

# Data and Image Models

*Maneesh Agrawala*

CS 558: Visualization  
Winter 2005

Lecture adapted from Hanrahan 2004

## The big picture

---

**task**

**data**

physical type  
int, float, etc.  
abstract type  
nominal, ordinal, etc.

**domain**

metadata  
semantics  
conceptual model

**processing  
algorithms**

**mapping**

visual encoding  
visual metaphor

**image**

visual channel  
retinal variables

[based on slide from Munzner]

# Topics

---

Properties of data or information

Properties of the image

Mapping data to images



Jacques Bertin

# Data

## Taxonomy by data type

---

- 1D (sets and sequences)
- Temporal
- 2D (maps)
- 3D (shapes)
- nD (relational)
- Trees (hierarchies)
- Networks (graphs)

Are there others?

The eyes have it: A task by data type taxonomy for information visualization [Schneiderman 96]

## Data models vs. Conceptual models

---

**Data models are mathematical abstractions**

- Sets with operations on them
- Example: integers with + and  $\times$  operators

**Conceptual models are mental constructions**

- Include semantics and support reasoning
- Example: navigating through a city using landmarks

**Examples (data vs. conceptual)**

- 1D vs. Time
- nD vs. Space

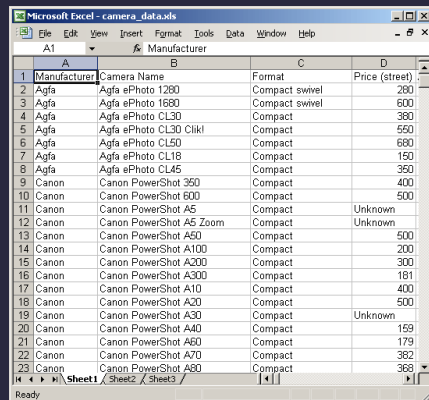
# Relational data model

Records are fixed-length tuples

Each column (attribute) of tuple has a domain (type)

Relation is schema and a table of tuples

Database is a collection of relations



The screenshot shows a Microsoft Excel spreadsheet titled "camera\_data.xls". The spreadsheet contains a table with the following columns: Manufacturer, Camera Name, Format, and Price (street). The data is as follows:

Manufacturer	Camera Name	Format	Price (street)
Agfa	Agfa ePhoto 1280	Compact swivel	280
Agfa	Agfa ePhoto 1680	Compact swivel	600
Agfa	Agfa ePhoto CL30	Compact	380
Agfa	Agfa ePhoto CL30 Click	Compact	560
Agfa	Agfa ePhoto CL50	Compact	680
Agfa	Agfa ePhoto CL16	Compact	150
Agfa	Agfa ePhoto CL45	Compact	350
Canon	Canon PowerShot 350	Compact	400
Canon	Canon PowerShot 600	Compact	500
Canon	Canon PowerShot A5	Compact	Unknown
Canon	Canon PowerShot A5 Zoom	Compact	Unknown
Canon	Canon PowerShot A50	Compact	500
Canon	Canon PowerShot A100	Compact	200
Canon	Canon PowerShot A200	Compact	300
Canon	Canon PowerShot A300	Compact	181
Canon	Canon PowerShot A10	Compact	400
Canon	Canon PowerShot A30	Compact	500
Canon	Canon PowerShot A30	Compact	Unknown
Canon	Canon PowerShot A40	Compact	159
Canon	Canon PowerShot A60	Compact	179
Canon	Canon PowerShot A70	Compact	382
Canon	Canon PowerShot A80	Compact	368

Example: Digital Cameras

# Relational algebra [Codd]

Data transformations (SQL)

- Selection (SELECT)
- Projection (WHERE)
- Sorting (ORDER BY)
- Aggregation (GROUP BY, SUM, MIN, ...)
- Set operations (UNION, ...)
- Join (INNER JOIN)

# Statistical data model

Variables or measurements (~ float attribute)

Categories or factors (~ int attribute)

Observations or cases (~ record)

