Collision and Contact Basics

Constraints

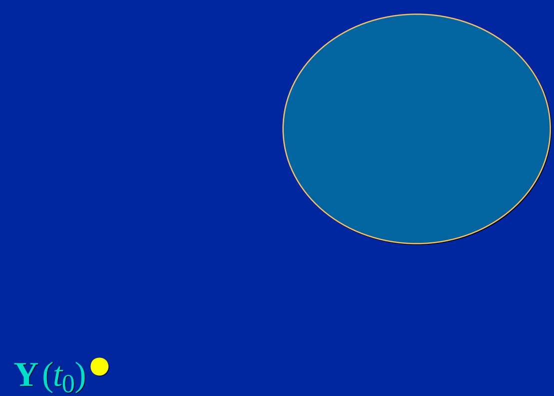
We want rigid bodies to behave as solid objects, and not interpenetrate. By applying constraint forces between contacting bodies, we prevent interpenetration from occurring. We need to:

a) Detect interpenetration

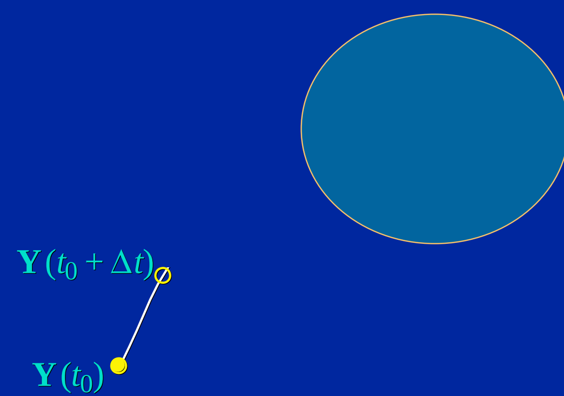
b) Determine contact points

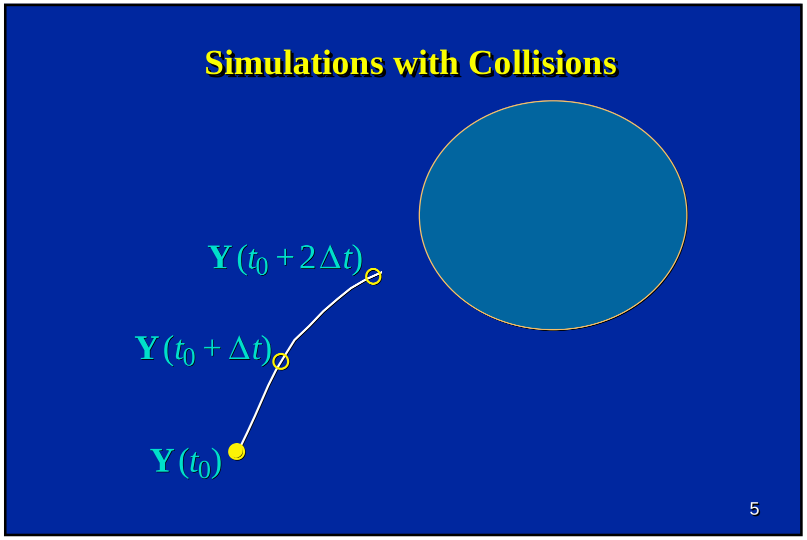
c) Compute constraint forces

