

# CSE-571 Robotics

## Bayes Filters

## Bayes Filters: Framework

- **Given:**
  - Stream of observations  $z$  and action data  $u$ :  

$$d_t = \{u_1, z_2 \dots, u_{t-1}, z_t\}$$
  - Sensor model  $P(z|x)$ .
  - Action model  $P(x|u, x')$ .
  - Prior probability of the system state  $P(x)$ .
- **Wanted:**
  - Estimate of the state  $X$  of a dynamical system.
  - The posterior of the state is also called **Belief**:

$$Bel(x_t) = P(x_t | u_1, z_2 \dots, u_{t-1}, z_t)$$

## Bayes Filters

$z$  = observation  
 $u$  = action  
 $x$  = state

$$Bel(x_t) = P(x_t | u_1, z_1 \dots, u_t, z_t)$$

**Bayes**  $= \eta P(z_t | x_t, u_1, z_1, \dots, u_t) P(x_t | u_1, z_1, \dots, u_t)$

**Markov**  $= \eta P(z_t | x_t) P(x_t | u_1, z_1, \dots, u_t)$

**Total prob.**  
 $= \eta P(z_t | x_t) \int P(x_t | u_1, z_1, \dots, u_t, x_{t-1}) P(x_{t-1} | u_1, z_1, \dots, u_t) dx_{t-1}$

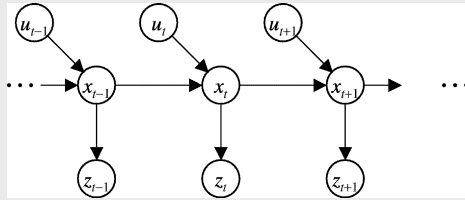
**Markov**  $= \eta P(z_t | x_t) \int P(x_t | u_t, x_{t-1}) P(x_{t-1} | u_1, z_1, \dots, u_t) dx_{t-1}$

$$= \eta P(z_t | x_t) \int P(x_t | u_t, x_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

$$Bel(x_t) = \eta P(z_t | x_t) \int P(x_t | u_t, x_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

1. Algorithm **Bayes\_filter**(  $Bel(x), d$  ):
2.  $n=0$
3. If  $d$  is a **perceptual** data item  $z$  then
4. For all  $x$  do
5.  $Bel'(x) = P(z | x) Bel(x)$
6.  $\eta = \eta + Bel'(x)$
7. For all  $x$  do
8.  $Bel'(x) = \eta^{-1} Bel'(x)$
9. Else if  $d$  is an **action** data item  $u$  then
10. For all  $x$  do
11.  $Bel'(x) = \int P(x | u, x') Bel(x') dx'$
12. Return  $Bel'(x)$

## Markov Assumption



$$p(z_t | x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t | x_t)$$

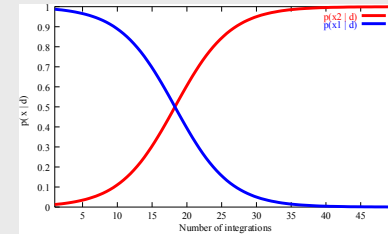
$$p(x_t | x_{1:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t | x_{t-1}, u_t)$$

### Underlying Assumptions

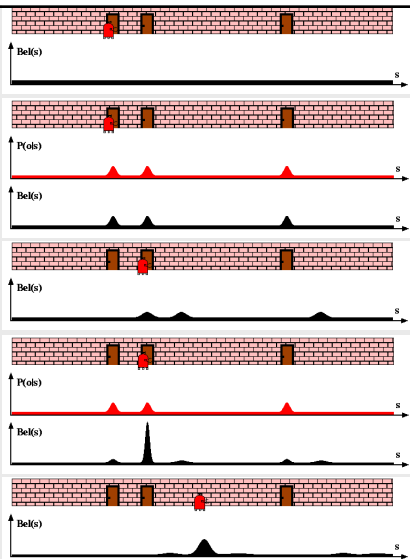
- Static world
- Independent noise
- Perfect model, no approximation errors

## Dynamic Environments

- Two possible locations  $x_1$  and  $x_2$
- $P(x_1) = 0.99$
- $P(z|x_2) = 0.09$   $P(z|x_1) = 0.07$



## Bayes Filters for Robot Localization



## Representations for Bayesian Robot Localization

### Discrete approaches ('95)

- Topological representation ('95)
  - uncertainty handling (POMDPs)
  - occas. global localization, recovery
- Grid-based, metric representation ('96)
  - global localization, recovery

### Particle filters ('99)

- sample-based representation
- global localization, recovery

### Kalman filters (late-80s)

- Gaussians, unimodal
- approximately linear models
- position tracking

AI

### Multi-hypothesis ('00)

- multiple Kalman filters
- global localization, recovery

Robotics

## Bayes Filters are Familiar!

$$Bel(x_t) = \eta P(z_t | x_t) \int P(x_t | u_t, x_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- Partially Observable Markov Decision Processes (POMDPs)

## Summary

- Bayes rule allows us to compute probabilities that are hard to assess otherwise.
- Under the Markov assumption, recursive Bayesian updating can be used to efficiently combine evidence.
- Bayes filters are a probabilistic tool for estimating the state of dynamic systems.