Signed Distance Function Representation, Tracking, and Mapping

Tanner Schmidt

Overview

- Explicit and implicit surface representations
- SDF fusion
- SDF tracking
- Related research
  - KinectFusion
  - Patch Volumes
  - DART
  - DynamicFusion

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- **Explicit and implicit surface representations**
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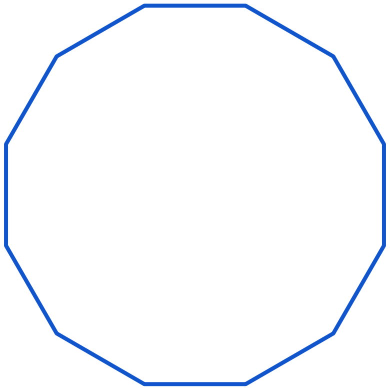
Explicit Surface Representations

- Geometry is stored **explicitly** as a list of points, triangles, or other geometric fragments
  - e.g. meshes, point clouds

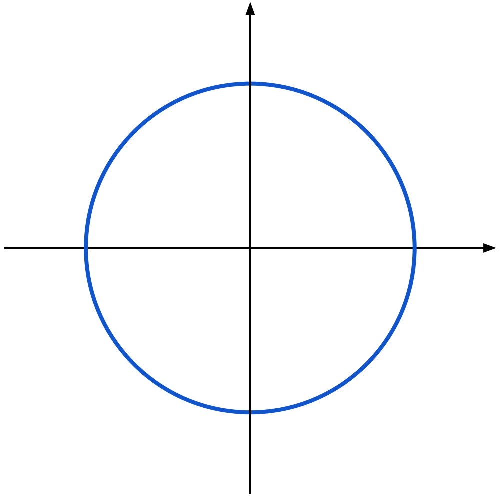


Vertices: [(x0, y0, z0), (x1, y1, z1), …, (xn, yn, zn)]

Indices: [(i0, i1), (i2, i3), …, (in-1, in)]

Implicit Surface Representation

- Geometry is not stored explicitly but rather defined as a level set of a function defined over the space in which the geometry is embedded
    - There are **parametric** representations:



$$f(x, y) = x^2 + y^2 - r^2$$

Implicit Surface Representation

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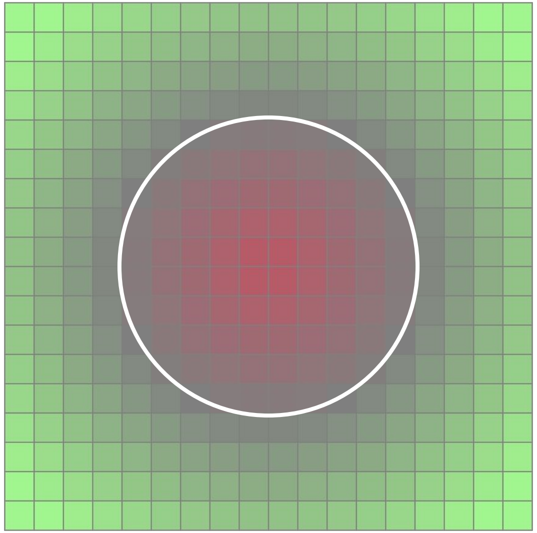
  - And there are **nonparametric** representations:



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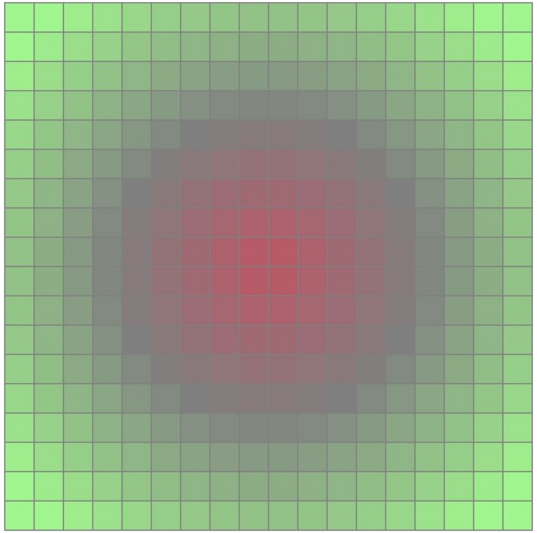
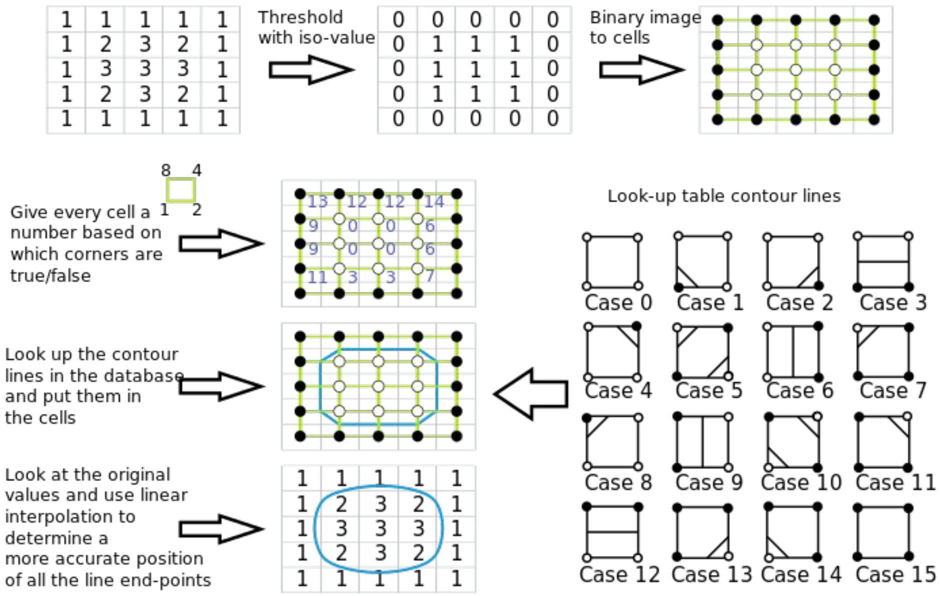


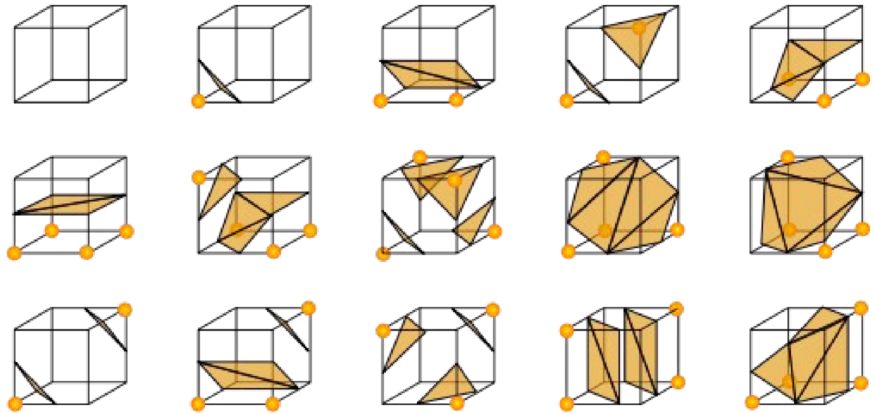
Implicit to Explicit Conversion

- In two dimensions, we can use an algorithm called **marching squares**



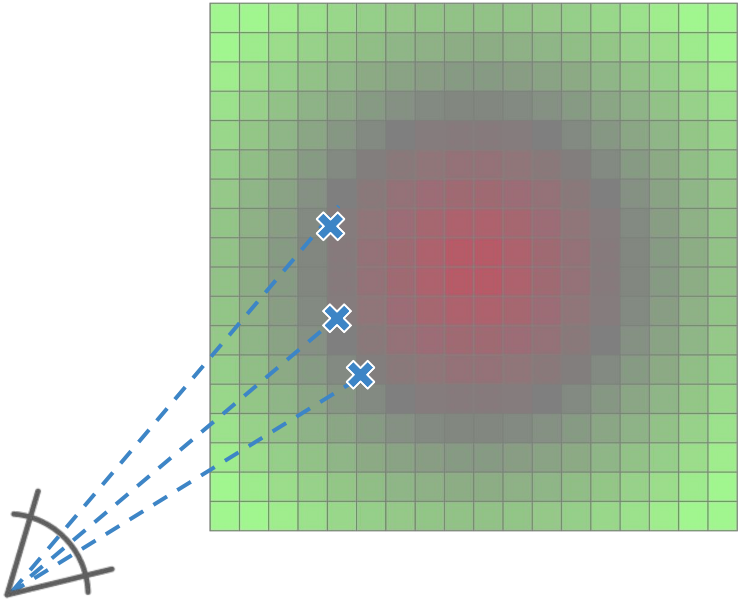
Implicit to Explicit Conversion in 3D

- Typically done using **marching cubes**, a 3D analogue to marching squares



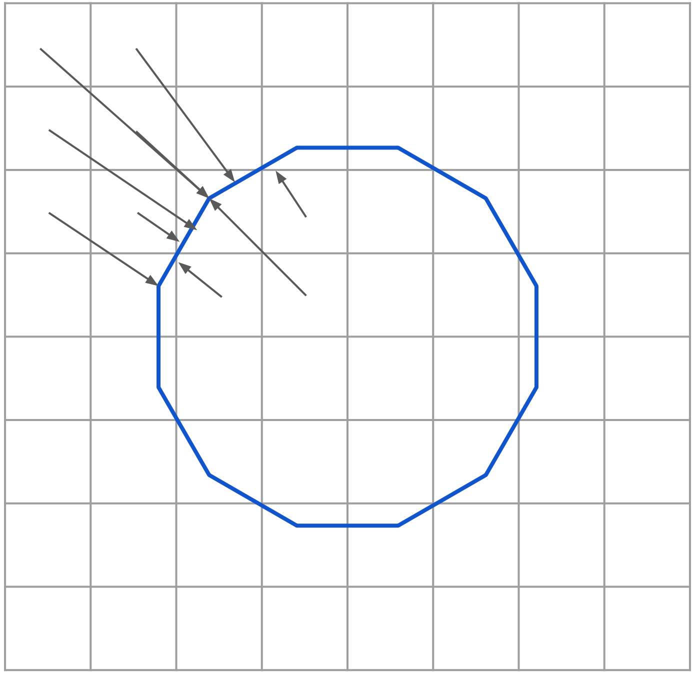
Implicit to Explicit Conversion in 3D

- Can also be done by **raycasting** for a view-dependent partial surface



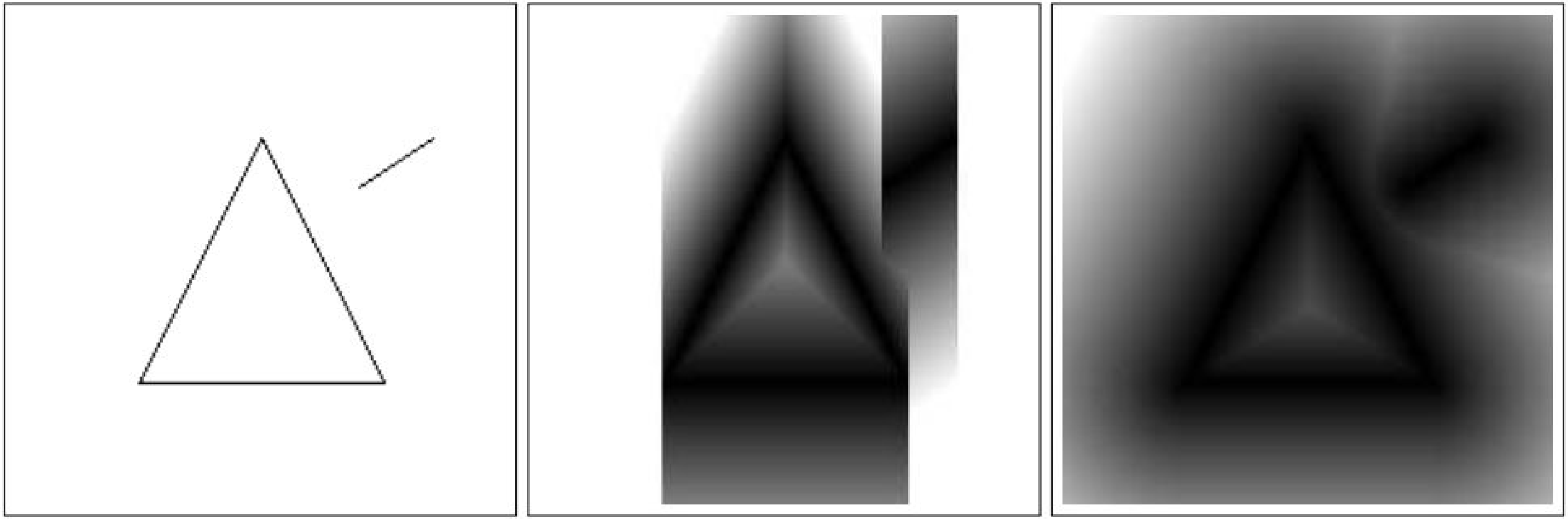
Explicit to Implicit Conversion

- Can be done by finding the closest point between each discrete location and any part of the geometry



Explicit to Implicit Conversion

- Can also be done with a **distance transform**



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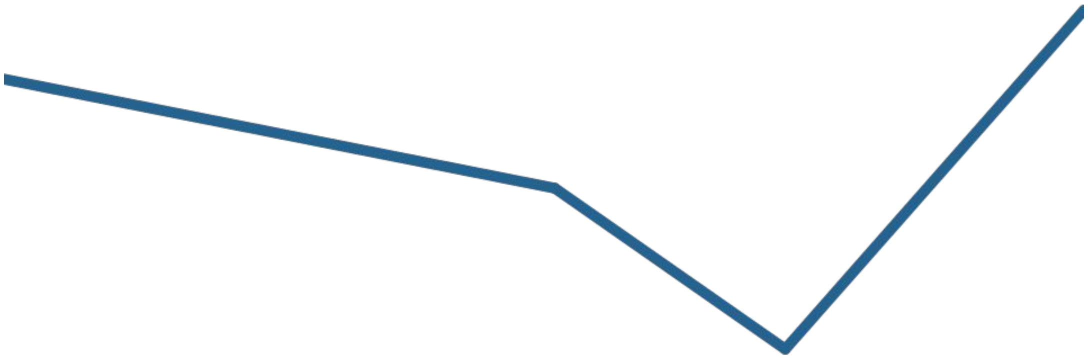
Signed Distance Function Fusion



