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# Proverb: The Probabilistic Cruciverbalist 

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#### Abstract

We attacked the problem of solving crossword puzzles by computer: given a set of clues and a crossword grid, try to maximize the number of words correctly filled in. After an analysis of a large collection of puzzles, we decided to use an open architecture in which independent programs specialize in solving specific types of clues, drawing on ideas from information retrieval, database search, and machine learning. Each expert module generates a (possibly empty) candidate list for each clue, and the lists are merged together and placed into the grid by a centralized solver. We used a probabilistic representation throughout the system as a common interchange language between subsystems and to drive the search for an optimal solution. Proverb, the complete system, averages $95.3 \%$ words correct and $98.1 \%$ letters correct in under 15 minutes per puzzle on a sample of 370 puzzles taken from the New York Times and several other puzzle sources. This corresponds to missing roughly 3 words or 4 letters on a daily $15 \times 15$ puzzle, making Proverb a better-than-average cruciverbalist (crossword solver).


## Introduction

Proverbs 022:021 That I might make thee know the certainty of the words of truth...
Crossword puzzles are attempted daily by millions of people, and require of the solver both an extensive knowledge of language, history and popular culture, and a search over possible answers to find a set that fits in the grid. This dual task, of answering natural language questions requiring shallow, broad knowledge, and of searching for an optimal set of answers for the grid, makes these puzzles an interesting challenge for artificial intelligence. In this paper, we describe Proverb, the first broad-coverage computer system for solving crossword puzzles ${ }^{1}$. While Proverb's performance is well below that of human champions, it exceeds that of casual human solvers, averaging over $95 \%$ words correct over a test set of 370 puzzles.

[^0]We will first describe the problem and provide some of the insights we gained from studying a large database of crossword puzzles, which motivated our design choices. We will then discuss our underlying probabilistic model and the architecture of Proverb, including how answers to clues are suggested by expert modules, and how we search for an optimal fit of these possible answers into the grid. Finally, we will present the system's performance on a large test suite of daily crossword puzzles, as well as on 1998 tournament puzzles.

## The Crossword Solving Problem

The solution to a crossword puzzle is a set of interlocking words (targets) written across and down a square grid. The solver is presented with an empty grid and a set of clues; each clue suggests its corresponding target. Some clue-target pairs are relatively direct: $\prec$ Florida fruit [6]: orange $\succ^{2}$, while others are more oblique and based on word play: $\prec$ Where to get a date [4]: palm $\succ$. Clues are between one and at most a dozen or so words long, averaging about 2.5 words in a sample of clues we've collected.

To solve a crossword puzzle by computer, we assume that we have both the grid and the clues in machine readable form, ignoring the special formatting and unusual marks that sometimes appear in crosswords. The crossword solving problem will be the task of returning a grid of letters, given the numbered clues and a labeled grid.

In this work, we focus on American-style crosswords, as opposed to British-style or cryptic crosswords. By convention, all targets are at least 3 letters in length and long targets can be constructed by stringing multiple words together: $\prec$ Don't say another word [13]: buttonyourlip $\succ$. Each empty square in the grid must be part of a down target and an across target.

As this is largely a new problem domain, distinct from crossword-puzzle creation (Ginsberg et al. 1990), we wondered how hard crossword solving really was.

[^1]|  | Puzzles |  |  |
| :--- | ---: | ---: | ---: |
| Source | CWDB | Train | Test |
| New York Times (NYT) | 792 | 10 | 70 |
| Los Angeles Times (LAT) | 439 | 10 | 50 |
| USA Today (USA) | 864 | 10 | 50 |
| Creator's Syndicate (CS) | 207 | 10 | 50 |
| CrosSynergy Syndicate (CSS) | 302 | 10 | 50 |
| Universal Crossword (UNI) | 262 | 10 | 50 |
| TV Guide (TVG) | 0 | 10 | 50 |
| Dell | 969 | 0 | 0 |
| Riddler | 764 | 0 | 0 |
| Other | 543 | 0 | 0 |
| Total | $\mathbf{5 1 3 3}$ | $\mathbf{7 0}$ | $\mathbf{3 7 0}$ |

Table 1: Our Crossword Database (CWDB) was drawn from a number of machine-readable sources.