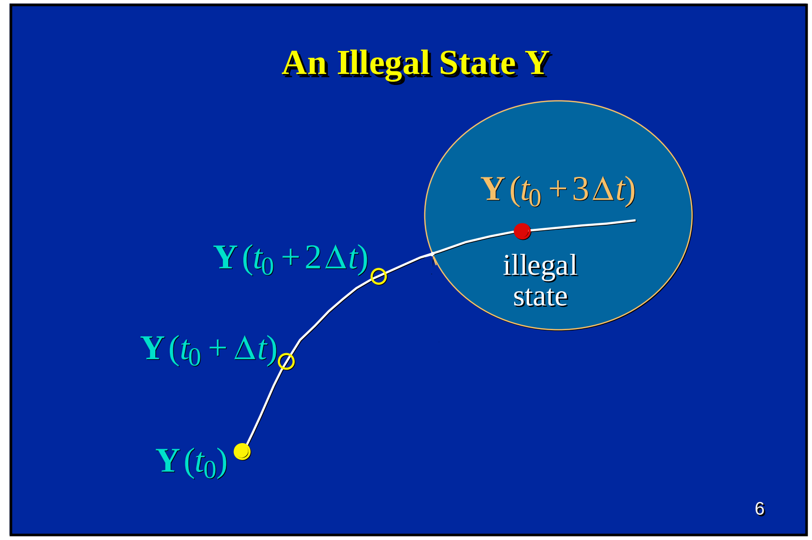
Simulations with Collisions

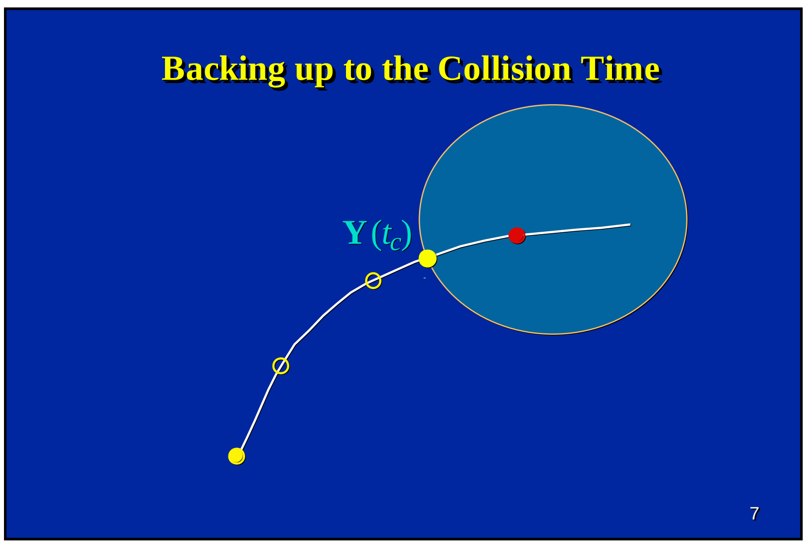


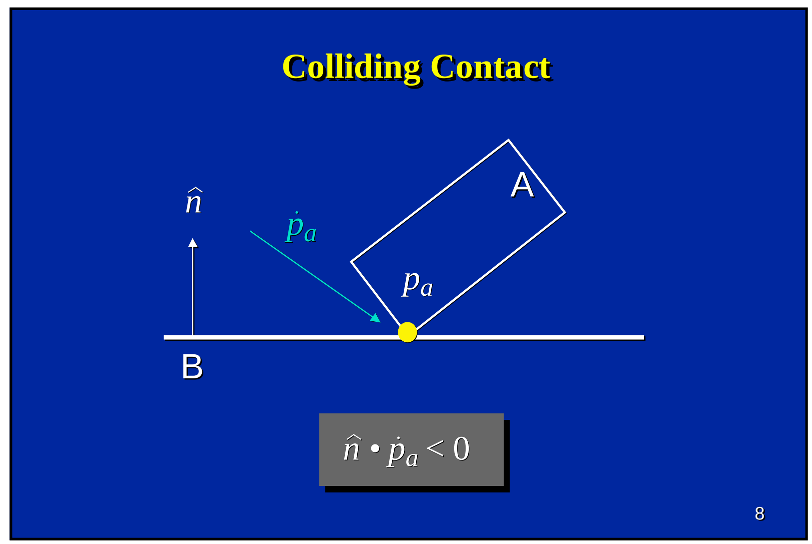
Simulations with Collisions



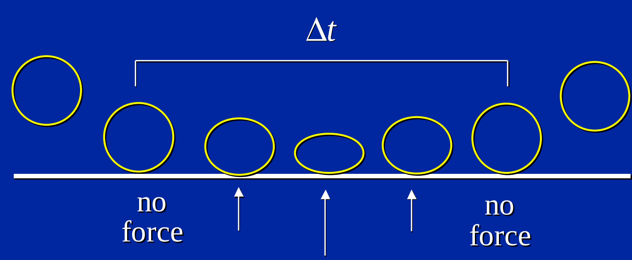

Simulations with Collisions
$\mathbf{Y}(t_0 + 2\Delta t)$
$\mathbf{Y}(t_0 + \Delta t)$
$\mathbf{Y}(t_0)$
5


An Illegal State Y
$\mathbf{Y}(t_0 + 3\Delta t)$
illegal state
$\mathbf{Y}(t_0 + 2\Delta t)$
$\mathbf{Y}(t_0 + \Delta t)$
$\mathbf{Y}(t_0)$
6