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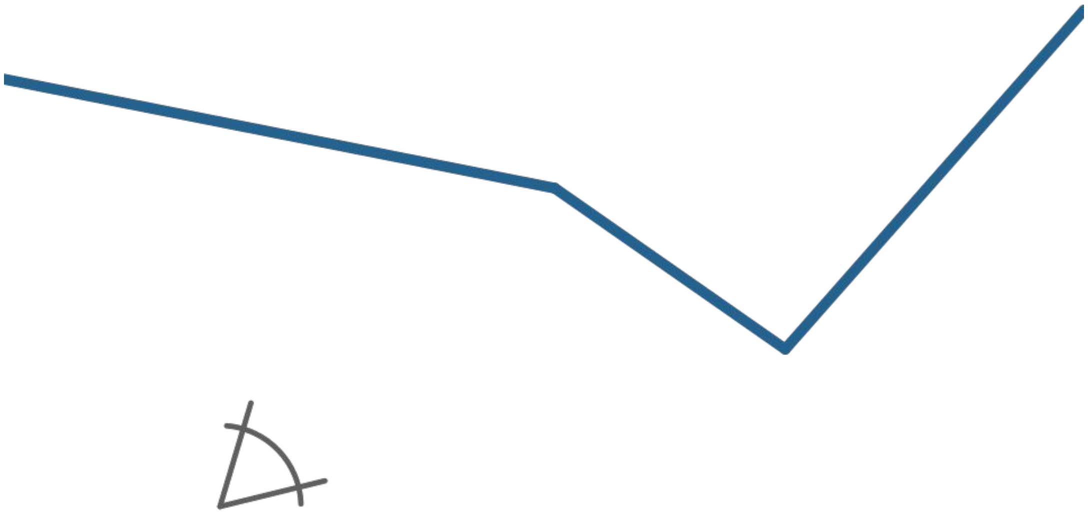

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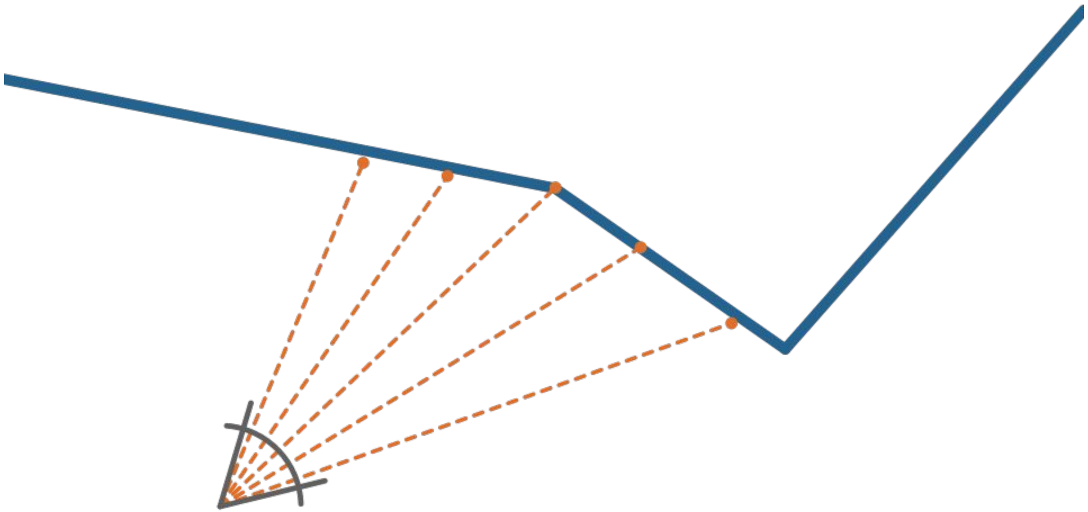

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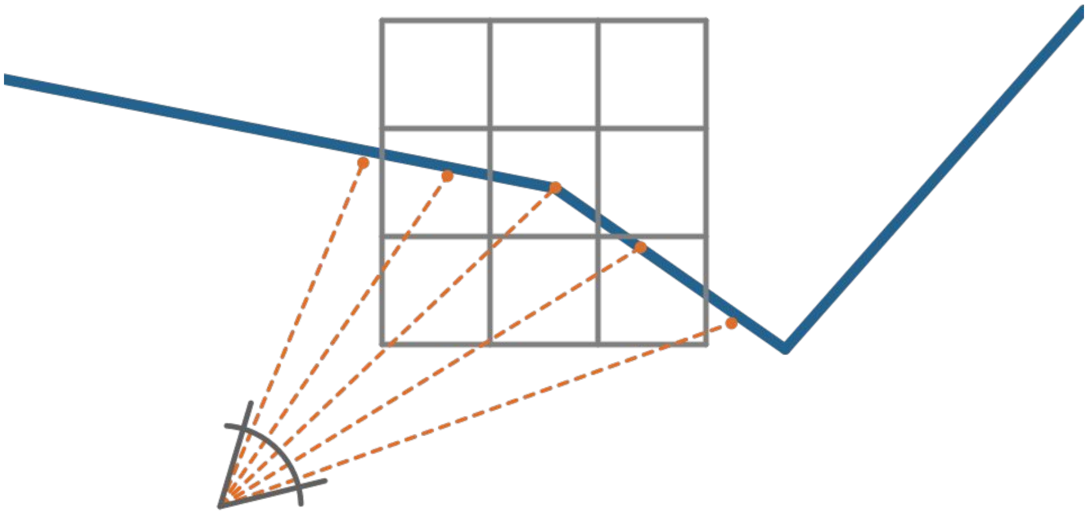

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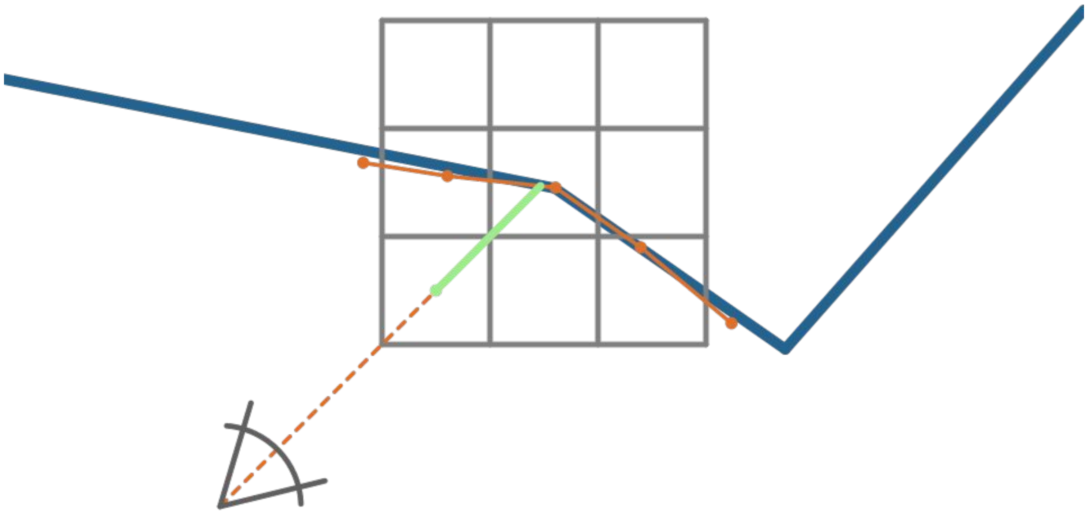

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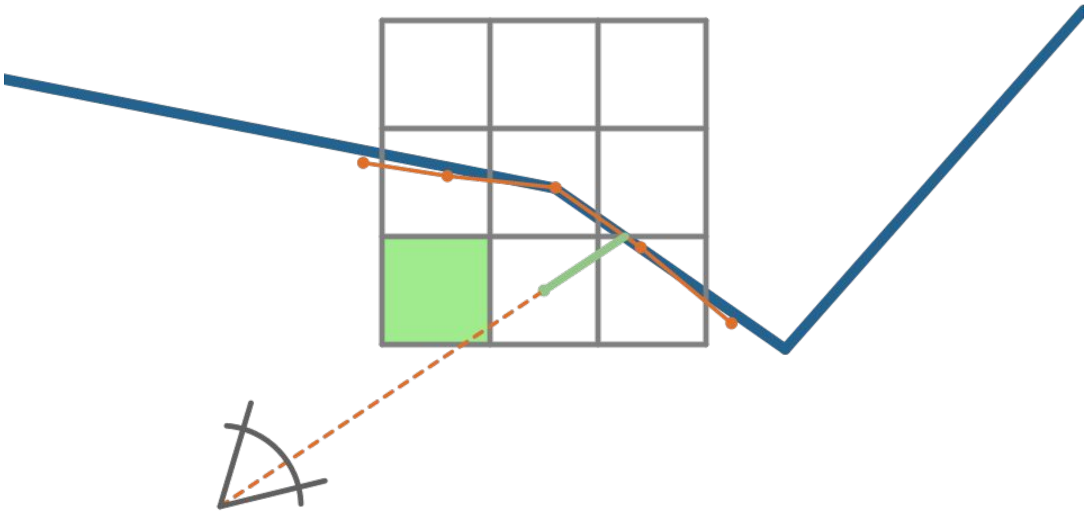

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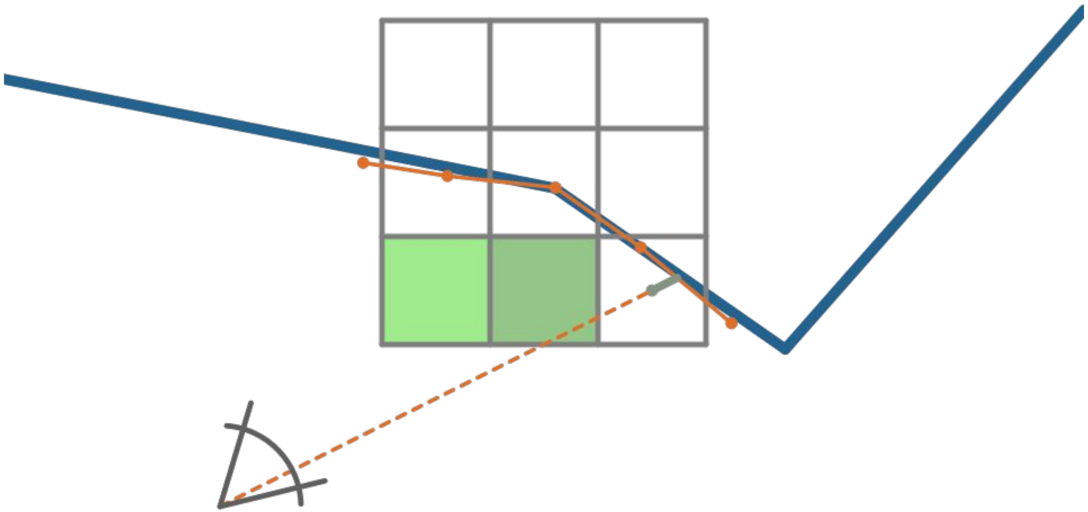

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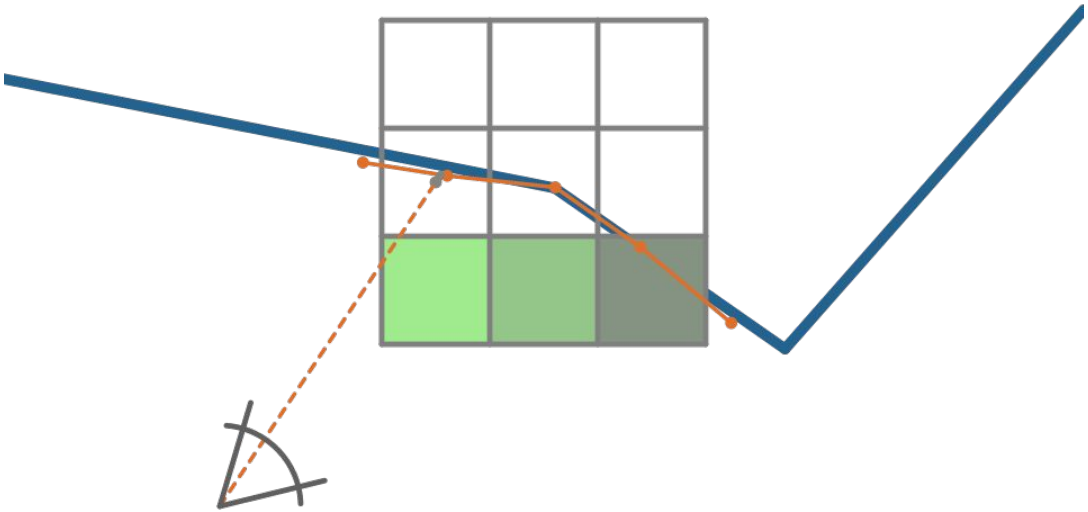

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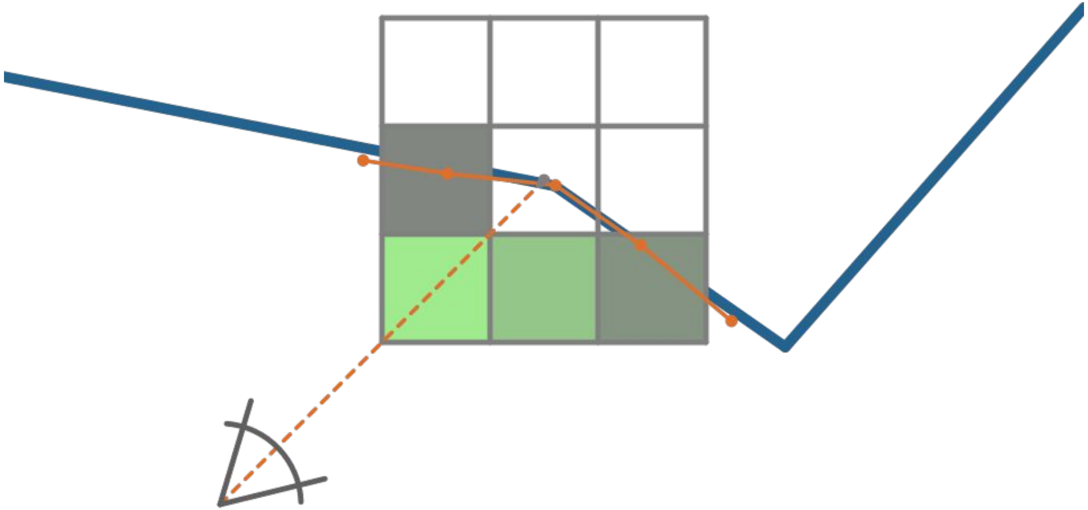

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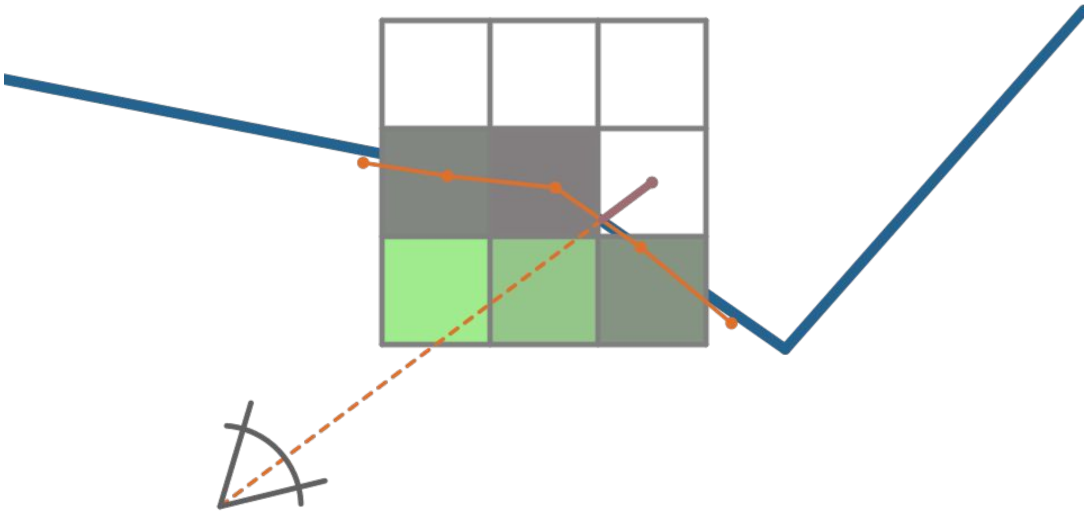