To gain some insight into the problem, we studied a large corpus of existing puzzles. We collected 5133 crossword puzzles from a variety of sources, summarized in Table 1. Several are online versions of daily print newspaper puzzles (The New York Times, The Los Angeles Times, The USA Today, TV Guide), from online sites featuring puzzles (Dell, Riddler) or from syndicates specifically producing for the online medium (Creator's Syndicate, CrosSynergy Syndicate). These puzzles constitute a crossword database (CWDB) of around 350,000 clue-target pairs, with over 250,000 of them unique, which served as a potent knowledge source for this project.

## Novelty

Human solvers improve with experience, in part because particular clues and targets tend to recur. For example, many human solvers will recognize $\prec$ Great Lake [4]: erie $\succ$ to be a common clue-target pair in many puzzles ${ }^{3}$. Our CWDB corresponds to the number of puzzles that would be encountered by a human over a fourteen-year period, at a rate of one puzzle a day.

What percentage of targets and clues in a new puzzle presented to our system will be in the existing database-how novel are crossword puzzles? In Figure 1, we graph the probability of novel targets, clues, clue-target pairs, and clue words as we increase the number of elements in the database.

After randomizing, we looked at subsets of the database ranging from 5,000 clues to almost 350,000 . For each subset, we calculated the percentage of the particular item (target, clue, clue-target, clue word) that are unique. This is an estimate for the likeli-

[^2]

Figure 1: Clue and target novelty decreases with the size of the CWDB. Given all 350,000 clues, we would expect a new puzzle to contain $34 \%$ previously seen clue-target pairs.
hood of the next item being novel. Given the complete database ( 344,921 clues) and a new puzzle, we would expect to have seen $91 \%$ of targets, $50 \%$ of clues, and $34 \%$ of clue-target pairs. We would also expect to have seen $96 \%$ of the words appearing in the clues. The CWDB clearly contains a tremendous amount of useful domain-specific information.

## The New York Times Crossword Puzzle

The New York Times (NYT) crossword is considered by many to be the premiere daily puzzle. NYT editors attempt to make the puzzles increase in difficulty from easy on Monday to very difficult on Saturday and Sunday. We hoped that studying the Monday-to-Saturday trends in the puzzles might provide insight into what makes a puzzle hard for humans.

In Table 2, we show how the distributions of clue types change day by day. For example, note that some "easier" clues, such as fill-in-the-blank clues $\prec$ Mai _-_ [3]: tai $\succ$ ) get less and less common as the week goes on. In addition, clues with a trailing question mark ( $\prec$ T.V. Series? [15]: sonyrcamagnovox $\succ$ ), which is often a sign of a themed or pun clue, get more common. The distribution of target lengths also varies, with words in the 6 to 10 letter range becoming much more common from Monday to Saturday. Sunday is not included in the table as it is a bit of an outlier on some of these scales, partly due to the fact that the puzzles are larger (up to $23 \times 23$, as opposed to $15 \times 15$ for the other days).

## Categories of Clues

In the common syntactic categories shown in Table 2, such as fill-in-the-blank and quoted phrases, clue structure leads to simple ways to answer those clues. For example, given the clue $\prec \ldots \ldots$ miss [5]: hitor $\succ$, we

