Interactive Control of Rigid Body Motion

Motion

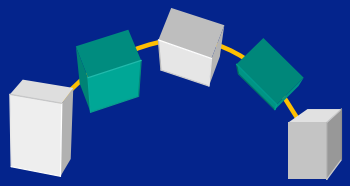
Keyframing

Interpolates motion from "key" positions

Simulation

Solves equations to compute motion

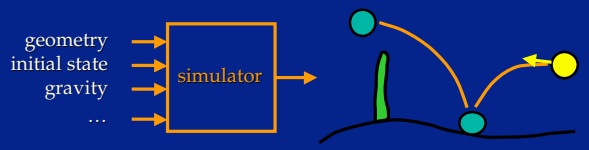
Keyframing



Perfect control over the motion

Realistic look difficult to achieve

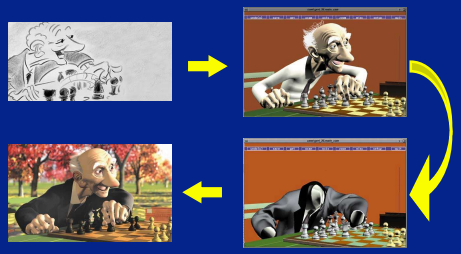
Simulation



Perfect realism of the motion

The motion is difficult to control

Production



Story dictates the motion

Artists must control the behavior

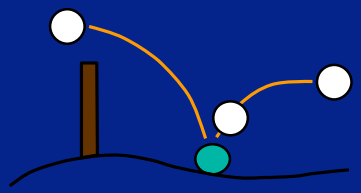
Rigid Body Motion

Interactive Control

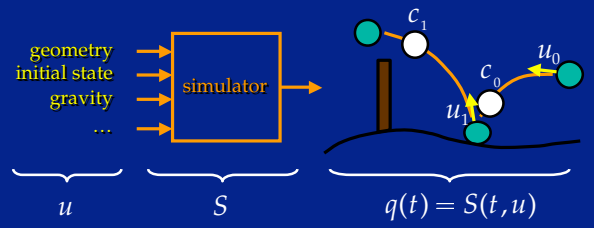
- Extended rigid body dynamics
- Differential control

Problem Statement

Compute a rigid body motion that achieves the desired goals



Problem Statement



Compute control parameters u such that $q(t_i) = c_i$ at times $t_i = t_0, \ldots, t_n$

Main Challenges

The function is nonlinear

- Motion is a solution to nonlinear DEs

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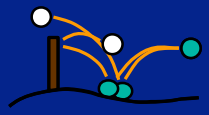

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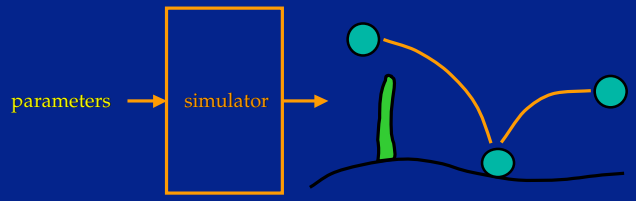
Artist must control the behavior

Extended Dynamics

Plausible dynamics instead of correct



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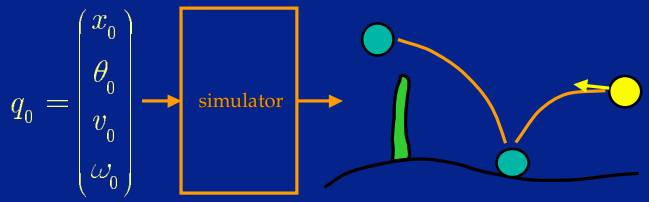

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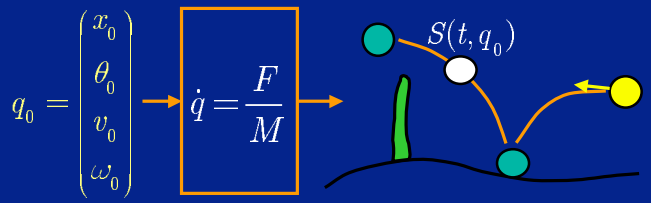

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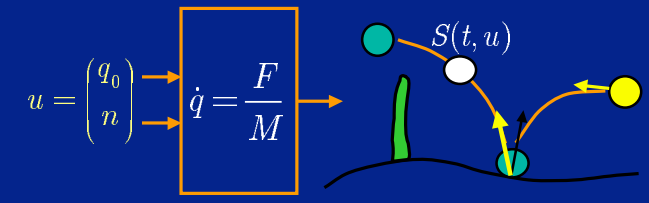