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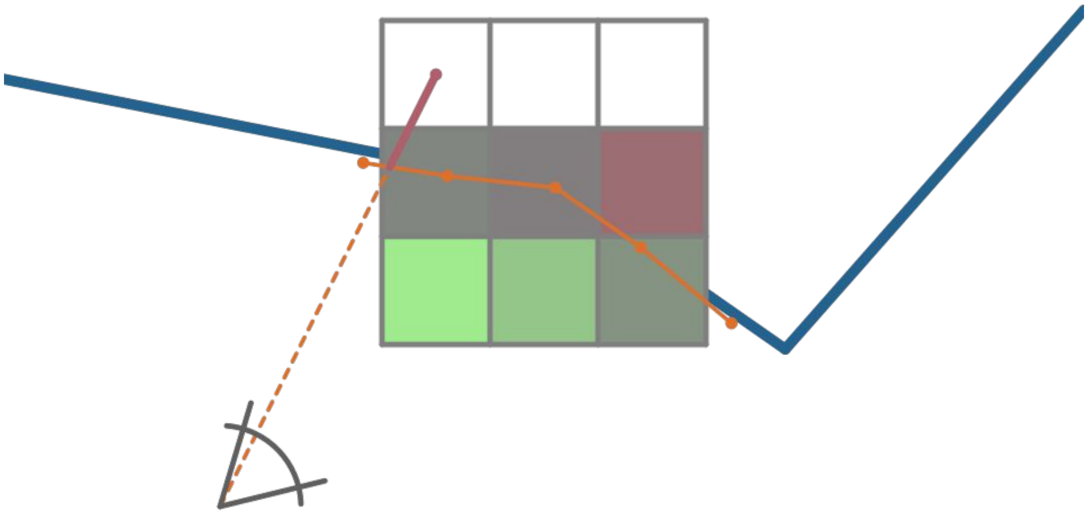

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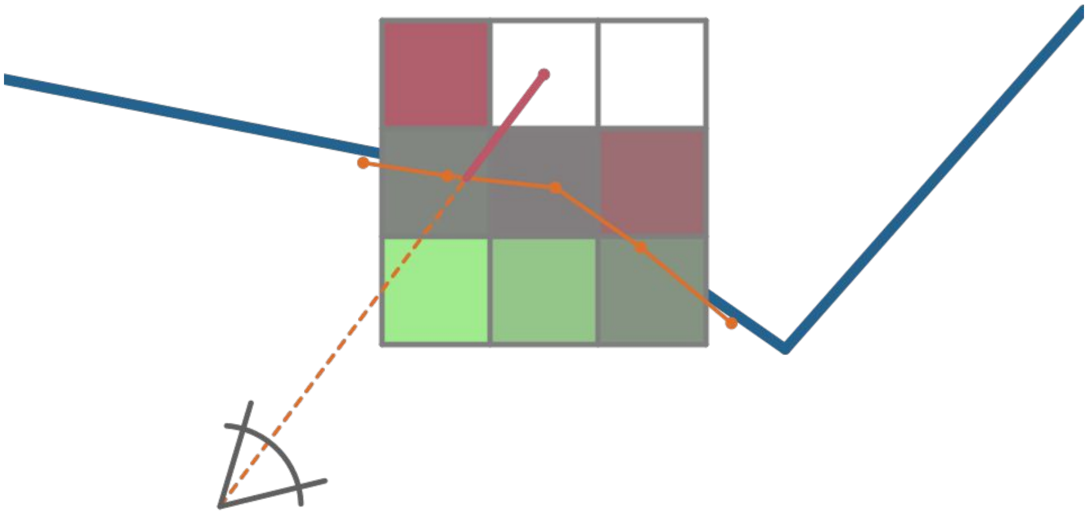

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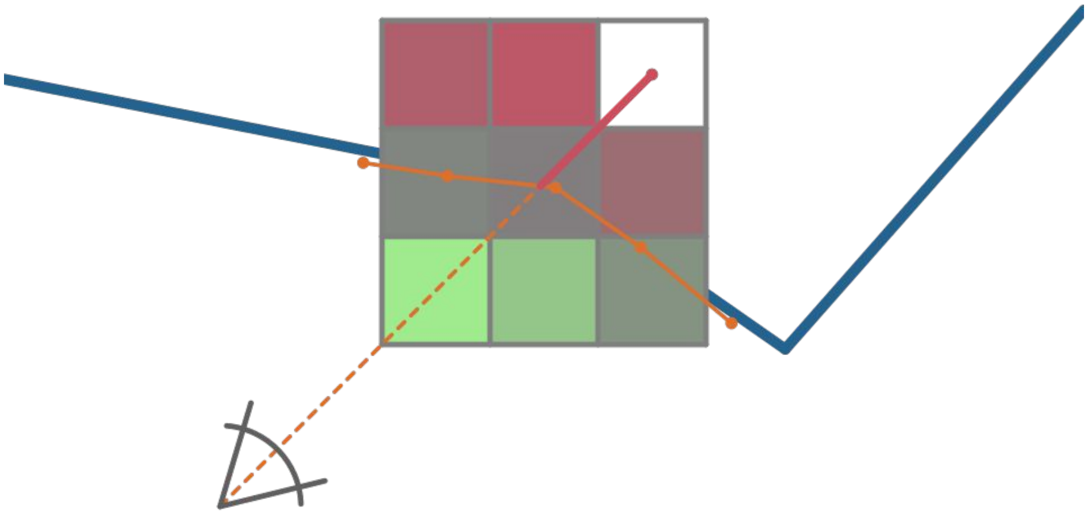

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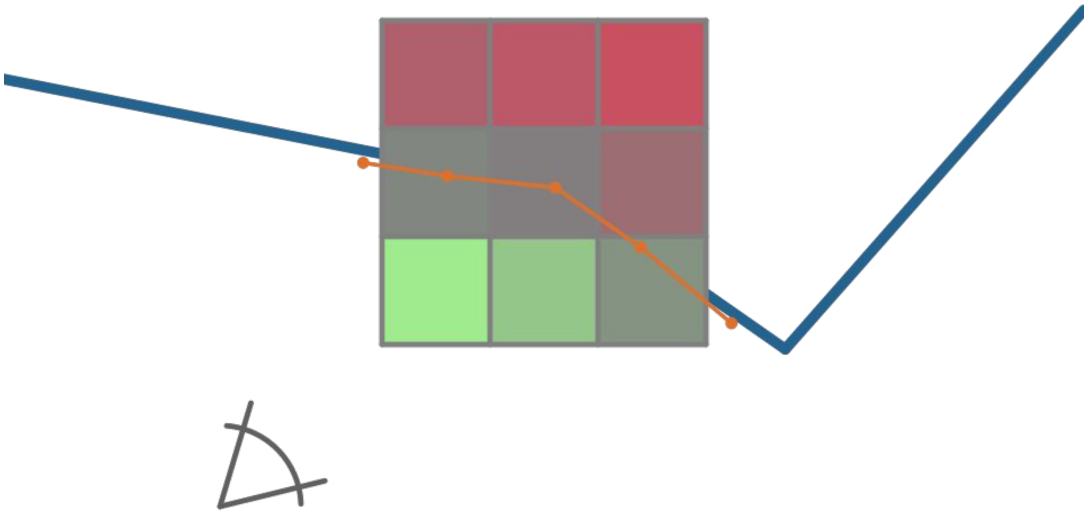

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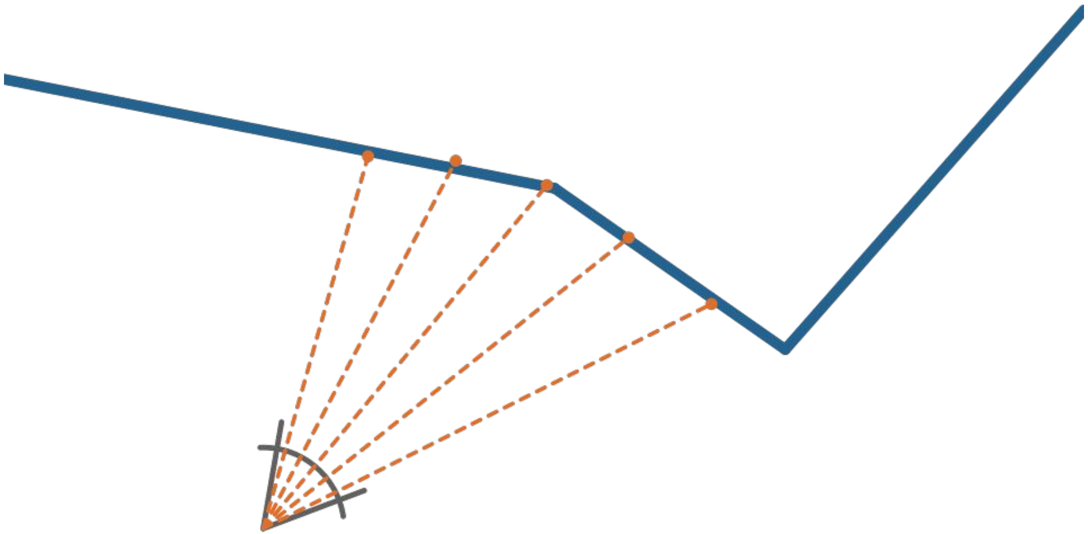

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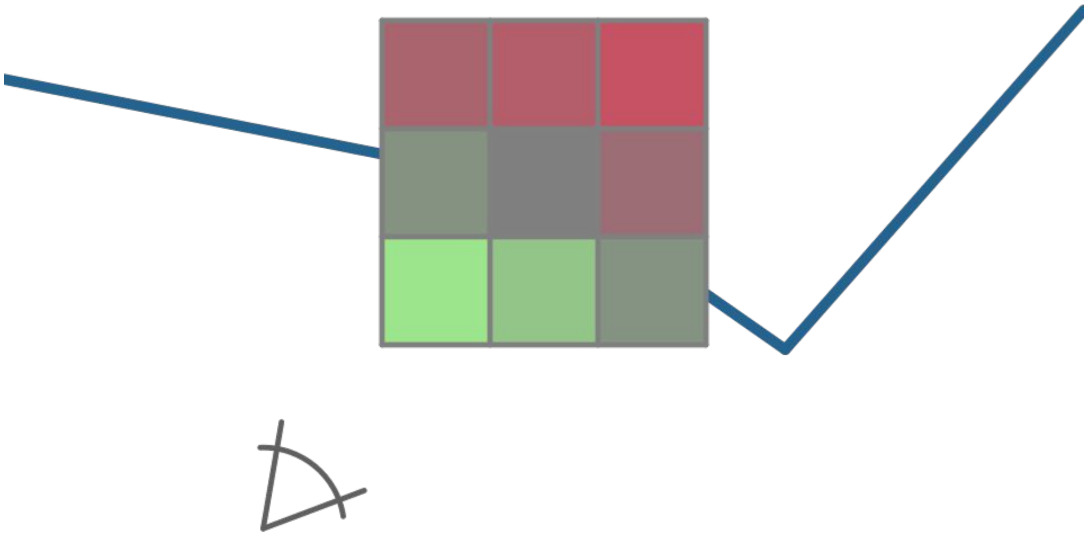

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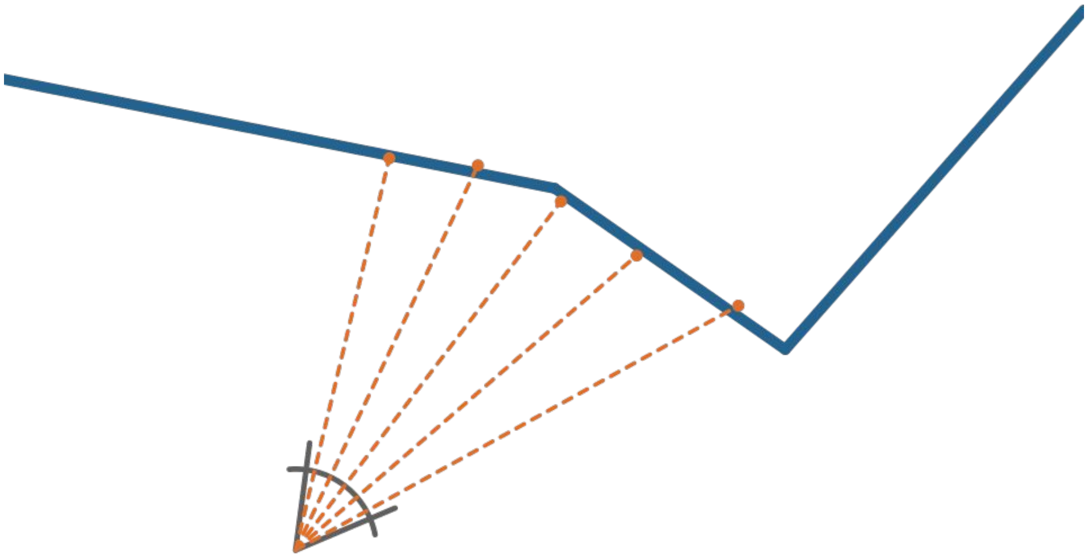

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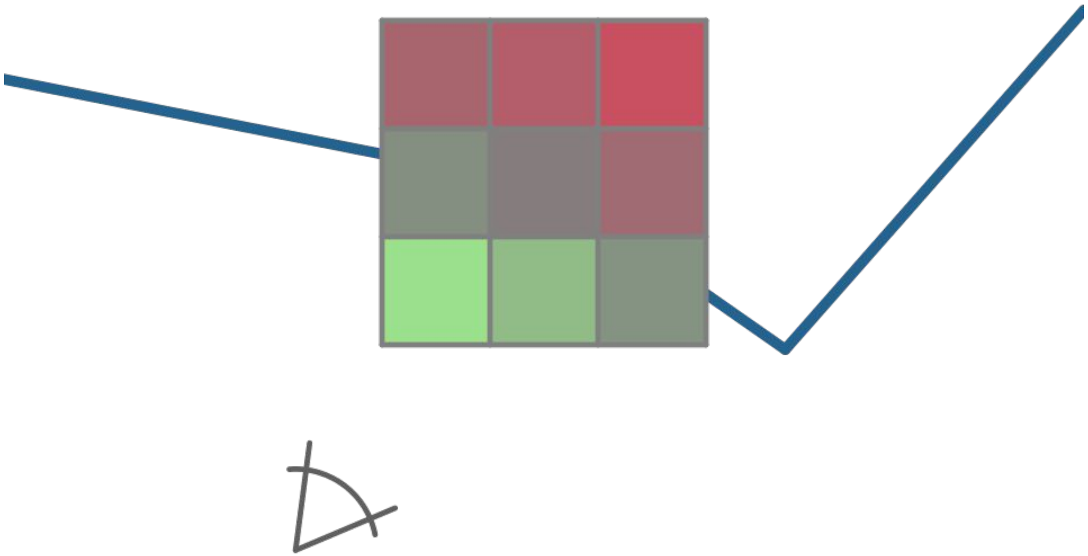