ID	Case	Species_No	Species	Petal_width	Petal_length	Sepal_width	Sepal_length
1	1	1	1 I. Setosa	2	14	33	50
2	2	1	3 I. Versicolour	24	56	31	67
3	3	1	2 I. Versicolour	13	45	28	57
4	4	2	1 I. Setosa	2	10	36	46
5	5	2	3 I. Versicolour	23	51	31	69
6	6	2	2 I. Versicolour	16	47	33	63
7	7	3	1 I. Setosa	2	16	31	48
8	8	3	3 I. Versicolour	20	52	30	65
9	9	3	2 I. Versicolour	14	47	32	70
10	10	4	1 I. Setosa	1	14	36	49
11	11	4	3 I. Versicolour	19	51	27	58
12	12	4	2 I. Versicolour	12	40	26	58
13	13	5	1 I. Setosa	2	13	32	44
14	14	5	3 I. Versicolour	17	45	25	49
15	15	5	2 I. Versicolour	10	33	23	50
16	16	6	1 I. Setosa	2	16	38	51
17	17	6	3 I. Versicolour	19	50	25	63
18	18	6	2 I. Versicolour	10	41	27	58
19	19	7	1 I. Setosa	2	16	30	50
20	20	7	3 I. Versicolour	18	49	27	63
21	21	7	2 I. Versicolour	15	45	29	60
22	22	8	1 I. Setosa	4	19	38	51
23	23	8	3 I. Versicolour	21	56	28	64
24	24	8	2 I. Versicolour	10	33	24	49
25	25	9	1 I. Setosa	2	14	30	49
26	26	9	3 I. Versicolour	19	51	27	58
27	27	9	2 I. Versicolour	14	39	27	52
28	28	10	1 I. Setosa	2	14	36	50
29	29	10	3 I. Versicolour	18	55	31	64
30	30	10	2 I. Versicolour	12	39	27	58
31	31	11	1 I. Setosa	4	15	34	54

Sepal and petal lengths and widths for three species of iris [Fisher].

Microsoft Excel - fischer.iris.2.xls

File Edit View Insert Format Tools Data Window Help

Type a question for help

ID	Case	Species	No	Species	Organ	Width	Length
1	1	1	I.	Setosa	Petal	2	14
2	2	1	I.	Setosa	Petal	24	56
3	3	1	I.	Setosa	Petal	13	45
4	4	1	I.	Setosa	Sepal	33	50
5	5	1	I.	Setosa	Sepal	31	67
6	6	1	I.	Setosa	Sepal	28	57
7	7	2	I.	Setosa	Petal	2	10
8	8	2	I.	Setosa	Petal	23	51
9	9	2	I.	Setosa	Petal	16	47
10	10	2	I.	Setosa	Sepal	36	46
11	11	2	I.	Setosa	Sepal	31	69
12	12	2	I.	Setosa	Sepal	33	63
13	13	3	I.	Setosa	Petal	2	16
14	14	3	I.	Setosa	Petal	20	52
15	15	3	I.	Setosa	Petal	14	47
16	16	3	I.	Setosa	Sepal	31	48
17	17	3	I.	Setosa	Sepal	30	65
18	18	3	I.	Setosa	Sepal	32	70
19	19	4	I.	Setosa	Petal	1	14
20	20	4	I.	Setosa	Petal	19	51
21	21	4	I.	Setosa	Petal	12	40
22	22	4	I.	Setosa	Sepal	36	49
23	23	4	I.	Setosa	Sepal	27	58
24	24	4	I.	Setosa	Sepal	26	58
25	25	5	I.	Setosa	Petal	2	13
26	26	5	I.	Setosa	Petal	17	45
27	27	5	I.	Setosa	Petal	10	33
28	28	5	I.	Setosa	Sepal	32	44
29	29	5	I.	Setosa	Sepal	25	49
30	30	5	I.	Setosa	Sepal	23	50
31	31	6	I.	Setosa	Petal	2	16

fischer.iris

Ready

Sepal and petal lengths and widths for three species of iris [Fisher].