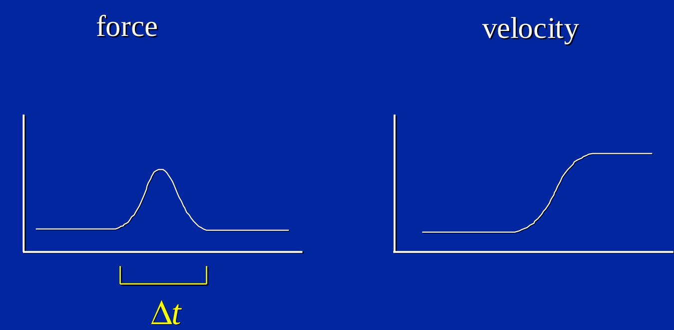

Backing up to the Collision Time
$\mathbf{Y}(t_c)$
7


Colliding Contact
$\hat{n}$
$\dot{p}_a$
A
p_a
B
$\hat{n} \bullet \dot{p}_a < 0$
8

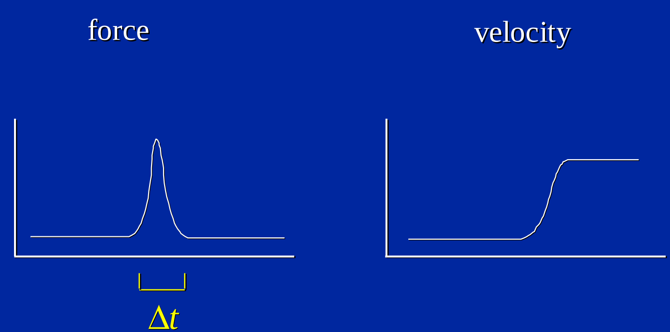
Collision Process



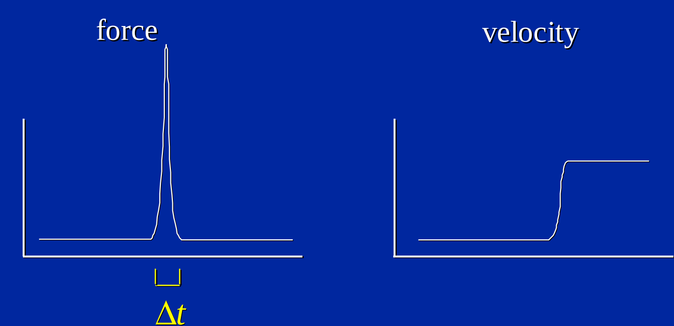
A Soft Collision



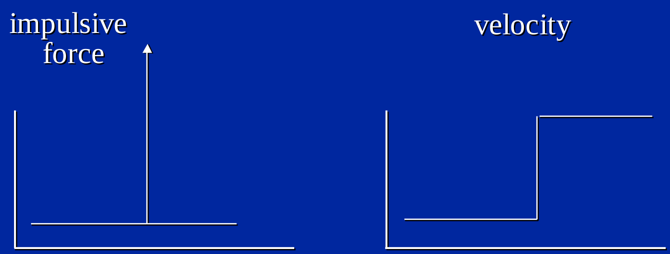
A Harder Collision



A Very Hard Collision



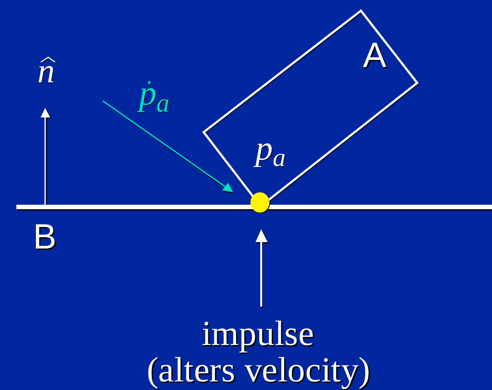
A Rigid Body Collision



$f_{imp} = \infty$

$\Delta t = 0$

Colliding Contact



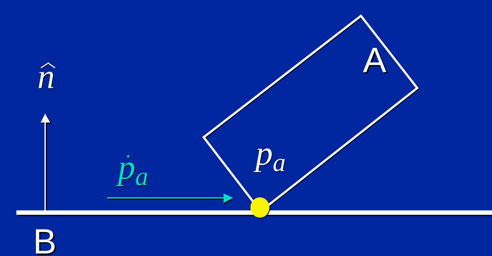
Impulse

$$F\Delta t = J$$