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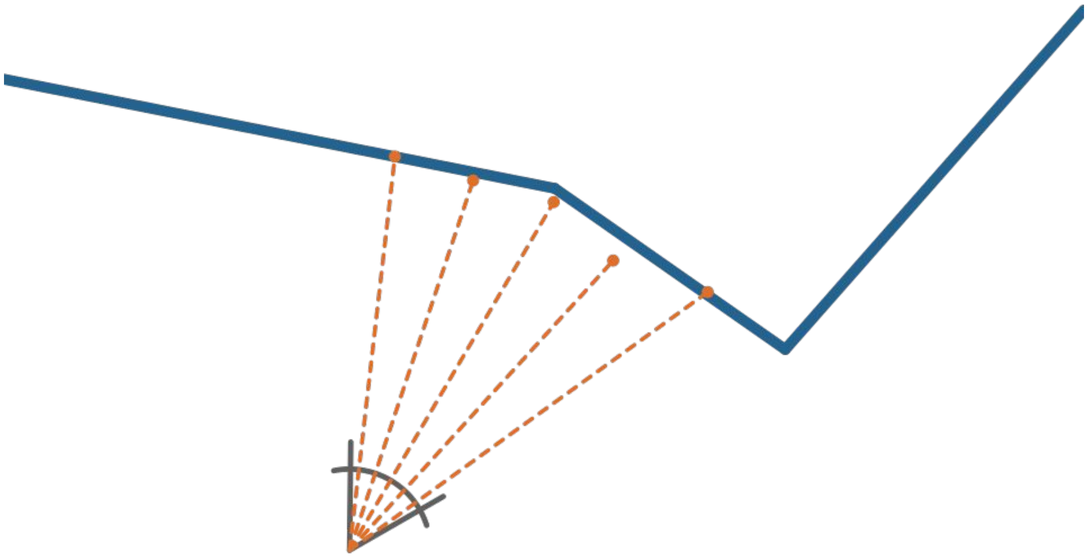

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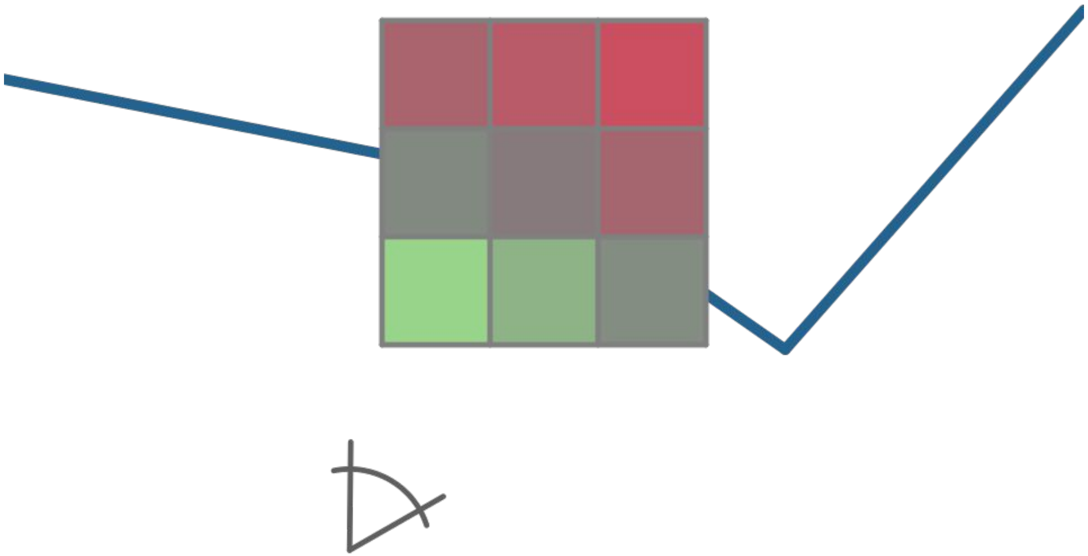

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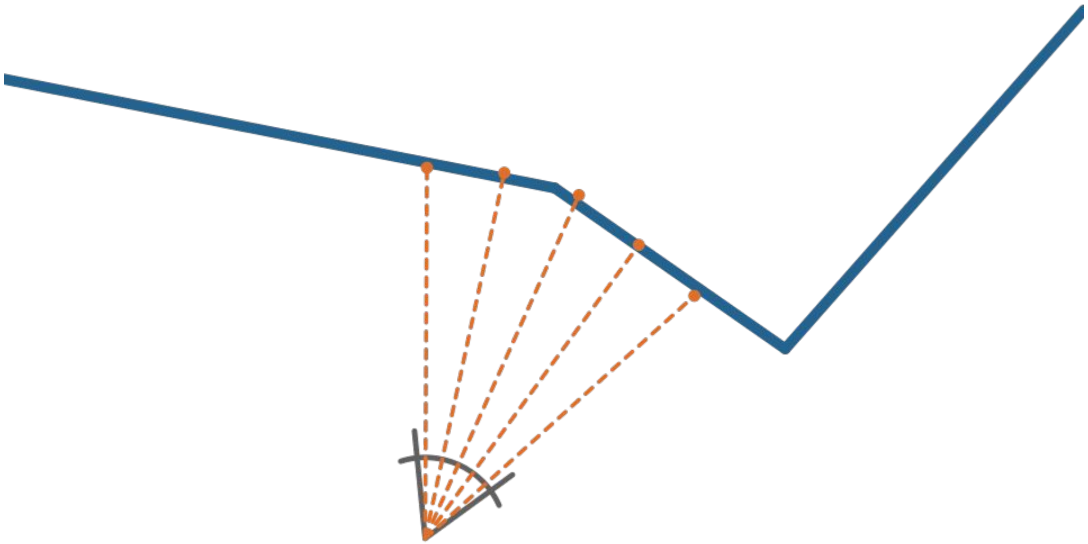

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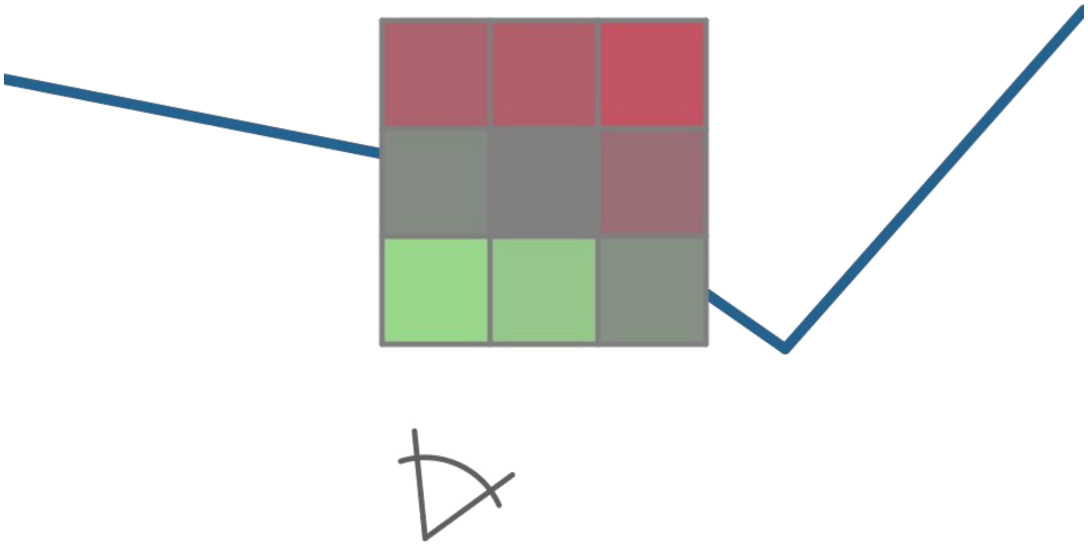

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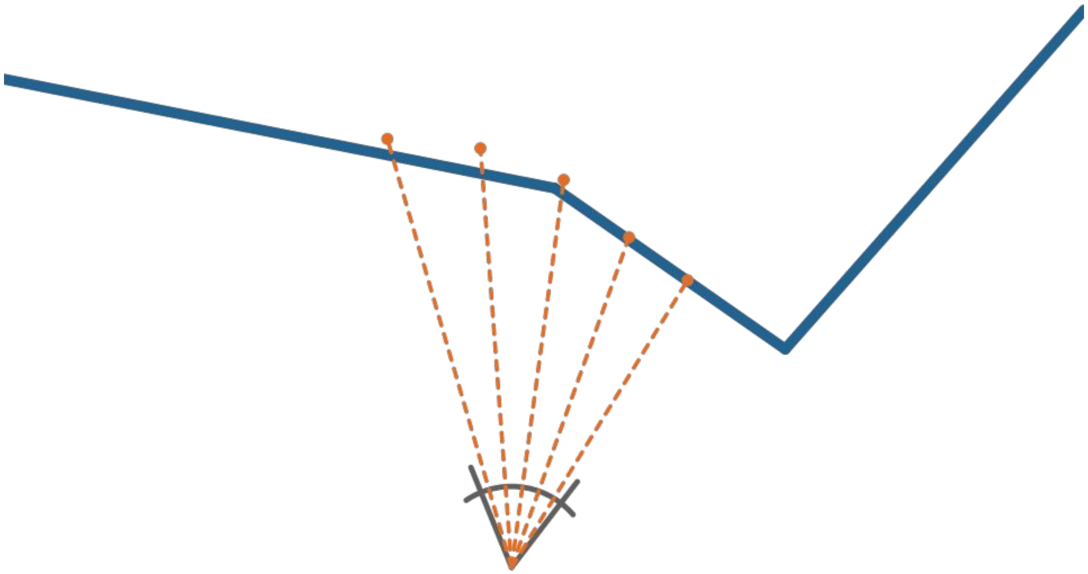

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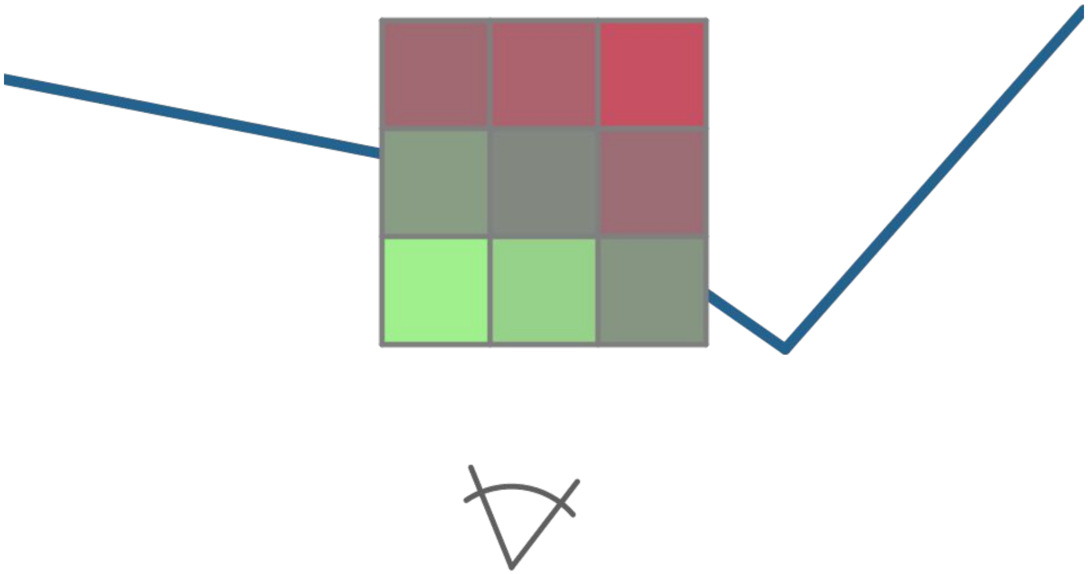

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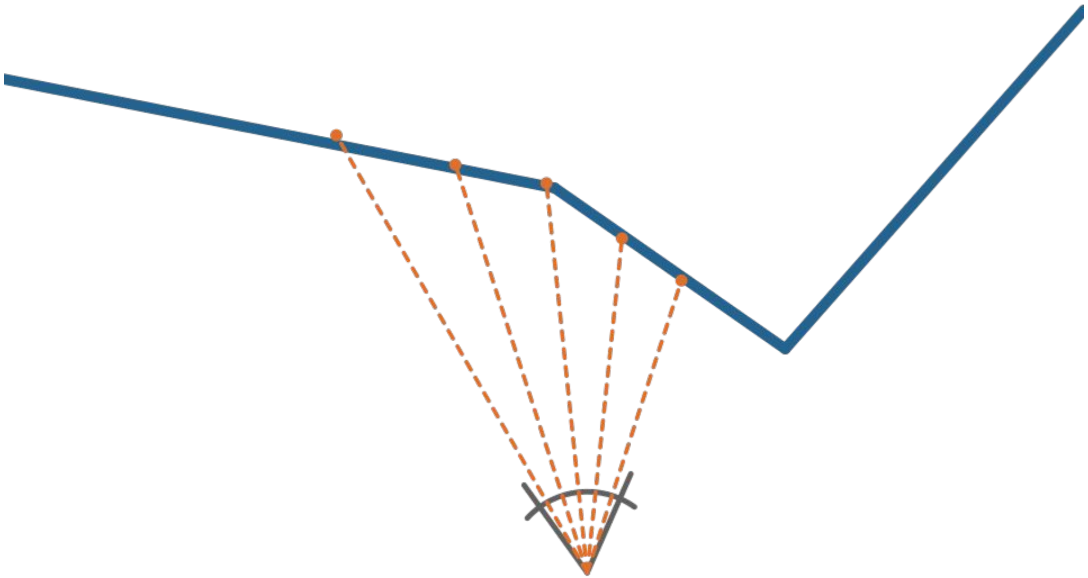

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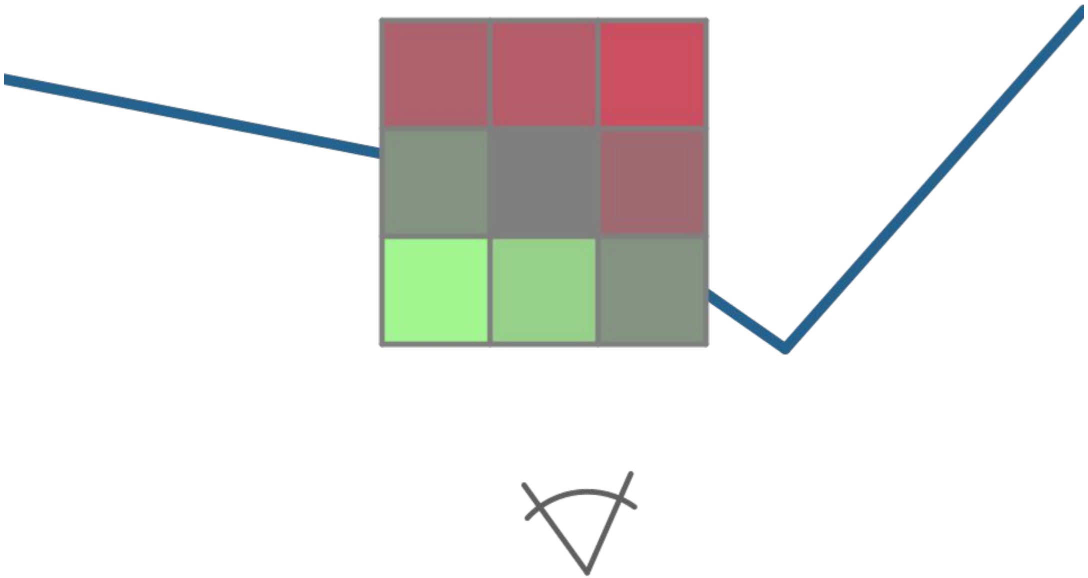

