CSE-571
Robotics

Bayes Filters

Bayes Filters: Framework

- **Given:**
  - Stream of observations *z* and action data *u:*

    $$d_t = \{u_1, z_2 \ldots, u_{t-1}, z_t\}$$

  - Sensor model *P(z|x).*
  - Action model *P(x|u,x').*
  - Prior probability of the system state *P(x).*
- **Wanted:**
  - Estimate of the state *X* of a dynamical system.
  - The posterior of the state is also called **Belief**:

    $$Bel(x_t) = P(x_t \mid u_1, z_2 \ldots, u_{t-1}, z_t)$$

Bayes Filters

z = observation
u = action
x = state

$$Bel(x_t) = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

Bayes

$$= \eta \; P(z_t \mid x_t, u_1, z_1, \ldots, u_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Markov

$$= \eta \; P(z_t \mid x_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Total prob.

$$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_1, z_1, \ldots, u_t, x_{t-1}) P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

Markov

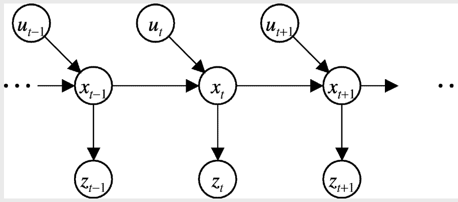
$$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

$$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

Algorithm **Bayes_filter**(*Bel(x),d*):
n=0
If *d* is a perceptual data item *z* then
    For all *x* do
        $Bel'(x) = P(z \mid x) Bel(x)$
        $\eta = \eta + Bel'(x)$
    For all *x* do
        $Bel'(x) = \eta^{-1} Bel'(x)$
Else if *d* is an action data item *u* then
    For all *x* do
        $Bel'(x) = \int P(x \mid u, x') \; Bel(x') \; dx'$
Return *Bel'(x)*

Markov Assumption



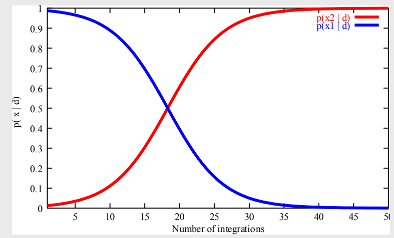
$$p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t \mid x_t)$$
$$p(x_t \mid x_{1:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$$

Underlying Assumptions

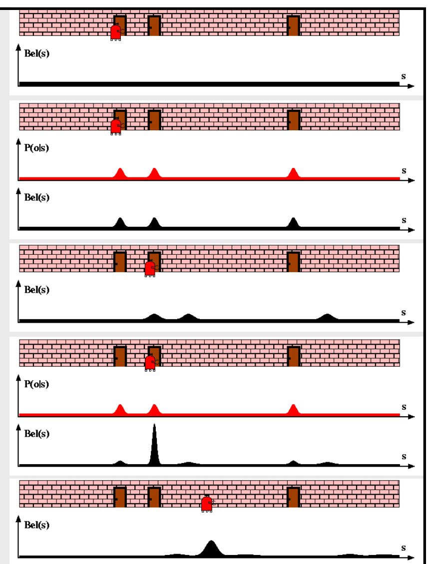
- Static world
- Independent noise
- Perfect model, no approximation errors

Dynamic Environments

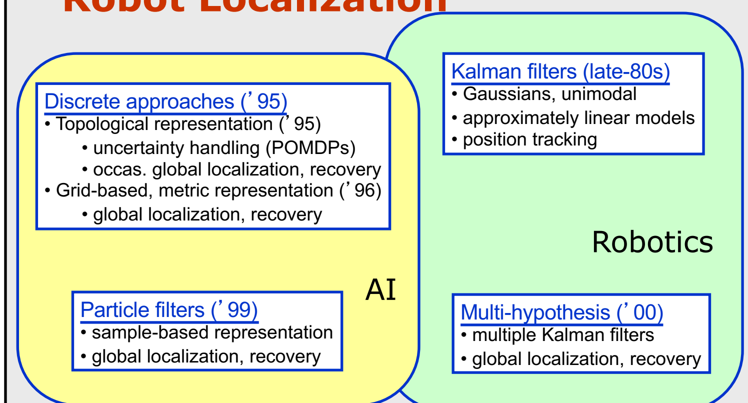
- Two possible locations x_1 and x_2
- $P(x_1)=0.99$
- $P(z|x_2)=0.09$ $P(z|x_1)=0.07$



Bayes Filters for Robot Localization



Representations for Bayesian Robot Localization



Bayes Filters are Familiar!

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- Partially Observable Markov Decision Processes (POMDPs)

Summary

- Bayes rule allows us to compute probabilities that are hard to assess otherwise.
- Under the Markov assumption, recursive Bayesian updating can be used to efficiently combine evidence.
- Bayes filters are a probabilistic tool for estimating the state of dynamic systems.