|  | Mon | Tue | Wed | Thu | Fri | Sat |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| \#puz | 89 | 92 | 90 | 91 | 91 | 87 |
| \#clues | 77.3 | 77.2 | 76.7 | 74.7 | 70.0 | 70.2 |
| 3 | 16.5 | 18.2 | 17.5 | 18.6 | 17.3 | 16.3 |
| $4-5$ | 64.6 | 61.1 | 62.5 | 54.7 | 44.2 | 40.2 |
| 6-10 | 15.8 | 17.7 | 16.9 | 23.1 | 35.2 | 41.7 |
| $11-15$ | 3.1 | 2.9 | 3.2 | 3.7 | 3.3 | 1.9 |
| Blank | 8.4 | 8.0 | 6.4 | 6.4 | 5.2 | 4.8 |
| Blank \& " | 3.1 | 3.1 | 2.7 | 2.2 | 2.0 | 1.7 |
| Single Word | 15.6 | 14.9 | 16.0 | 17.2 | 16.9 | 20.6 |
| (Year) | 1.4 | 1.6 | 1.9 | 2.1 | 2.5 | 2.7 |
| Final '?' | 0.8 | 1.2 | 2.5 | 3.2 | 3.5 | 2.6 |
| X, in a way | 0.0 | 0.1 | 0.2 | 0.4 | 0.6 | 0.8 |

Table 2: NYT clue statistics vary by day of week.
might scan through text sources looking for all 9-letter phrases that match on word boundaries and known letters. If encounter a clue such as $\prec$ Map abbr. [3]: rte $\succ$, we might want to return a list of likely abbreviations.

In addition, a number of non-syntactic, expert categories stand out, such as synonyms ( $\prec$ Covered [5]: awash $\succ$ ), kind-of ( $\prec$ Kind of duck or letter [4]: dead $\succ$ ), movies ( $\prec 1954$ mutant ants film [4]: them $\succ$ ), geography ( $\prec$ Frankfurt's river [4]: oder $\succ$ ), music ( $\prec$ 'Upside down' singer [4]: ross $\succ$ ) and literature ( $\prec$ Carroll character [5]: alice $\succ$ ).

There are also clues that do not fit simple pattern, but might be solved by existing information retrieval techniques ( $\prec$ Nebraska tribesman [4]: otoe $\succ$ ). Given the many different sources of information that can be brought to bear to solve different types of clues, this suggests a two-stage architecture for our solver: one consisting of a collection of special-purpose and general candidate-generation modules, and one that combines the results from these modules to generate a solution to the puzzle. This decentralized architecture allowed a relatively large group of contributors (approximately ten people) to contribute modules using techniques ranging from generic word lists to highly specific modules, from string matching to general-purpose information retrieval. The next section describes Proverb's modular design.

## Architecture

Figure 2 illustrates the components of Proverb. Given a puzzle, the Coordinator separates the clues from the grid and sends a copy of the clue list (with target lengths) to each Expert Module. The expert modules generate probability-weighted candidate lists, in isolation from the other grid constraints. Expert modules are free to return no candidates for any clues, or 10,000 for every one. The collection of candidate lists is then reweighted by the Merger to compensate for differences in module weighting, and combined into a single list of candidates for each clue. Finally, the Solver takes these weighted lists and searches for the best solution it can find that also satisfies the grid con-


Figure 2: Proverb consists of a set of independent communicating programs written in Java, C, C++, and Perl.
straints.
The Implicit Distribution Modules are used by the solver, and are described in a later section.

## The Probabilistic Model

To unify the candidate-generation modules, it is important to first understand our underlying assumptions about the crossword-puzzle problem. First, assume that crossword puzzles are created by repeatedly choosing words for the slots according to a particular creator's distribution (ignore clues and crossing constraints for now). After choosing the words, if the crossing constraints are satisfied, then the creator keeps the puzzle. Otherwise, the creator draws again. Normalizing to account for all the illegal puzzles generated gives us a probability distribution over legal puzzles.

Now, suppose that for each slot in a puzzle, we had a probability distribution over possible words for the slot given the clue. Then, we could try to solve one of a number of probabilistic optimization problems to produce the "best" fill of the grid. In our work, we define "best" as the puzzle with the maximum expected number of targets in common with the creator's solution: the maximum expected overlap. We will discuss this optimization more in a following section, but for now it is important only to see that we would like to think of candidate generation as establishing probability distributions over possible solutions.

We will next discuss how individual modules can create approximations to these distributions, how we can combine them into a unified distributions, and then finally how we can search to find a good solution to the optimization problem.