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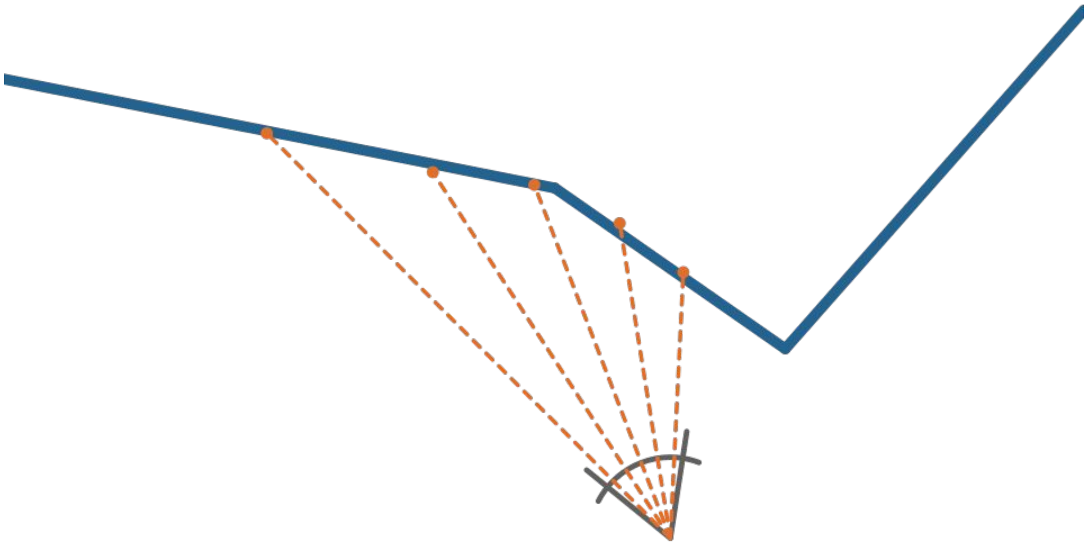

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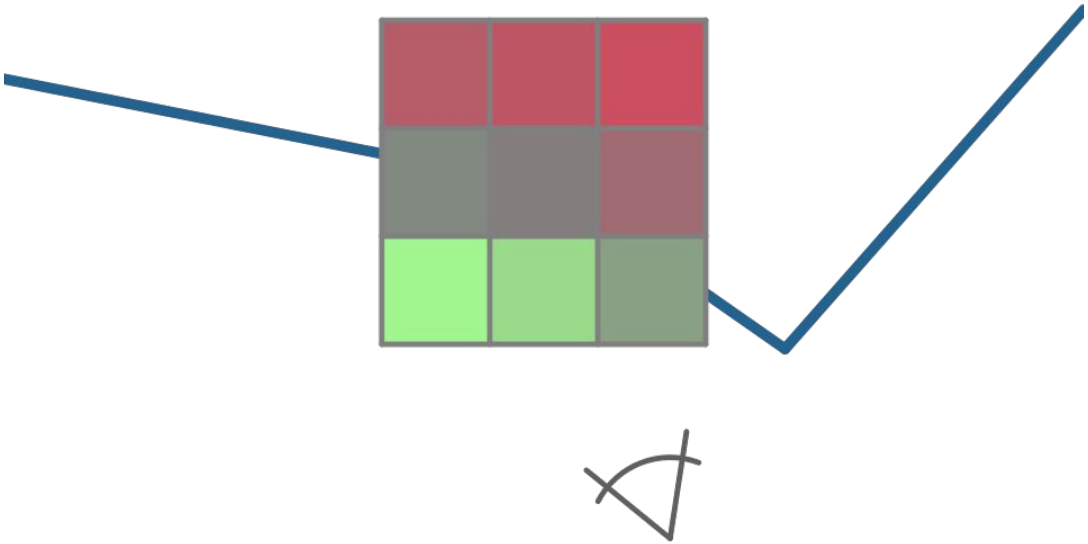

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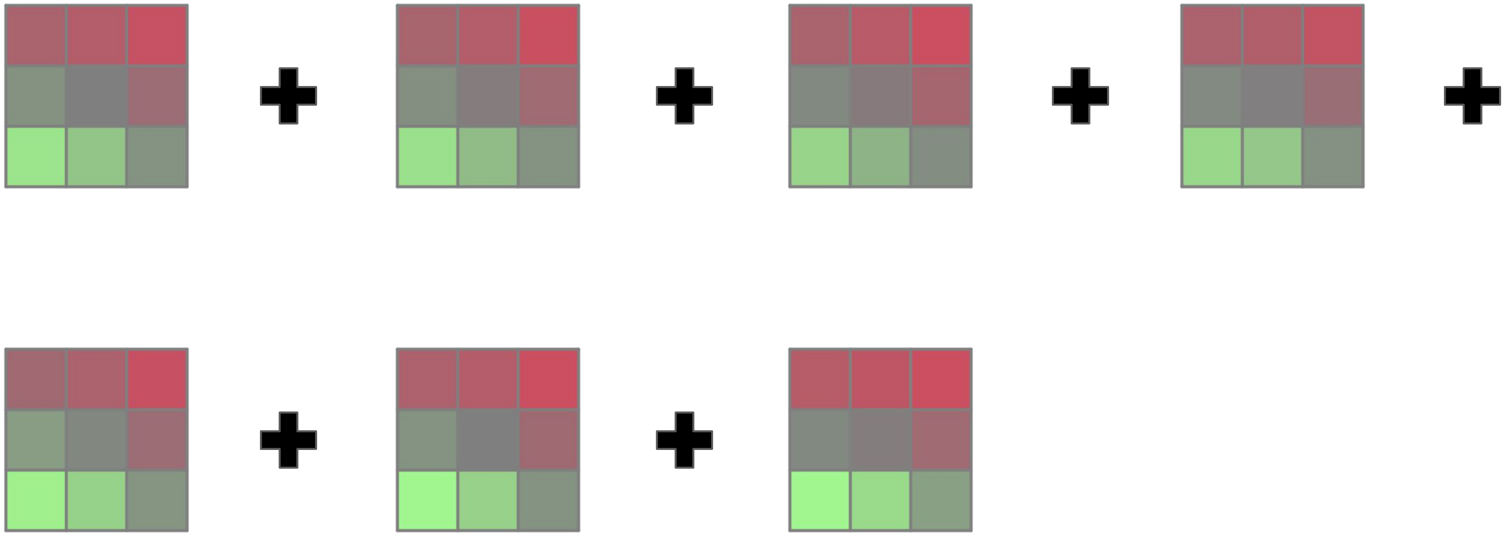

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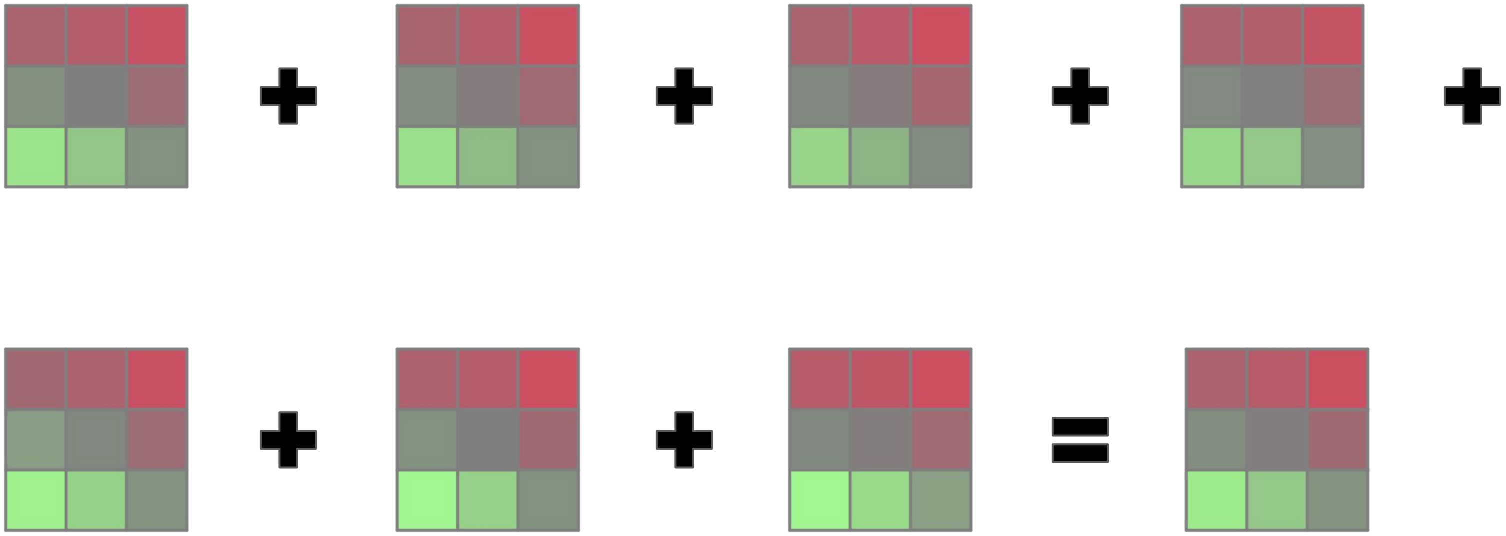

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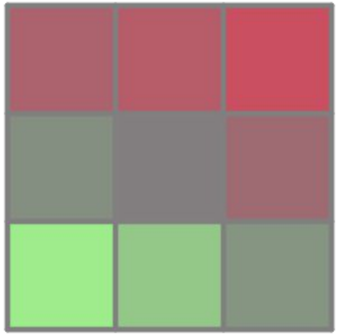

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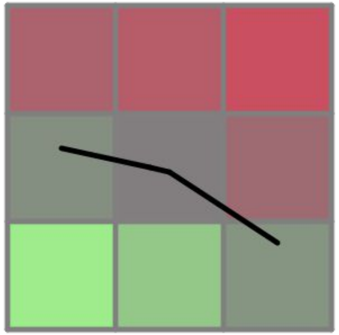

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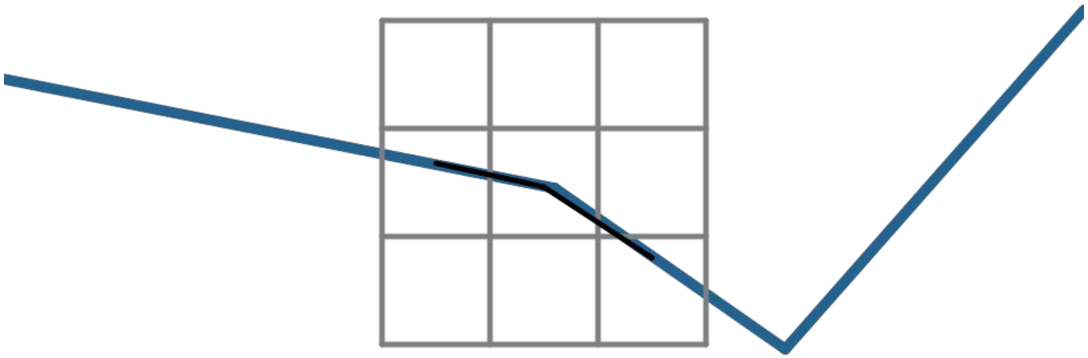
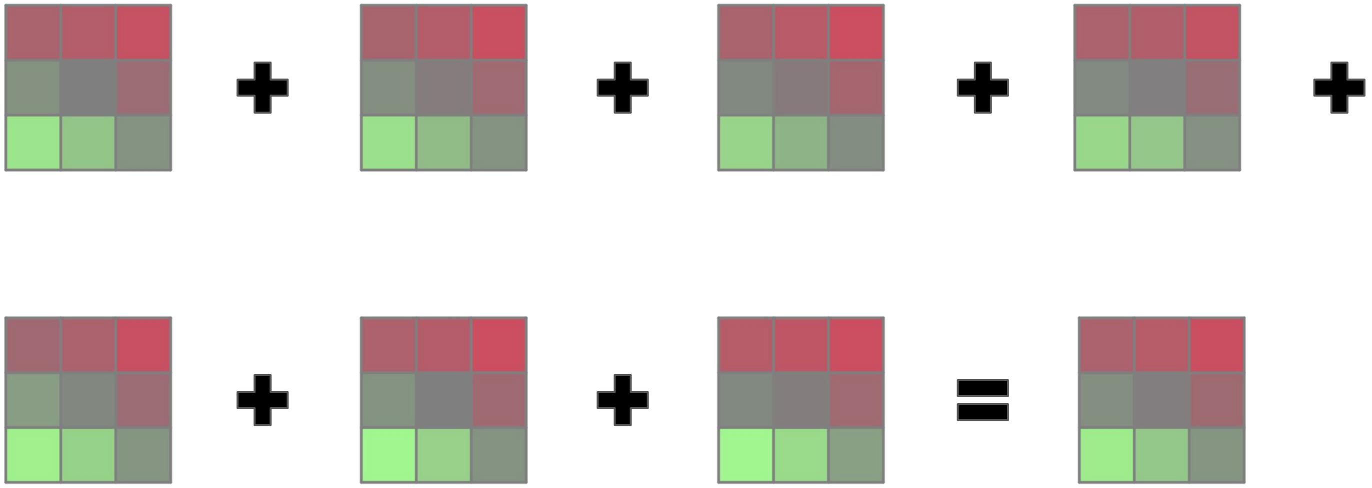


Signed Distance Function Fusion

- This addition requires the per-frame projected truncated signed distance volumes to be globally registered



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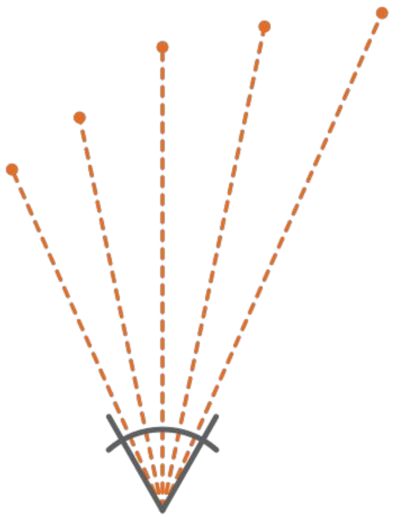

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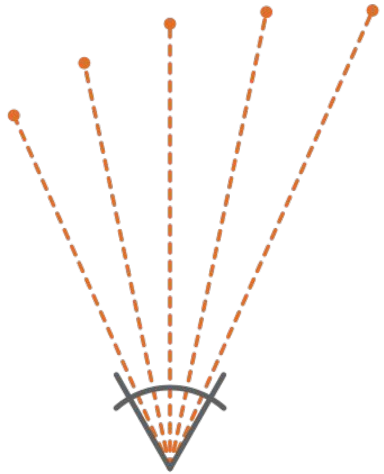

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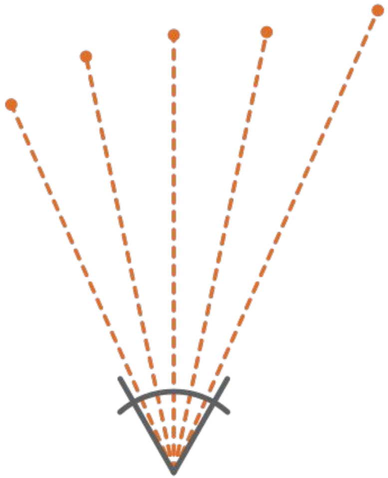