$$\frac{\Delta v}{\Delta t} = \frac{F}{M}$$

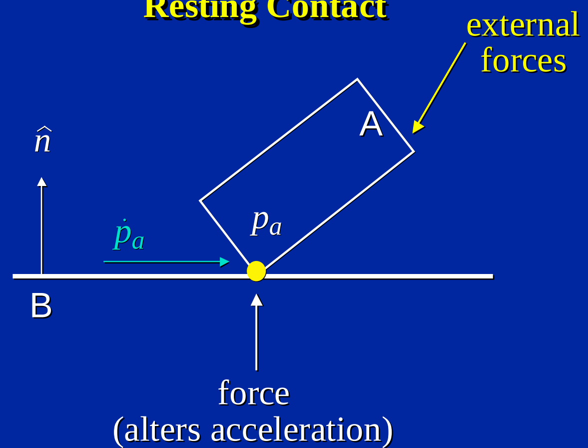
$$\Delta v = \frac{J}{M}$$

Resting Contact

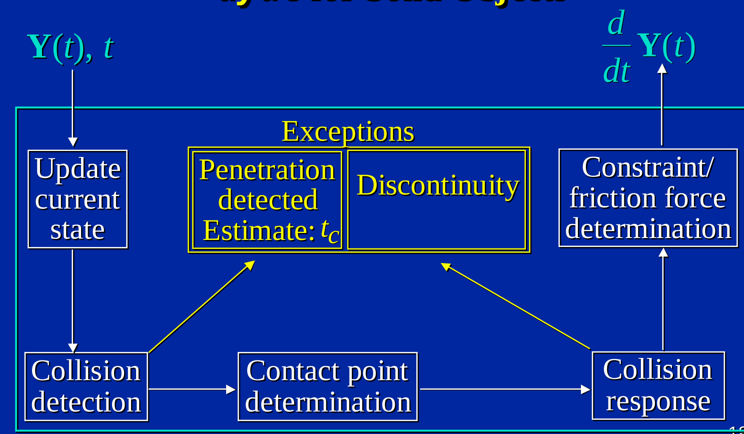


$$\hat{n} \bullet \dot{p}_a = 0$$

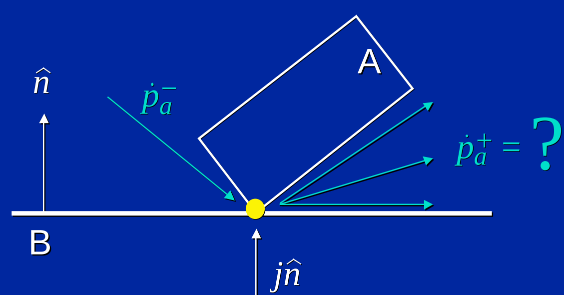
Resting Contact



dydt for Solid Objects

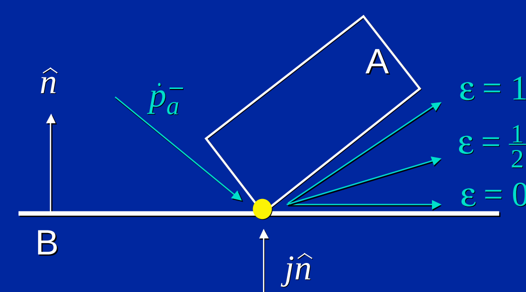


Computing Impulses



Coefficient of Restitution

$$\hat{n} \cdot \dot{p}_a^+ = -\varepsilon(\hat{n} \cdot \dot{p}_a^-)$$



Computing j

$$v_a^+(t_0) = v_a^-(t_0) + \frac{j\hat{n}(t_0)}{M_a}$$

$$\omega_a^+(t_0) = \omega_a^-(t_0) + I_a^{-1}\left(r_a \times j\hat{n}(t_0)\right)$$