Microsoft Excel - fischer.iris.3.xls

File Edit View Insert Format Tools Data Window Help

Type a question for help

ID	Case	Species	Organ	Measure	Value
1	1	1	I.	Setosa	petal width 2
2	2	1	I.	Setosa	petal width 24
3	3	1	I.	Setosa	petal width 13
4	4	1	I.	Setosa	petal length 14
5	5	1	I.	Setosa	petal length 56
6	6	1	I.	Setosa	petal length 45
7	7	2	I.	Setosa	sepal width 33
8	8	2	I.	Setosa	sepal width 31
9	9	2	I.	Setosa	sepal width 28
10	10	2	I.	Setosa	sepal length 50
11	11	2	I.	Setosa	sepal length 67
12	12	2	I.	Setosa	sepal length 57
13	13	3	I.	Setosa	petal width 2
14	14	3	I.	Setosa	petal width 23
15	15	3	I.	Setosa	petal width 16
16	16	3	I.	Setosa	petal length 10
17	17	3	I.	Setosa	petal length 51
18	18	3	I.	Setosa	petal length 47
19	19	3	I.	Setosa	sepal width 36
20	20	3	I.	Setosa	sepal width 31
21	21	3	I.	Setosa	sepal width 33
22	22	3	I.	Setosa	sepal length 46
23	23	3	I.	Setosa	sepal length 69
24	24	3	I.	Setosa	sepal length 63
25	25	5	I.	Setosa	petal width 2
26	26	5	I.	Setosa	petal width 20
27	27	5	I.	Setosa	petal width 14
28	28	5	I.	Setosa	petal length 16
29	29	5	I.	Setosa	petal length 52
30	30	5	I.	Setosa	petal length 47
31	31	6	I.	Setosa	sepal width 31

Sheet1

Ready

Sepal and petal lengths and widths for three species of iris [Fisher].

	I. Setosa				I. Virginica				I. Versicolor			
	petal		sepal		petal		sepal		petal		sepal	
	length	width	length	width	length	width	length	width	length	width	length	width
1	14	2	50	33	56	24	67	31	45	13	57	28
2	10	2	46	36	51	23	69	31	47	16	63	33
3	16	2	48	31	52	20	65	30	47	14	70	32
4	14	1	49	36	51	19	58	27	40	12	58	26
5	13	2	44	32	45	17	49	25	33	10	50	23
6	16	2	51	38	50	19	63	25	41	10	58	27
7	16	2	50	30	49	18	63	27	45	15	60	29
8	19	4	51	38	56	21	64	28	33	10	49	24
9	14	2	49	30	51	19	58	27	39	14	52	27
10	14	2	50	36	55	18	64	31	39	12	58	27
11	15	4	54	34	50	15	60	22	42	15	59	30
12	14	2	55	42	57	23	69	32	44	13	63	23
13	14	2	44	29	49	20	56	28	49	15	63	25
14	14	1	48	30	58	18	67	25	30	11	51	25
15	17	3	57	38	54	21	69	31	36	13	56	29
16	15	4	51	37	61	25	72	36	44	14	66	30
17	13	2	55	35	55	21	68	30	50	17	67	30
18	13	2	44	30	56	22	64	28	45	15	62	22
19	16	2	47	32	51	15	63	28	46	14	61	30
20	12	2	50	32	59	23	68	32	39	11	56	25
21	11	1	43	30	54	22	62	34	45	15	64	22

Format of the data in Appendix 14, pp. 365-366

Chambers, Cleveland, Kleiner, Tukey, *Graphical Methods for Data Analysis*

## Types

### Physical types

- Characterized by storage
- Characterized by machine operations

Example:

bool, short, int32, float, double, string, ...

### Abstract types

- Characterized by methods/attributes
- Organized into a class hierarchy

Example:

nominal, ordinal, cardinal, ...,  
plants, animals, metazoans, ...

# Measurements

## N - Nominal (labels)

- Fruits: Apples, oranges, ...