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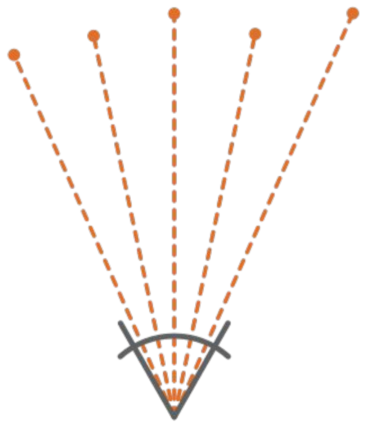

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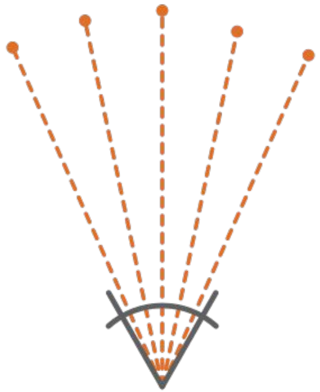

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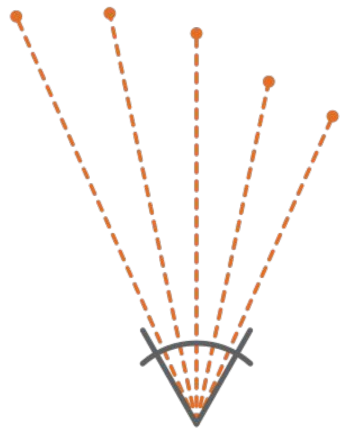

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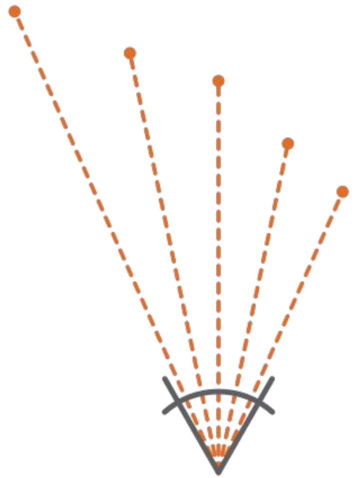

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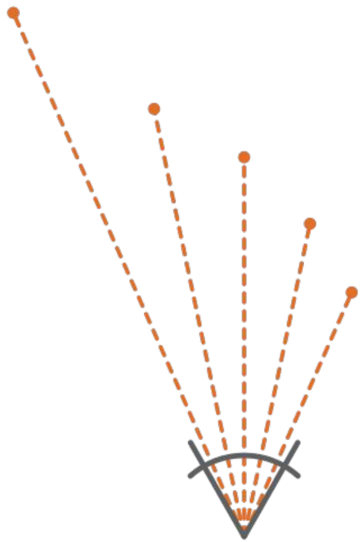
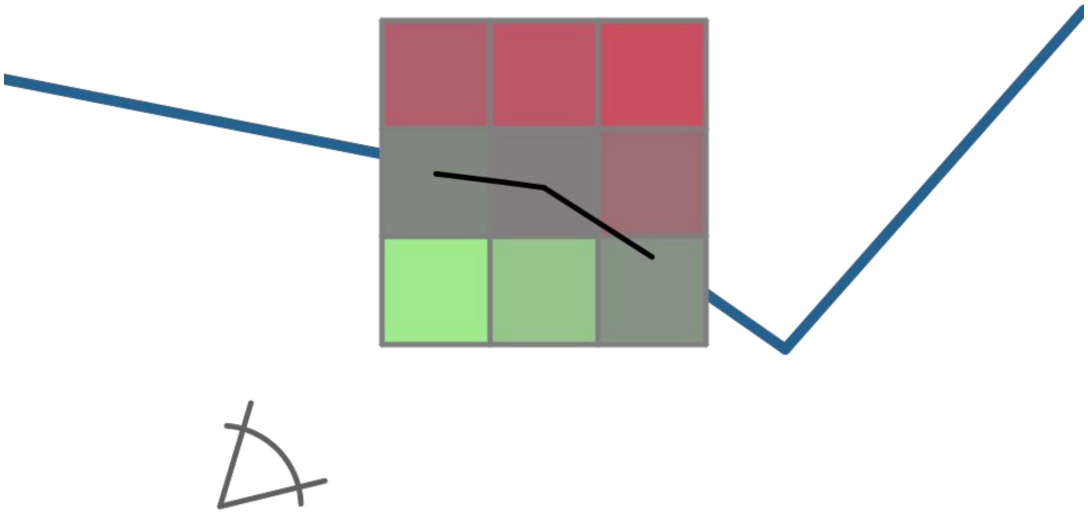


Point-plane Iterative Closest Point (ICP)



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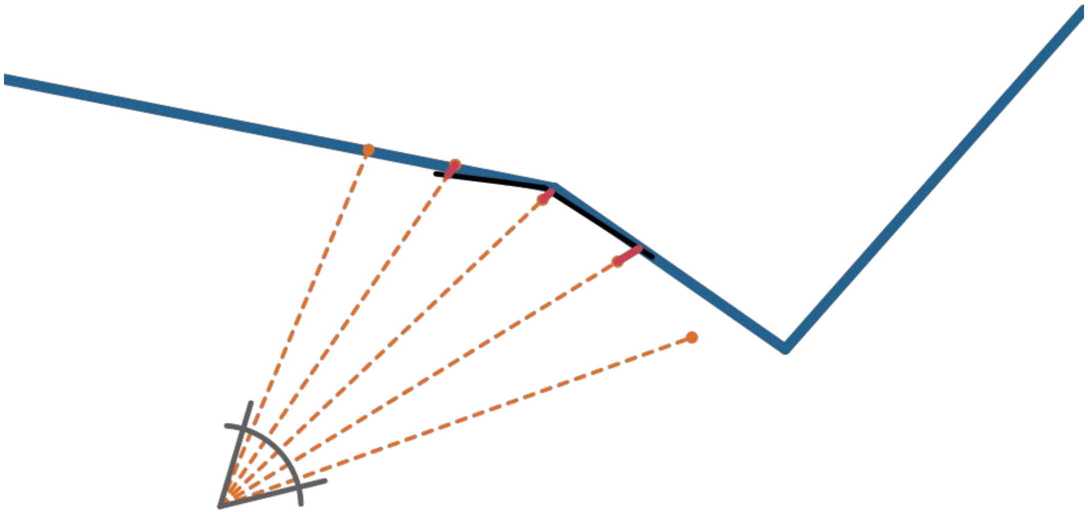

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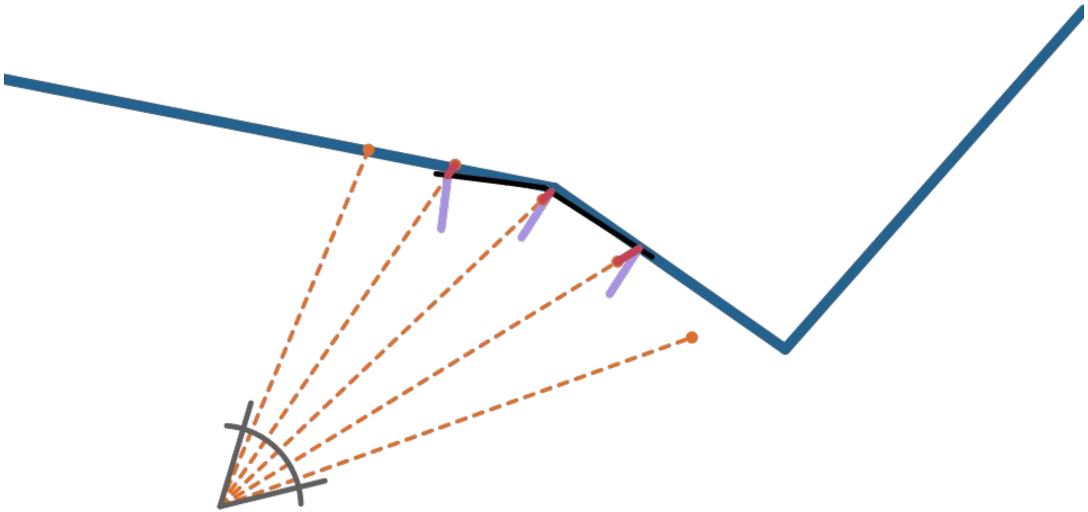

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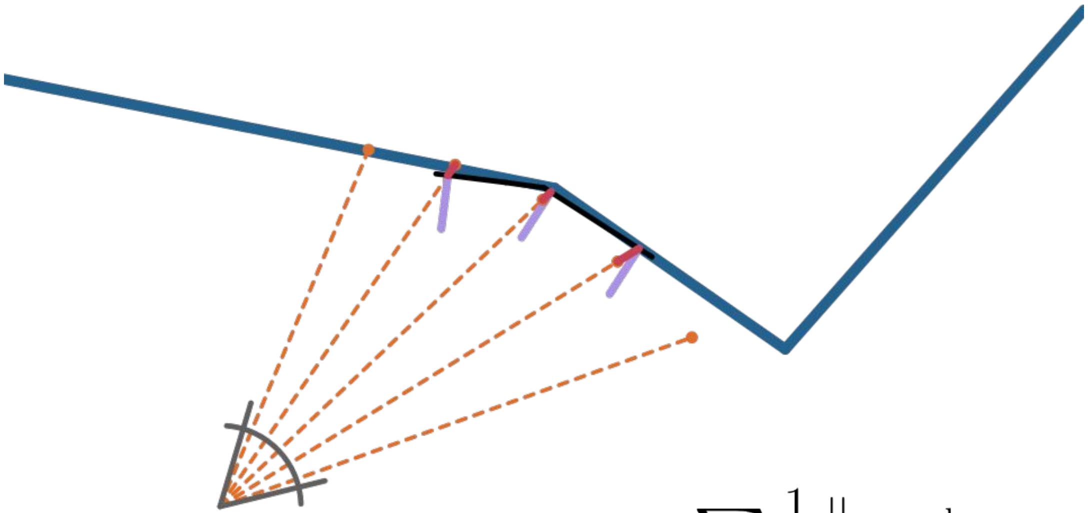
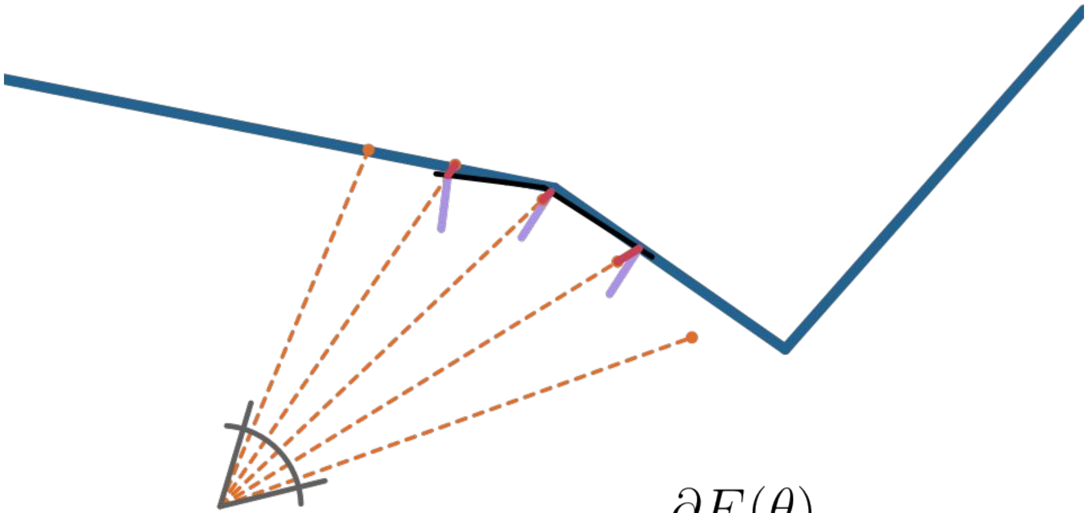


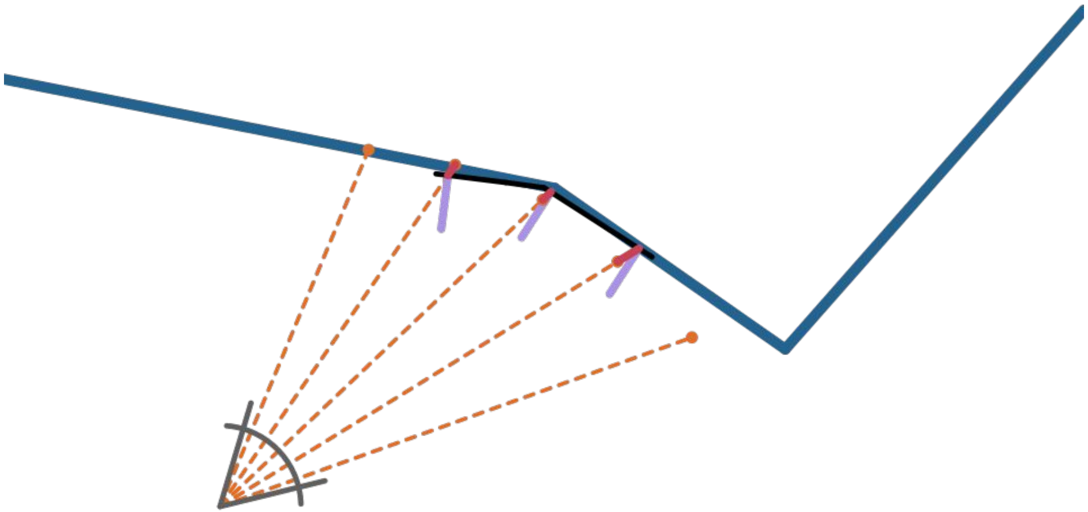
$$E(\theta) = \sum_i \frac{1}{2} \left\| \underline{n_i^{\text{pred}}} \cdot (T_{gl}(\theta) \underline{x_i^{\text{obs}} - x_i^{\text{pred}}}) \right\|_2^2$$

Point-plane Iterative Closest Point (ICP)



$$\frac{\partial E(\theta)}{\partial \theta} = \sum_i n_i^{\mathrm{pred}} \cdot \frac{\partial T_{gl}(\theta) x_i^{\mathrm{obs}}}{\partial \theta}$$

Point-plane Iterative Closest Point (ICP)



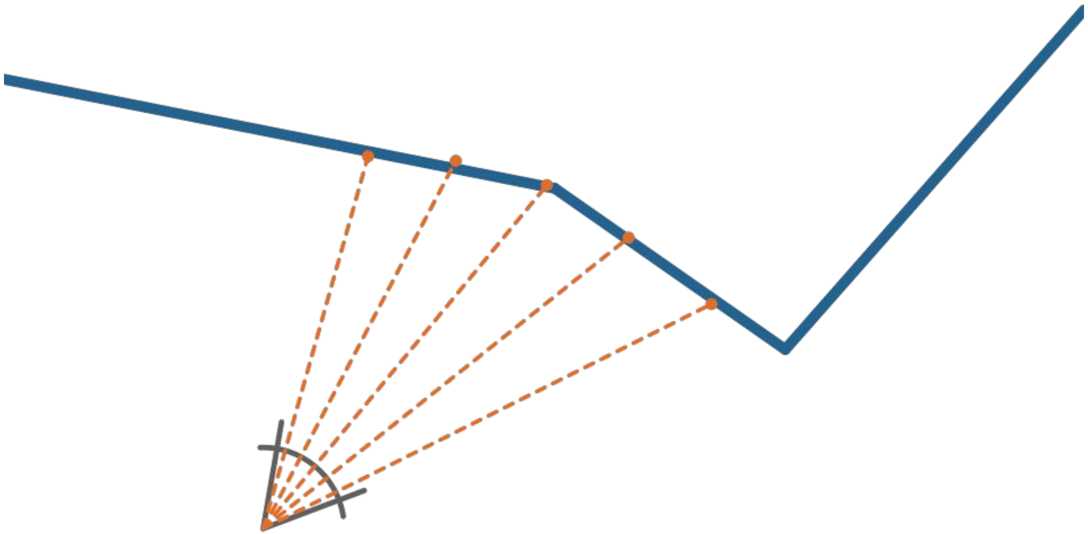
$$\frac{\partial E(\theta \oplus \Delta\theta)}{\partial \Delta\theta} = \sum_i n_i^{\text{pred}} \cdot \frac{\partial}{\partial \Delta\theta} T_{gl}(\theta \oplus \Delta\theta) x_i^{\text{obs}}$$