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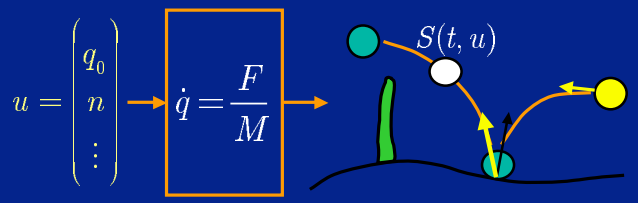
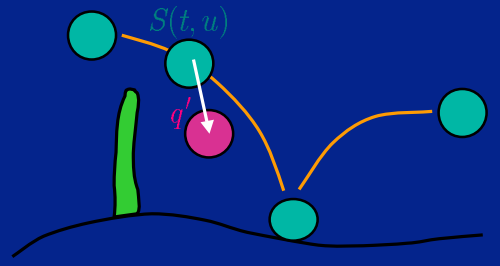
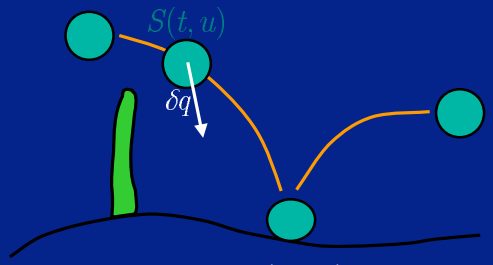


Differential Control



$$q' = S(t,u)$$

Differential Control



$$\delta q = \frac{\partial S(t,u)}{\partial u}\,\delta u$$

V

Interactive Control

1) *Evaluate* δc_i *for current parameters* u
2) *Compute* δu *such that* $\delta c_i = \frac{\partial S(t_i,u)}{\partial u}\,\delta u$
3) *Update parameters* $u' = u + \varepsilon\,\delta u$
4) *Repeat with* $u = u'$

Important Details

1) Evaluate δc_i for current parameters u
2) Compute δu such that $\delta c_i = \frac{\partial S(t_i, u)}{\partial u} \delta u$
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4) Repeat with $u = u'$

Simulation must be fast

Polygonal bodies

Important Details

1) Evaluate δc_i for current parameters u
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Derivative evaluation

- Finite differences are slow and inaccurate
- Specialized automatic differentiation technique

Important Details

1) Evaluate δc_i for current parameters u
2) Compute δu such that $\delta c_i = \frac{\partial S(t_i, u)}{\partial u} \delta u$
3) Update parameters $u' = u + \varepsilon\, \delta u$
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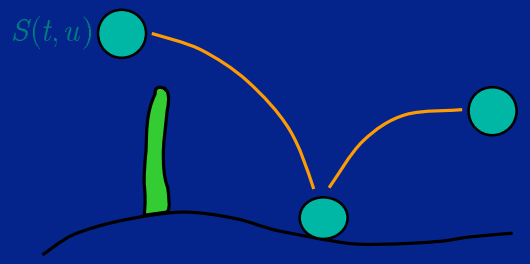
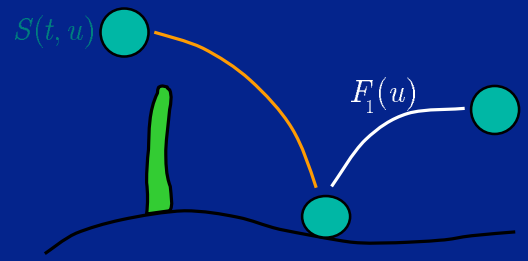
Convergence