## O - Ordered

- Days: Mon, Tue, Wed, Thu, Fri, Sat, Sun
- Quality of meat: Grade A, AA, AAA

## Q - Interval (Location of zero arbitrary)

- Periods of time: second, minute, ...

## Q - Ratio (zero fixed)

- Counts
- Physical measurement: Kelvin, ...

S. S. Stevens, On the theory of scales of measurements, 1946

Microsoft Excel - fischer.iris.2.colored.xls

	A	B	C	D	E	F	G	H	I	J
1	ID	Case	Species_No	Species	Organ	Width	Length			
2	1	1	1	I. Setosa	Petal	2	14			
3	2	1	3	I. Versicolour	Petal	24	56			
4	3	1	2	I. Versicolour	Petal	13	45			
5	4	1	1	I. Setosa	Sepal	33	50			
6	5	1	3	I. Versicolour	Sepal	31	67			
7	6	1	2	I. Versicolour	Sepal	28	57			
8	7	2	1	I. Setosa	Petal	2	10			
9	8	2	3	I. Versicolour	Petal	23	51			
10	9	2	2	I. Versicolour	Petal	16	47			
11	10	2	1	I. Setosa	Sepal	36	46			
12	11	2	3	I. Versicolour	Sepal	31	69			
13	12	2	2	I. Versicolour	Sepal	33	63			
14	13	3	1	I. Setosa	Petal	2	16			
15	14	3	3	I. Versicolour	Petal	20	52			
16	15	3	2	I. Versicolour	Petal	14	47			
17	16	3	1	I. Setosa	Sepal	31	48			
18	17	3	3	I. Versicolour	Sepal	30	65			
19	18	3	2	I. Versicolour	Sepal	32	70			
20	19	4	1	I. Setosa	Petal	1	14			
21	20	4	3	I. Versicolour	Petal	19	51			
22	21	4	2	I. Versicolour	Petal	12	40			
23	22	4	1	I. Setosa	Sepal	36	49			
24	23	4	3	I. Versicolour	Sepal	27	58			
25	24	4	2	I. Versicolour	Sepal	26	58			
26	25	5	1	I. Setosa	Petal	2	13			
27	26	5	3	I. Versicolour	Petal	17	45			
28	27	5	2	I. Versicolour	Petal	10	33			
29	28	5	1	I. Setosa	Sepal	32	44			
30	29	5	3	I. Versicolour	Sepal	25	49			
31	30	5	2	I. Versicolour	Sepal	23	50			
32	31	6	1	I. Setosa	Petal	2	16			

Legend: Q (red), O (green), N (blue)



# Dimensions and measures

---

## Independent vs. dependent variables

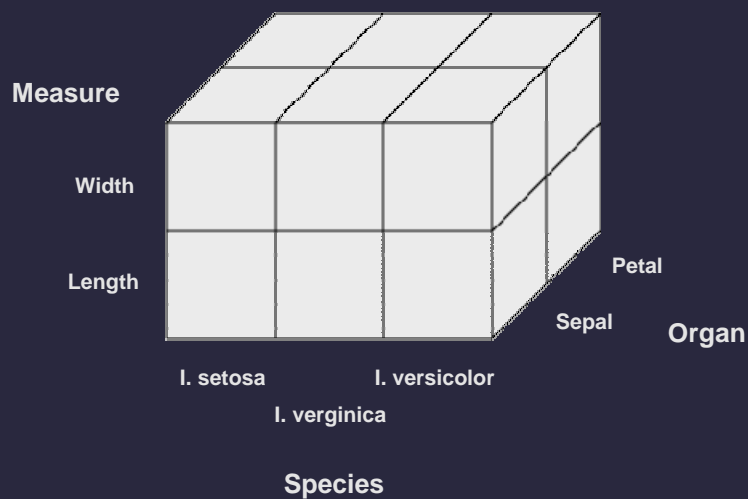
- Example:  $y = f(x,a)$
- Dimensions:  $\text{Domain}(x) \times \text{Domain}(a)$
- Measures:  $\text{Range}(y)$

