Projections

Brian Curless
CSE 557
Fall 2015

Reading

Required:

- Shirley, Ch. 7, Sec. 8.2, Ch. 18

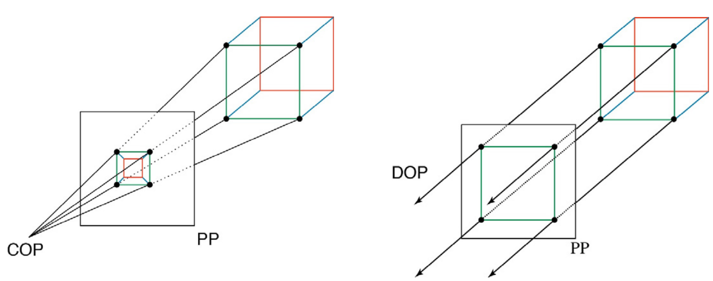
Further reading:

- Foley, et al, Chapter 5.6 and Chapter 6
- David F. Rogers and J. Alan Adams, *Mathematical Elements for Computer Graphics*, 2nd Ed., McGraw-Hill, New York, 1990, Chapter 2.
- I. E. Sutherland, R. F. Sproull, and R. A. Schumacker, A characterization of ten hidden surface algorithms, *ACM Computing Surveys* 6(1): 1-55, March 1974.

Projections

Projections transform points in n-space to m-space, where $m < n$.

In 3-D, we map points from 3-space to the **projection plane** (PP) (a.k.a., image plane) along **projectors** (a.k.a., viewing rays) emanating from the center of projection (COP):

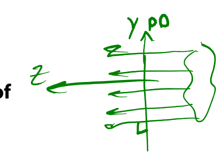


There are two basic types of projections:

- Perspective – distance from COP to PP finite
- Parallel – distance from COP to PP infinite

Parallel projections

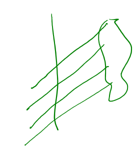
For parallel projections, we specify a **direction of projection** (DOP) instead of a COP.



orthographic

There are two types of parallel projections:

- **Orthographic projection** – DOP perpendicular to PP
- **Oblique projection** – DOP not perpendicular to PP



oblique

We can write orthographic projection onto the z=0 plane with a simple matrix.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Normally, we do not drop the z value right away. Why not?
z - buffering

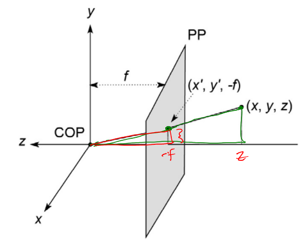
Properties of parallel projection

Properties of parallel projection:

- Not realistic looking
- Good for exact measurements
- Are actually a kind of affine transformation
  - Parallel lines remain parallel
  - Ratios are preserved
  - Angles not (in general) preserved
- Most often used in CAD, architectural drawings, etc., where taking exact measurement is important

Derivation of perspective projection

Consider the projection of a point onto the projection plane:



The distance f is called the **focal length** of the pinhole camera, as it is essentially equivalent to the focal length of a lens system in a camera.

By similar triangles, we can compute how much the x and y coordinates are scaled to get x' and y':

$$\frac{y'}{y} = \frac{f}{-z} \Rightarrow y' = \frac{-f}{z} y \qquad x' = \frac{-f}{z} x$$

Homogeneous coordinates revisited

Remember how we said that affine transformations work with the last coordinate always set to one.

What happens if the coordinate is not one?

We divide all the coordinates by w:

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \rightarrow \begin{bmatrix} x/w \\ y/w \\ z/w \\ 1 \end{bmatrix}$$

If $w = 1$, then nothing changes.

Sometimes we call this division step the "perspective divide."

Homogeneous coordinates and perspective projection

Now we can re-write the perspective projection as a matrix equation:

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/f \end{bmatrix}$$

After division by w, we get:

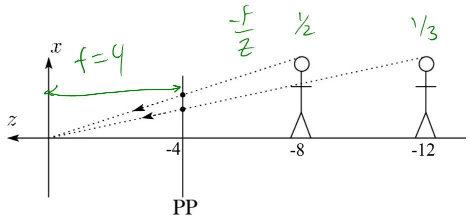
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{f}{z}x \\ -\frac{f}{z}y \\ 1 \end{bmatrix}$$

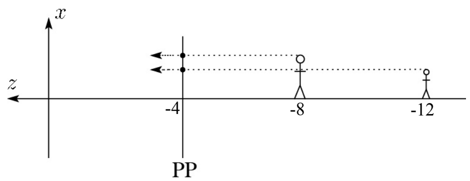
Again, projection implies dropping the z coordinate to give a 2D image, but we usually keep it around a little while longer.

Projective normalization

After applying the perspective transformation and dividing by *w*, we are free to do a simple parallel projection to get the 2D image.

What does this imply about the shape of things after the perspective transformation + divide?

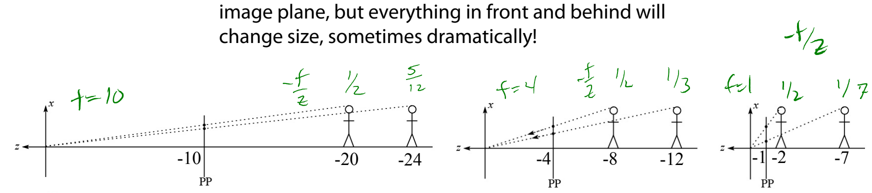


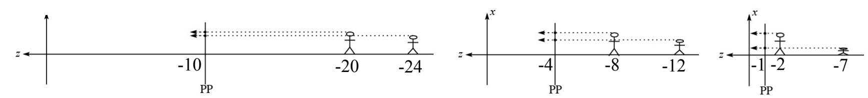


Dolly and zoom

Now imagine starting far away from your subject, with a long focal length ("zoomed in") and then walk toward your subject ("dolly in") while reducing the zoom.

You can keep the subject of constant size on the image plane, but everything in front and behind will change size, sometimes dramatically!

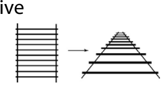




Properties of perspective projections

The perspective projection is an example of a **projective transformation**.

Here are some properties of projective transformations:



- Lines map to lines
- Parallel lines do <u>not</u> necessarily remain parallel
- Ratios are <u>not</u> preserved

One of the advantages of perspective projection is that size varies inversely with distance – looks realistic.

A disadvantage is that we can't judge distances as exactly as we can with parallel projections.