Point-plane Iterative Closest Point (ICP)

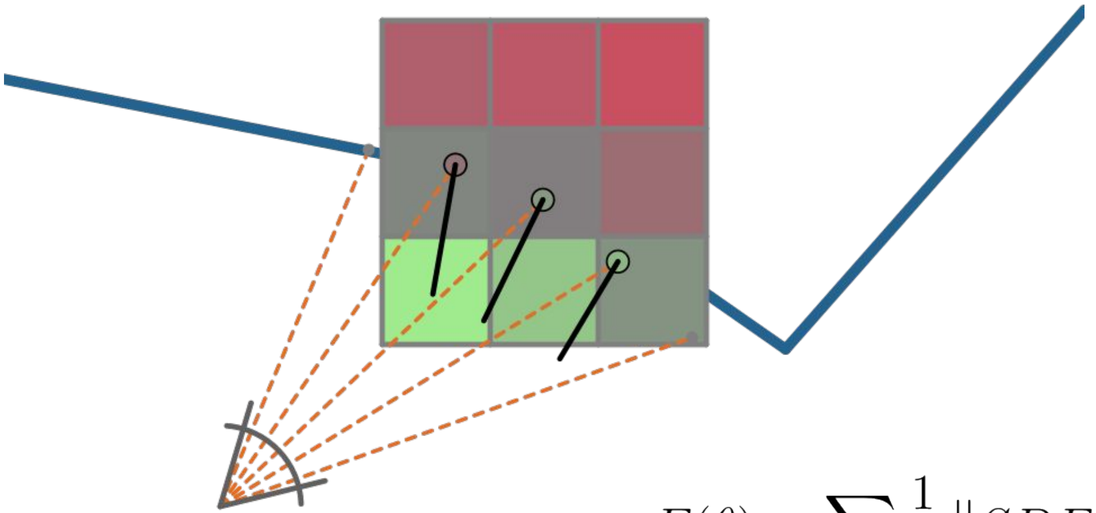
$$\frac{\partial}{\partial \Delta\theta} T(\theta \oplus \Delta\theta) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & z & -y \\ 0 & 1 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{bmatrix}$$

$$T(\theta \oplus \Delta\theta) = e^{\Delta\theta} T(\theta)$$

Point-plane Iterative Closest Point (ICP)

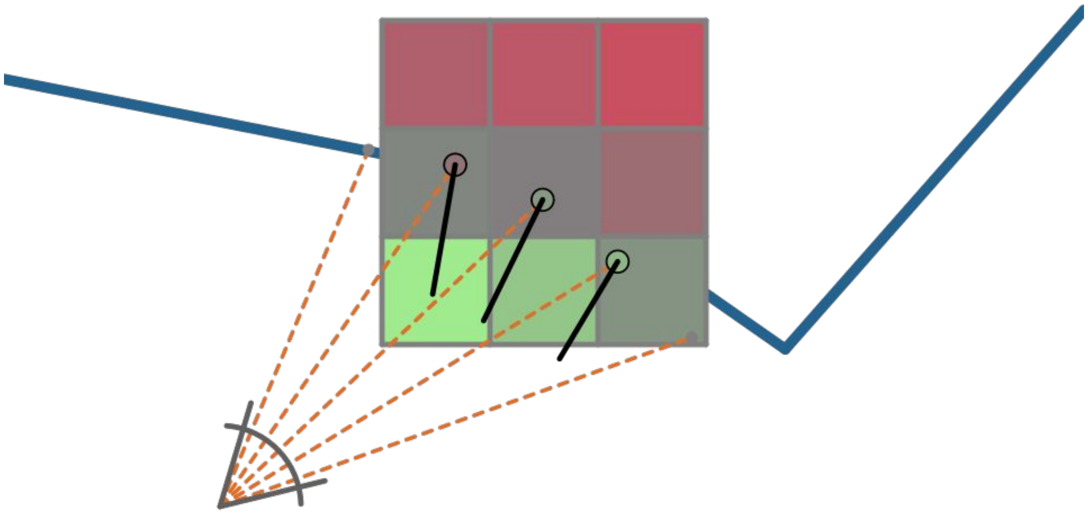


Direct Signed Distance Function Tracking



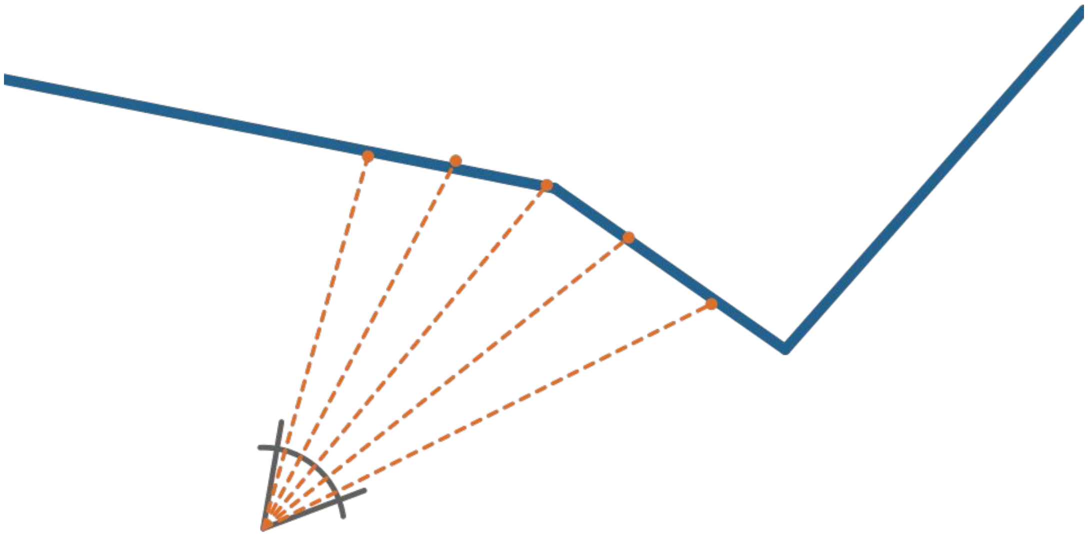
$$E(\theta) = \sum_i \frac{1}{2} \left\| SDF(T_{gl}(\theta) x_i^{\text{obs}}) \right\|_2^2$$

Direct Signed Distance Function Tracking



$$\frac{\partial E(\theta \oplus \Delta\theta)}{\partial \Delta\theta} = \sum_i \nabla SDF(T_{gl^{t-1}}(\theta) x_i^{\text{obs}}) R_{gl^{t-1}} \frac{\partial T_{l^{t-1}l^t}(\Delta\theta)}{\partial \Delta\theta}$$

Direct Signed Distance Function Tracking



Online fusion

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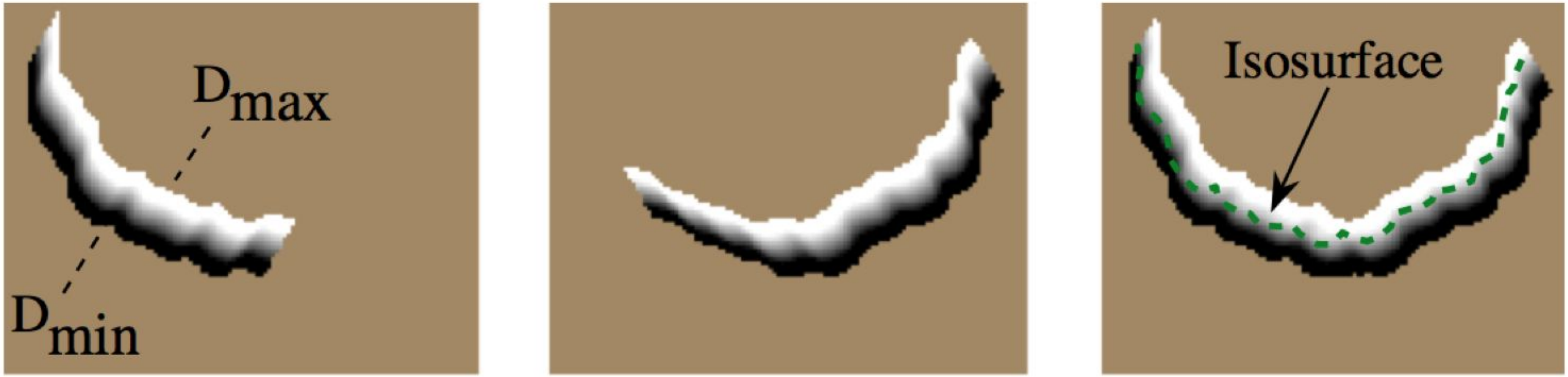
- We must maintain a **running average** SDF value at each cell
- Each cell stores both an SDF value and a weight

$$SDF^{0:t} \leftarrow \frac{SDF^{0:t-1} w^{0:t-1} + SDF^t w^t}{w^{0:t-1} + w^t}$$

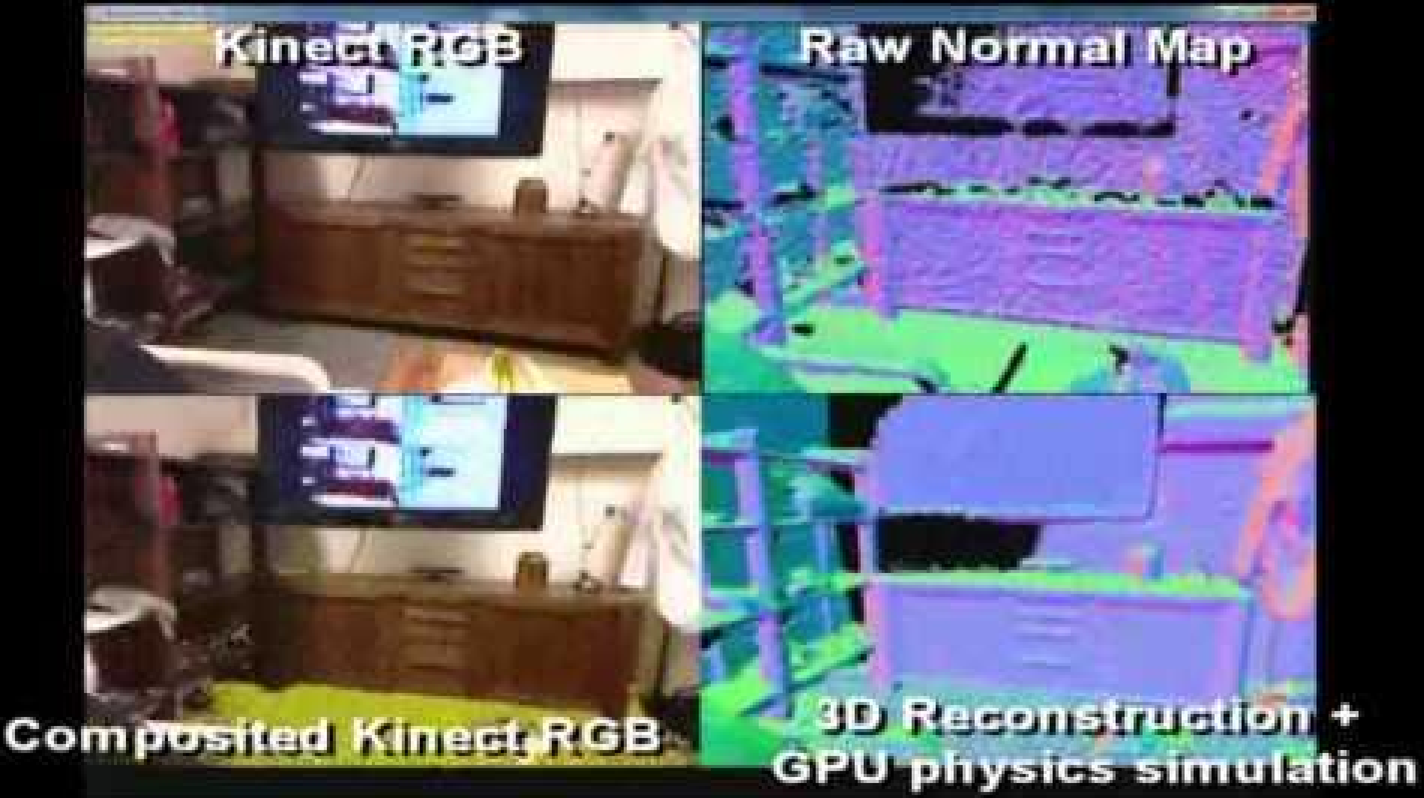
$$w^{0:t} \leftarrow w^{0:t-1} + w^t$$

Truncated Signed Distance Function



Overview

- Explicit and implicit surface representations
- SDF fusion
- SDF tracking
- **Related research**
    - **KinectFusion**
    - PatchVolumes
    - DART
    - DynamicFusion


Kinect RGB
Raw Normal Map
Composited Kinect RGB
3D Reconstruction +
GPU physics simulation

Overview

- Explicit and implicit surface representations
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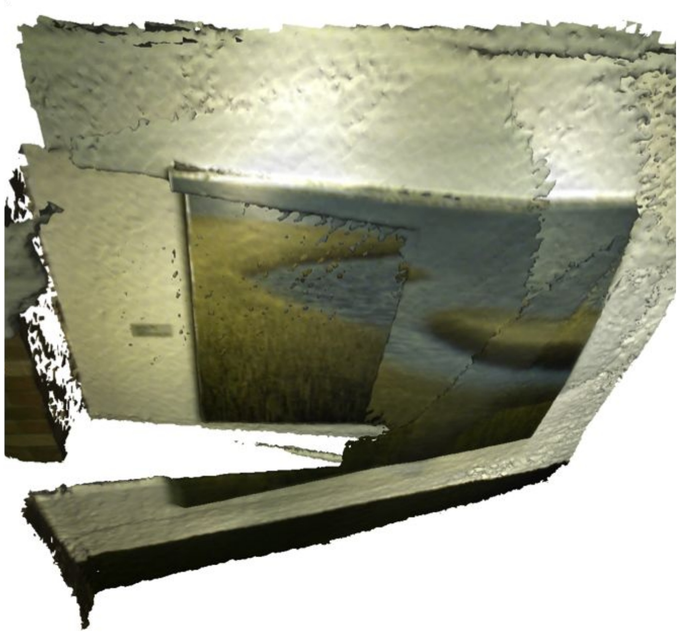
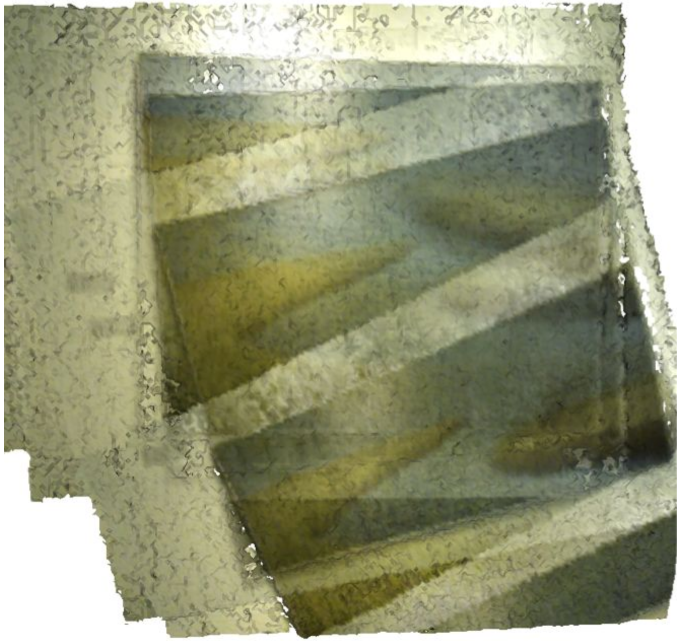
Tracking Failure