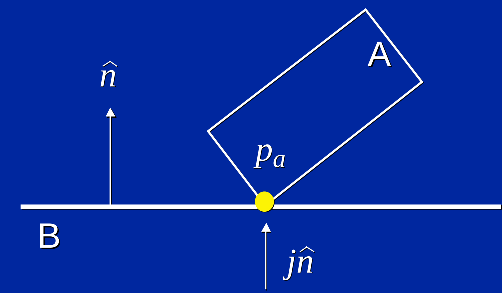
$$\dot{p}_a^+(t_0) = v_a^+(t_0) + \omega_a^+(t_0) \times r_a$$

$$\Downarrow$$

$$\dot{p}_a^+(t_0) = aj + b$$

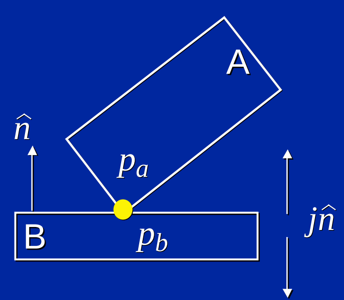
Computing *j*

$$\hat{n} \cdot \dot{p}_a^+ = -\varepsilon\left(\hat{n} \cdot \dot{p}_a^-\right) \longrightarrow cj + b = d$$



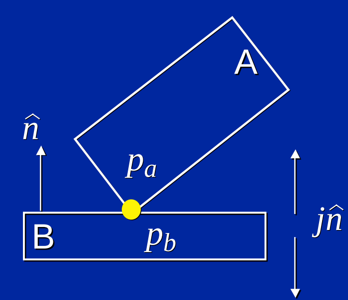
Computing *j*

$$\hat{n} \cdot (\dot{p}_a^+ - \dot{p}_b^+) = -\varepsilon\left(\hat{n} \cdot (\dot{p}_a^- - \dot{p}_b^-)\right)$$

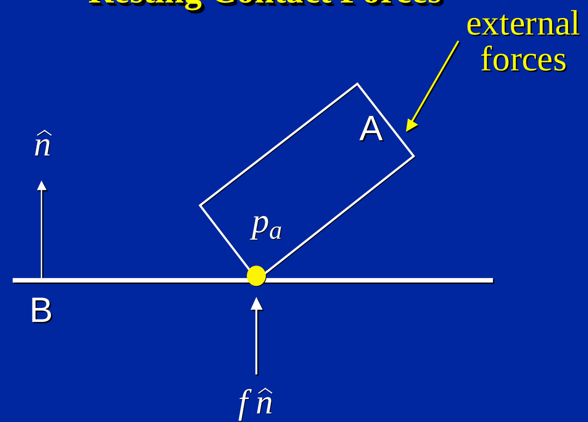


Computing *j*

$$\hat{n} \cdot (\dot{p}_a^+ - \dot{p}_b^+) = -\varepsilon\left(\hat{n} \cdot (\dot{p}_a^- - \dot{p}_b^-)\right) \longrightarrow cj + b = d$$



Resting Contact Forces



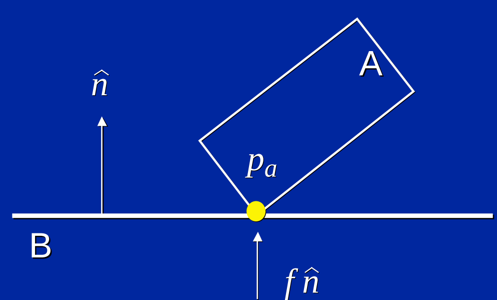
Conditions on the Constraint Force

To avoid inter-penetration, the force strength f must prevent the vertex p_a from accelerating downwards. If B is fixed, this is written as

$$\hat{n} \cdot \ddot{p}_a \geq 0$$

Computing f

$$\hat{n} \cdot \ddot{p}_a \geq 0 \longrightarrow af + b \geq 0$$



Conditions on the Constraint Force

To prevent the constraint force from holding bodies together, the force must be repulsive:

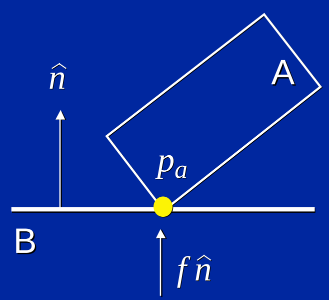
$$f \geq 0$$

Does the above, along with

$$\hat{n} \cdot \ddot{p}_a \geq 0 \longrightarrow af + b \geq 0$$

sufficiently constrain f?