## Common techniques for summarizing data

- GroupBy dimensions
- Aggregate measures

# Data cube

---



## Summary of basic properties

---

- **Multidimensional**
  - Number of columns (dimensions)
- **Type**
  - Nominal, ordinal, quantitative
- **Cardinality (levels)**
  - Number of values possible within a column

**Image**

# Visual language is a sign system

Image is perceived as a set of signs

Sender encodes information in these signs

Receiver decodes information from these signs

## Visual variables

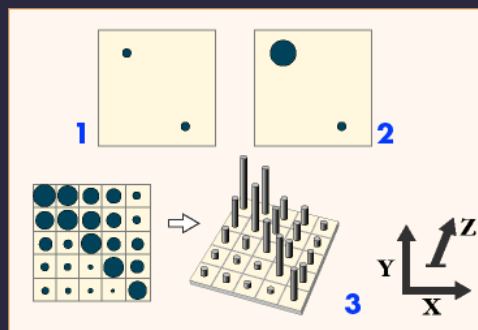
J. Bertin, Semiology of Graphics, 1967

[x,y]

- Position

[z]

- Size
- Value
- Color
- Texture
- Orientation
- Shape



Note: Bertin does not consider 3D or time

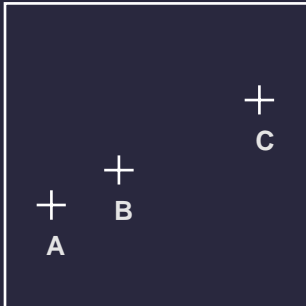
Note: Card and Mackinlay extend the number of vars.

**LES VARIABLES DE L'IMAGE**

	POINTS			LIGNES			ZONES	
XY 2 DIMENSIONS DU PLAN	x	x	x	/	~	/	14-15-9	2-18-2
Z TAILLE	█	█	█	/	~	/	10-21-2	1-21-1
VALEUR	█	█	█	/	~	/	█	█
<b>LES VARIABLES DE SÉPARATION DES IMAGES</b>								
GRAIN	█	█	█	/	~	/	█	█
COULEUR	█	█	█	/	~	/	█	█
ORIENTATION	█	█	█	/	~	/	█	█
FORME	█	█	█	/	~	/	█	█

[Bertin, Semiology of Graphics, 1983]

## Information in position



1. A, B, C are distinguishable
2. B is between A and C.
3. BC is twice as long as AB.

"Resemblance, order and proportional are the three signfields in graphics." - Bertin

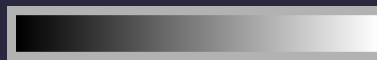
## Information in color and value

Value is perceived as ordered

∴ Encode ordinal variables (O)



∴ Encode continuous variables (Q) [not as well]



Hue is normally perceived as unordered

∴ Encode nominal variables (N) using color



## Bertins' "Levels of Organization"

Position	N	O	Q	N Nominal O Ordered Q Quantitative  Note: $Q < O < N$
Size	N	O	Q	
Value	N	O	q	
Texture	N	o		Note: Bertin actually breaks visual variables down into differentiating ( $\neq$ ) and associating ( $\equiv$ )
Color	N			
Orientation	N			
Shape	N			

# Encoding rules

## Depicting information

---

2 variables can be mapped directly to space

- Graphs, maps, ...

Other variables must be a visual variables

- Color, size, shape, ...

What about data sets with many variables?

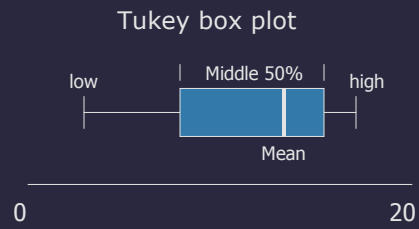
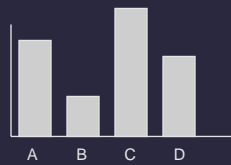
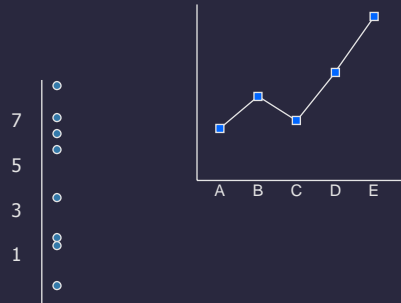
- How many variables may be shown?
- What are the best ways to show those variables?