Color-only Tracking

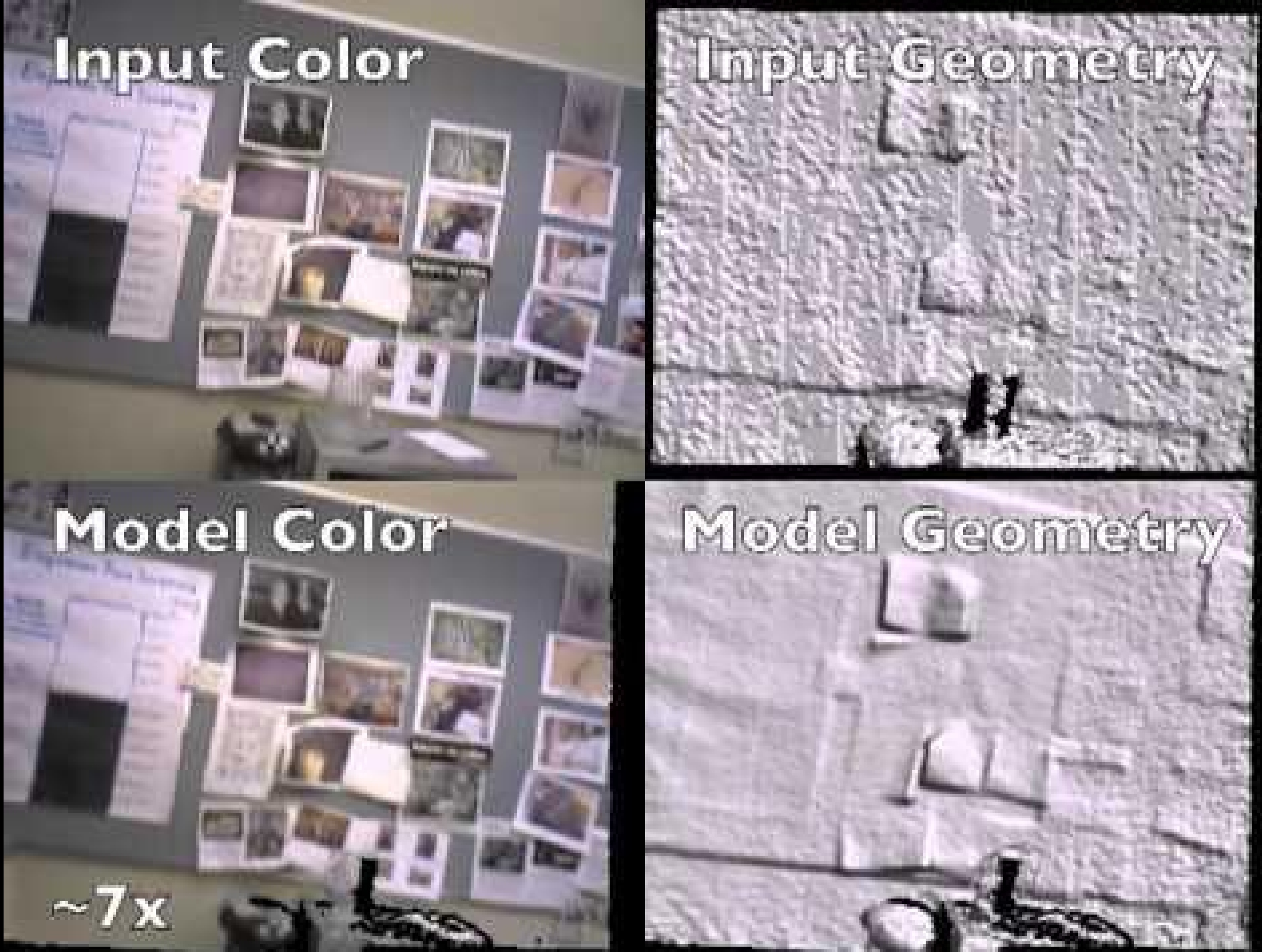


Depth-only Tracking

Loop Closure

Without Loop Closure

With Loop Closure


Input Color
Input Geometry
Model Color
Model Geometry
~7x

Overview

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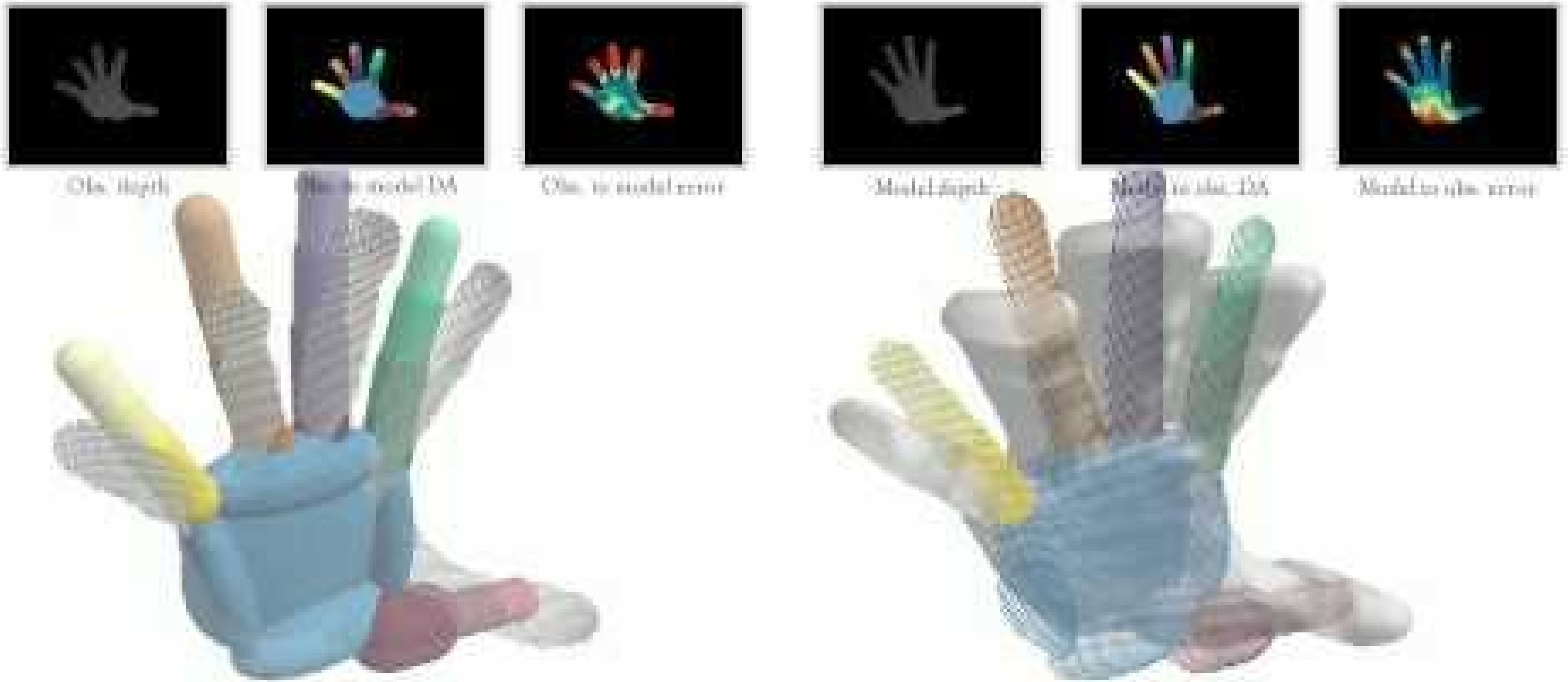
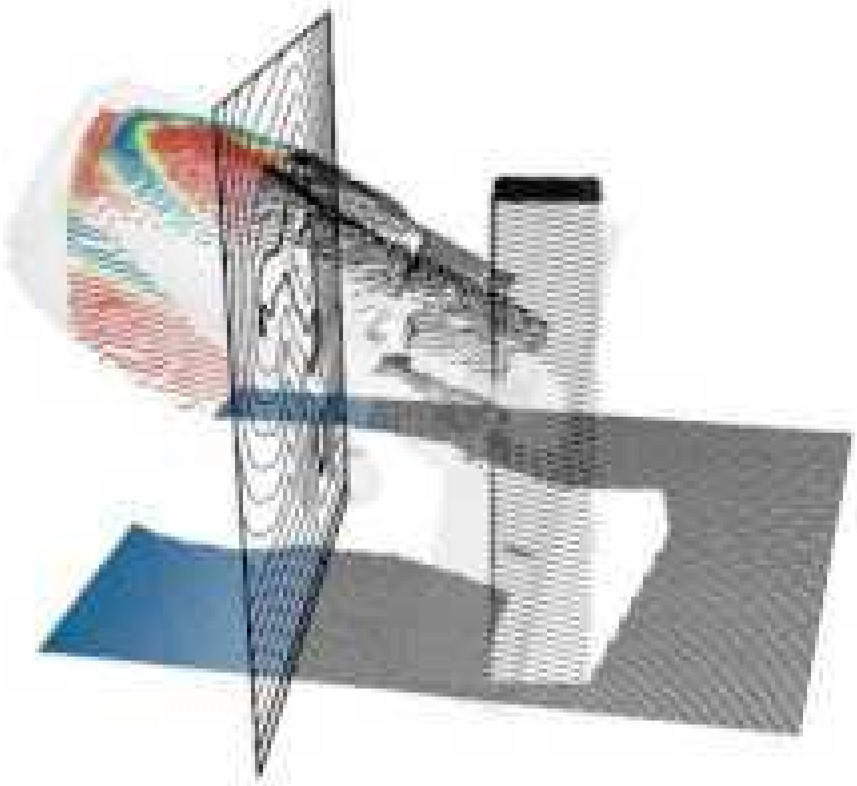
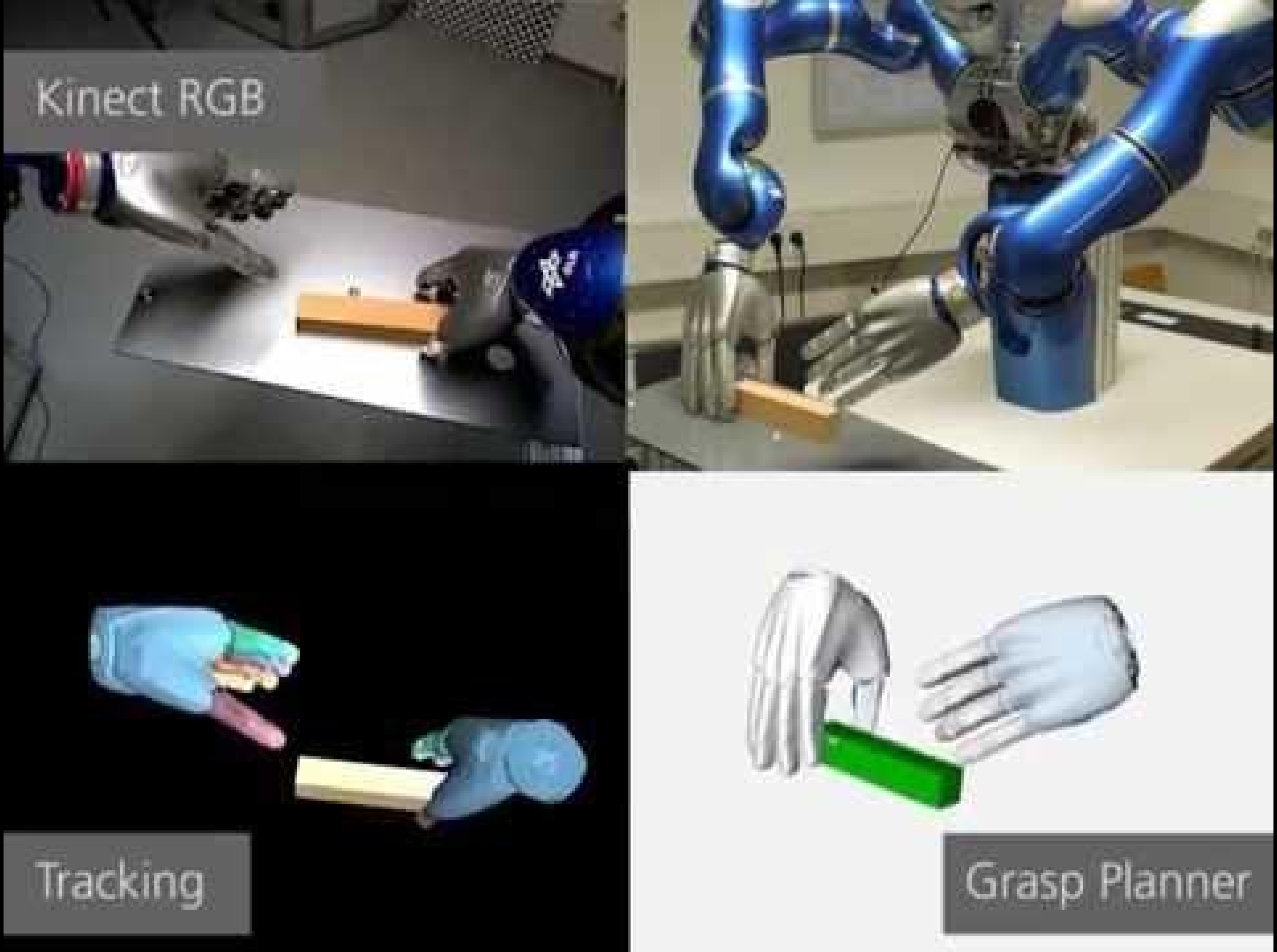


- The full objective function is given by:

$$\hat{\theta} = \arg\min_{\theta} \sum_{u \in \Omega} \mathrm{SDF}_{\mathrm{mdl}}\left(\mathbf{x}_{u}; \theta\right)^{2} + \lambda \sum_{u \in \Omega} \mathrm{SDF}_{\mathrm{obs}}\left(\tilde{\mathbf{x}}_{u}(\theta); D\right)^{2}$$




Kinect RGB
Tracking
Grasp Planner

Overview

- Explicit and implicit surface representations
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    - PatchVolumes
    - DART
    - **DynamicFusion**


Live Input Depth Map
Live Model Output
Live RG age (unused)
Canonical Model Reconstruction
Warped Model