Workless Constraint Force



Either

$$af + b = 0$$
$$f \geq 0$$

or

$$af + b > 0$$
$$f = 0$$

Conditions on the Constraint Force

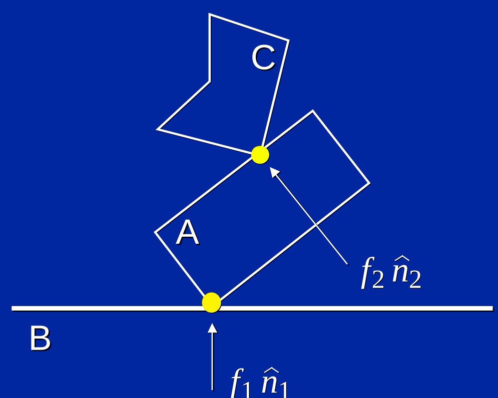
To make f be workless, we use the condition

$$f \cdot (af + b) = 0$$

The full set of conditions is

$$af + b \geq 0$$
$$f \geq 0$$
$$f \cdot (af + b) = 0$$

Multiple Contact Points



Conditions on f_1

Non-penetration:

$$a_{11} f_1 + a_{12} f_2 + b_1 \geq 0$$

Repulsive:

$$f_1 \geq 0$$

Workless:

$$f_1 \cdot (a_{11} f_1 + a_{12} f_2 + b_1) = 0$$

Quadratic Program for f_1 and f_2

Non-penetration:

$$a_{11} f_1 + a_{12} f_2 + b_1 \ge 0$$
$$a_{21} f_1 + a_{22} f_2 + b_2 \ge 0$$

Repulsive:

$$f_1 \ge 0$$
$$f_2 \ge 0$$

Workless:

$$f_1 \cdot (a_{11} f_1 + a_{12} f_2 + b_1) = 0$$
$$f_2 \cdot (a_{21} f_1 + a_{22} f_2 + b_2) = 0$$

In the Notes – Constraint Forces

Derivations of the non-penetration constraints for contacting polyhedra.

Derivations and code for computing the a_{ij} and b_i coefficients.

Code for computing and applying the constraint forces $f_i \hat{n}_i$.

Quadratic Programs with Equality Constraints

Non-penetration:

$$a_{11} f_1 + a_{12} f_2 + b_1 = 0$$
$$a_{21} f_1 + a_{22} f_2 + b_2 \ge 0$$

Repulsive:

~~$f_1 \ge 0$~~

$$f_2 \ge 0$$

Workless:

$$f_1 \cdot (a_{11} f_1 + a_{12} f_2 + b_1) = 0 \quad \text{(free)}$$
$$f_2 \cdot (a_{21} f_1 + a_{22} f_2 + b_2) = 0$$

Collision Detection

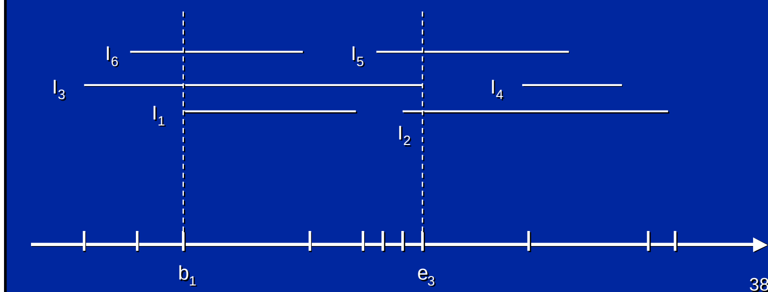
- **O(n^2) nature of the problem**
- **A number of ways to avoid quadratic performance:**
  - **Improve the constant by using bounding boxes**
  - **Use temporal coherence**

Bounding Box

- Axis aligned so intersection test is fast
- But still doing $O(n^2)$ work

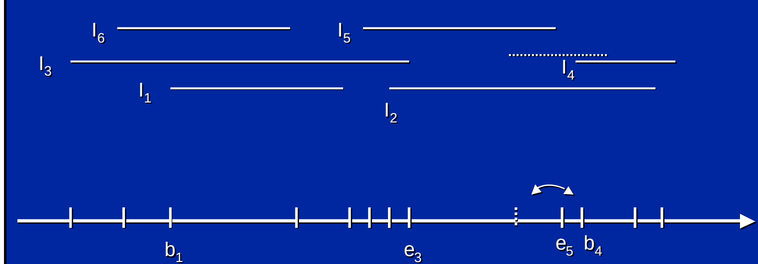
Sort and sweep algorithm

From scratch $O(n \log n + k)$



Sort and sweep algorithm

With coherence $O(n + c)$



Collision Detection of convex polyhedra

- Compute the separating plane
- Use coherence to avoid recomputing the separating plane
- If no separating exists polyhedra are intersecting

Updating the separating plane