- Interaction
- Local sampling

Discrete Domain

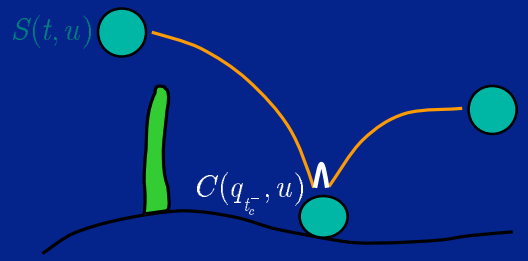


Discrete Domain



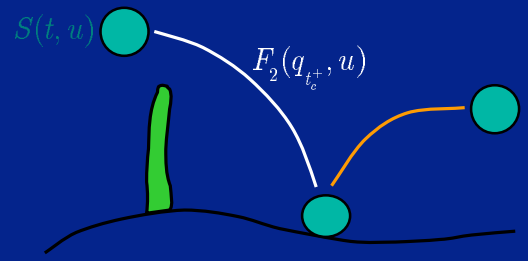
$$q_{t_c^-} = F_1(u)$$

Discrete Domain



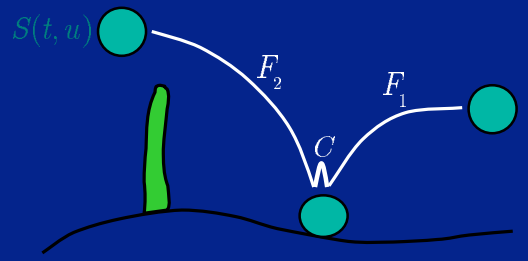
$$q_{t_c^+} = C(F_1(u), u)$$

Discrete Domain



$$S(t,u) = F_2(C(F_1(u), u), u)$$

Discrete Domain

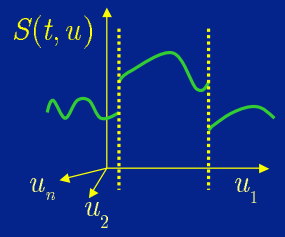


$$S(t,u) = F_2 \circ C \circ F_1$$

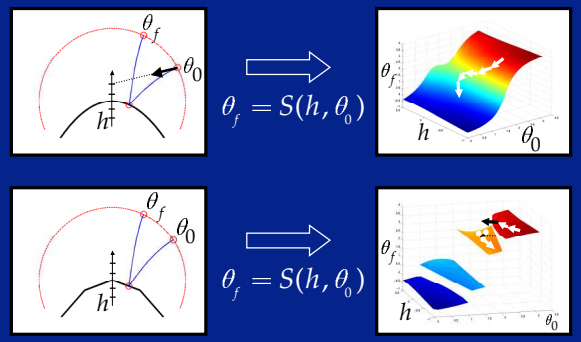
Main Challenge

Differential Control assumes S(t,u) is continuous with respect to u

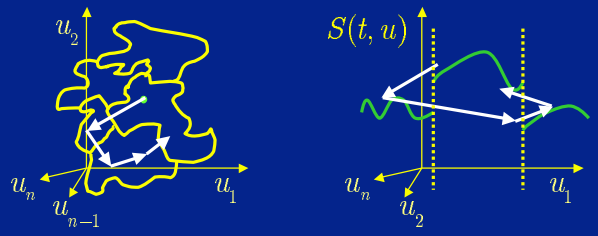
Discrete search/optimization



Convergence



Following Derivatives

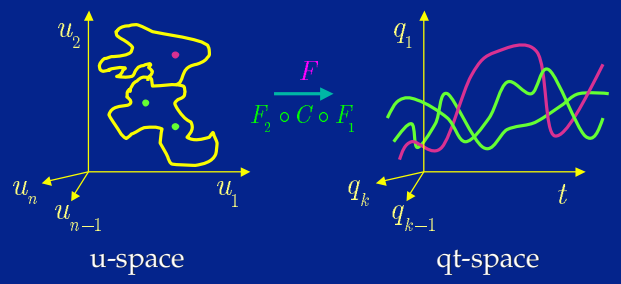


u-space

Following Derivatives

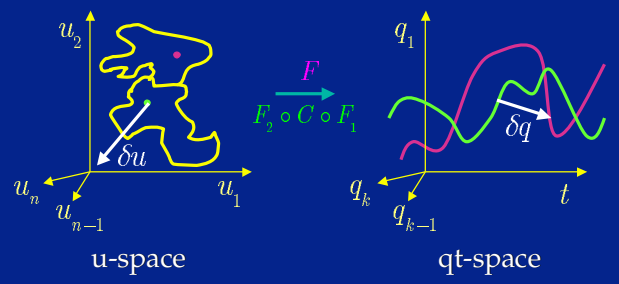
Works well for "local" discrete change

Discrete Domain



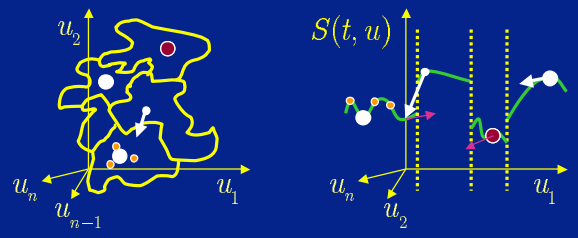
u-space qt-space

Discrete Search/Optimization



u-space qt-space

Sampling



u-space

Summary

Interactive Control

- Continuous optimization
  - *Differential Control*
- Discrete optimization
  - *Sampling*
  - *Randomized Path Planning*