[Exclude 3D and T for now ...]

# Univariate data

cases			
	A B C		
1			

variable

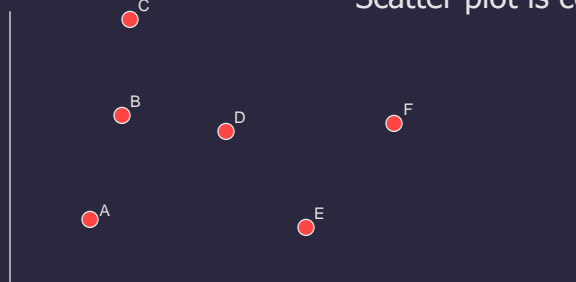


[based on slide from Stasko]

# Bivariate data

cases			
	A B C		
1			
2			

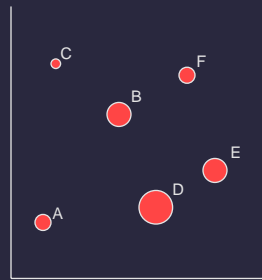
Scatter plot is common



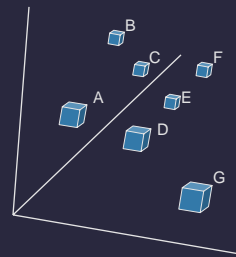
[based on slide from Stasko]

# Trivariate data

	A	B	C
1			
2			
3			

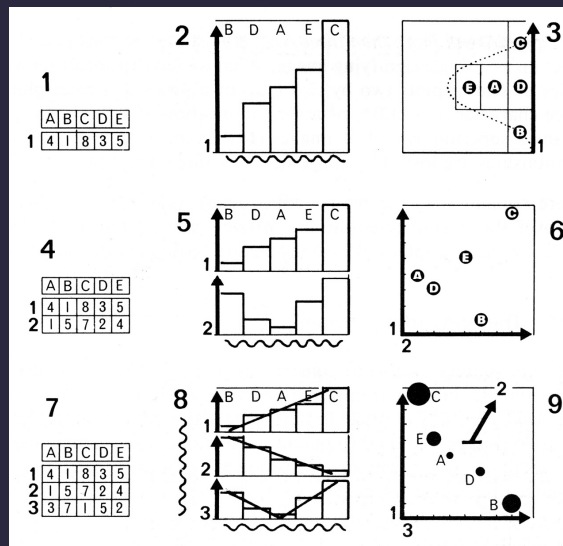


3D scatter plot is possible



[based on slide from Stasko]

# Large design space (visual metaphors)



[Bertin, Graphics and Graphic Info. Processing, 1981]



# Multidimensional data

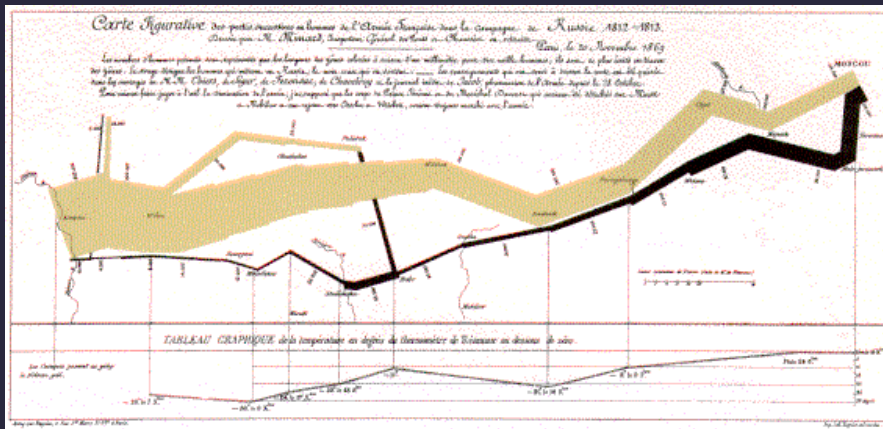
*“With up to three rows, a data table can be constructed directly as a single image ... However, an image has only three dimensions. And this barrier is impassible.”*

