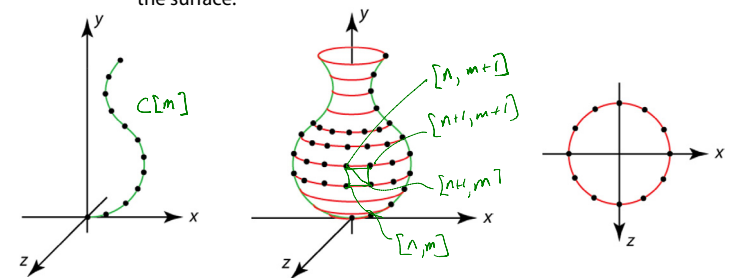
Let $R_y(\theta)$ be a rotation about the y -axis.

Find: A surface $S(u,v)$ which is $C(u)$ rotated about the y -axis, where $u, v \in [0, 1]$.

Solution: $S(u,v) = R_y(2\pi v)C(u)$

Constructing surfaces of revolution

We can sample in u and v to get a grid of points over the surface.



Suppose we sample:

- ♦ in u , to give $C[m]$ where $m \in [0..M-1]$
- ♦ in v , to give rotation angle $\theta[n] = 2\pi n/N$ where $n \in [0..N-1]$

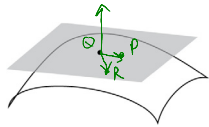
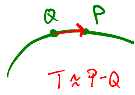
We can now write the surface as:

$$S[n,m] = R_y(2\pi n/N)C[m]$$

How would we turn this into a mesh of triangles?

How do we assign per-vertex normals?

Tangent vectors and tangent planes



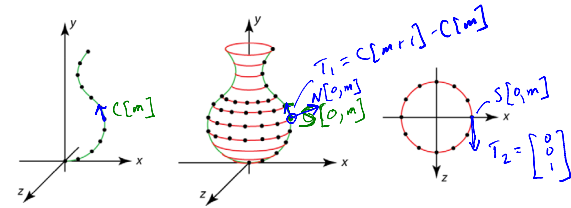
$$T_1 \approx P-Q$$

$$T_2 \approx R-Q$$

$$N = T_2 \times T_1$$

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Normals on a surface of revolution



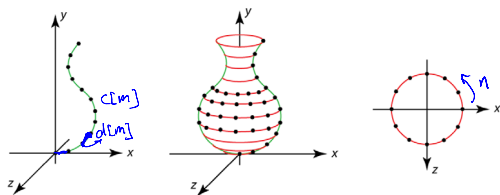
$$T_1 = C[m, r] - C[m]$$

$$\hat{N}[0, m] = \frac{T_1 \times T_2}{\|T_1 \times T_2\|}$$

$$N[n, m] = R\left[\frac{2\pi n}{N}\right] \hat{N}[0, m]$$

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Texture coordinates on a surface of revolution



$$v = \frac{m}{M}$$

$$u = \frac{n}{N}$$

$$v = \frac{\sum_{i=0}^{m-1} d[i]}{\sum_{i=0}^{M-1} d[i]} \text{ arc length}$$

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Triangle meshes

How should we generally represent triangle meshes?

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