Bertin

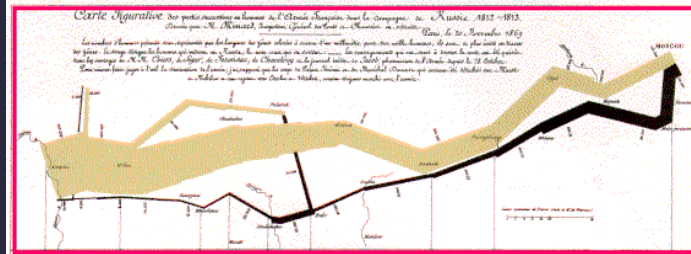
	A	B	C
1			
2			
3			
4			
5			
6			
7			
8			

# Composition/Decomposition

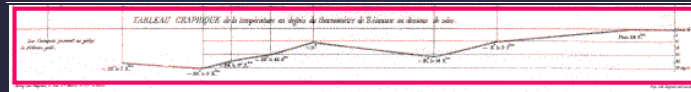
## Minard's 1869 Napoleon's march



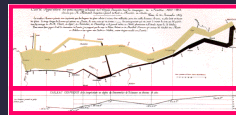
# Single axis composition



+



=



[based on slide from Mackinlay]

# Mark composition

temperature

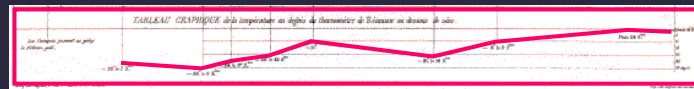
+

time

+

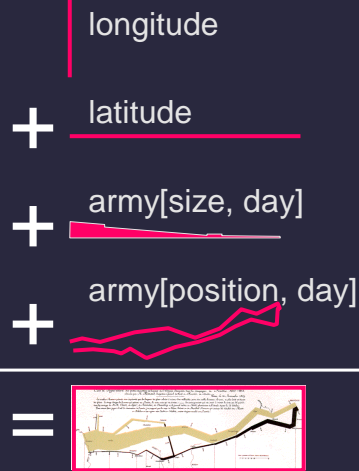
temp[day]

=

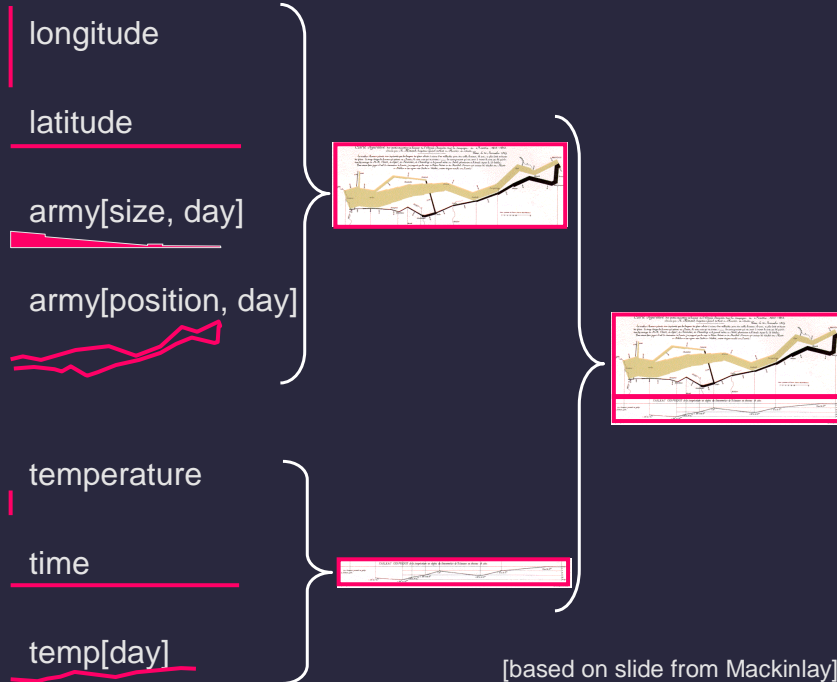


[based on slide from Mackinlay]

# Mark composition



[based on slide from Mackinlay]



[based on slide from Mackinlay]

# Automated design

J. Mackinlay's APT 86

## Combinatorics of encodings

---

### Challenge:

Pick the best encoding from the exponential number of possibilities  $(n+1)^8$

### Principle of Consistency:

The properties of the image should match the properties of the data.

### Principle of Importance Ordering:

Encode the most important information in the most effective way.

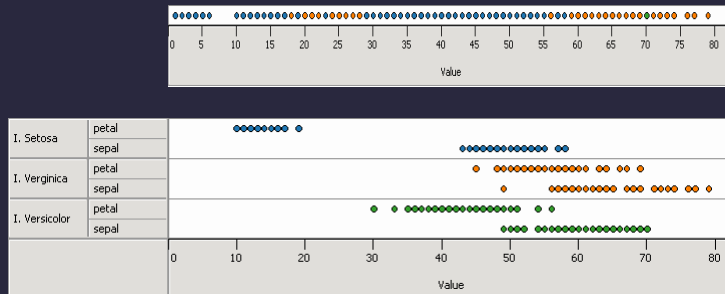
# Mackinlay's expressiveness criteria

## Expressiveness

A set of facts is expressible in a visual language if the sentences (i.e. the visualizations) in the language express all the facts in the set of data, and only the facts in the data.

## Cannot express the facts

A one-to-many (1 → N) relation cannot be expressed in a single horizontal dot plot because multiple tuples are mapped to the same position



## Expresses facts not in the data

A length is interpreted as a quantitative value;  
∴ Length of bar says something untrue about N data

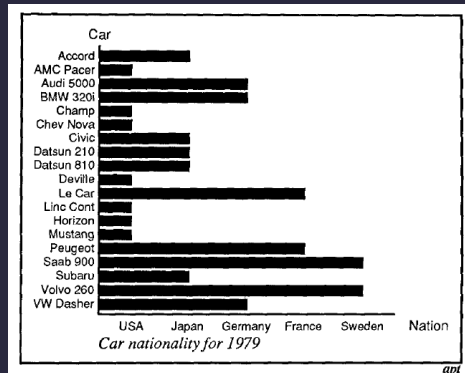


Fig. 11. Incorrect use of a bar chart for the *Nation* relation. The lengths of the bars suggest an ordering on the vertical axis, as if the USA cars were longer or better than the other cars, which is not true for the *Nation* relation.

[Mackinlay, APT, 1986]

## Mackinlay's effectiveness criteria

### Effectiveness

A visualization is more effective than another visualization if the information conveyed by one visualization is more readily perceived than the information in the other visualization.

Subject of next lecture

## Mackinlay ranking

---

QUANTITATIVE	ORDINAL	NOMINAL
Position	Position	Position
Length	Density (Val)	Color Hue
Angle	Color Sat	Texture
Slope	Color Hue	Connection
Area (Size)	Texture	Containment
Volume	Connection	Density (Val)
Density (Val)	Containment	Color Sat
Color Sat	Length	Shape
Color Hue	Angle	Length
Texture	Slope	Angle
Connection	Area (Size)	Slope
Containment	Volume	Area
Shape	Shape	Volume

Conjectured *effectiveness* of the encoding

## Mackinlay's design algorithm

---

- User formally specifies data model
- APT searches over design space
  - Tests expressiveness of each visual encoding
  - Generates image for encodings that pass test
  - Tests perceptual effectiveness of resulting image
- Outputs most effective visualization

## Summary

---

- **Formal specification**
  - Data model
  - Image model
  - Encodings mapping data to image
- **Choose expressive and effective encodings**
  - Formal test of expressiveness
  - Experimental tests of perceptual effectiveness