## Candidate-List Generation

The first step is to have each module generate candidates for each clue, given the target length. Each module returns a confidence score (how sure it is that
the answer lies in its list), and a weighted list of possible answers. For example, given the clue $\prec$ Farrow of 'Peyton Place' [3]: mia $\succ$, the movie module returns:

$$
\begin{aligned}
1.0: & 0.909091 \mathrm{mia}, 0.010101 \mathrm{tom}, 0.010101 \mathrm{kip}, \cdots \\
& \cdots, 0.010101 \text { ben, } 0.010101 \mathrm{peg}, 0.010101 \text { ray }
\end{aligned}
$$

The module returns a 1.0 confidence in its list, and gives higher weight to the person on the show with the given last name, while giving lower weight to other cast members.

Note that most of the modules will not be able to generate actual probabilities distributions for the targets, and will need to make approximations. The merging step discussed next will attempt to account for the error in these estimates by testing on training data, and adjusting scaling parameters to compensate. It is important for modules to be consistent, and to give more likely candidates more weight. Also, the better control a module exerts over the overall confidence score when uncertain, the more the merger will "trust" the module's predictions.
In all, we built 30 different modules, many of which are described briefly below. To get some sense of the contribution of the major modules, Table 3 summarizes performance on 70 puzzles, containing 5374 clues. These puzzles were drawn from the same sources as the test puzzles, ten from each. For each module, we list several measures of performance: the percentage of clues that the module guessed at, the percentage of the time the target was in the module's candidate list, the average length of the returned lists, and the percentage of clues the module "won"-it had the correct answer weighted higher than all other modules. This final statistic is an important measure of the module's contribution to the system. For example, the WordList-Big module generates over 100,000 words for some clues, so it often has the target in its list ( $97 \%$ of the time). However, since it generates so many, the individual weight given to the target is usually lower than that assigned by other modules, and, thus, it is the best predictor only $0.1 \%$ of the time.

Another way of looking at the contribution of the modules is to consider the probability assigned to each target given the clues. Ideally, we would like all targets to have probability 1 . In general, we want to maximize the product of the probabilities assigned to the targets, since this quantity is directly related to what the solver will be maximizing. In Figure 3, the top line represents the probability assigned by the Bigram module (described later). This probability is low for all targets, but very low for the hard targets. As we add groups of modules, the effect on the probabilities assigned to targets can be seen as a lowering of the curve, which corresponds to assigning more and more probability to the target. Note the large increase due to the Exact Match module. Finally, notice that there is a small segment that we do very poorly on-the targets that no module other than Bigram returns. We


Figure 3: The cumulative probability assigned as module groups are added shows that different types of modules make different contributions. Each line is sorted independently.
will later introduce extensions to the system that help with this range.

## Word List Modules

WordList, WordList-Big These modules ignore their clues and return all words of the correct length from several dictionaries. WordList contains a list of 655,000 terms from a wide variety of sources, including online texts, enclyopedias and dictionaries. WordList-Big contains everything in WordList, as well as many constructed 'terms', produced by combining related entries in databases. This includes combining first and last names, as well as merging adjacent words from clues in the CWDB. WordListBig contains over 2.1 million terms.
WordList-CWDB WordList-CWDB contains the 58,000 unique targets in the CWDB, and returns all targets of the appropriate length, regardless of the clue. It weights them with estimates of their "prior" probabilities as targets of arbitrary clues.

## CWDB-Specific Modules

Exact Match This module returns all targets of the correct length associated with this clue in the CWDB. Confidence is based on a Bayesian calculation involving the number of exact matches of correct and incorrect lengths.
Transformations This module learns a set of textual transformations which, when applied to clue-target pairs in the CWDB, generates other clue-target pairs in the database. When faced with a new clue, it applies all applicable transformations and returns the results, weighted based on the previous precision/recall of these transformations. Transformations in the database include single-word substitution, removing one phrase from the beginning or end of a clue and adding another phrase to the beginning

| Module | Guess | Acc | Len | Best |
| :--- | ---: | ---: | ---: | ---: |
| Bigram | 100.0 | 100.0 | - | 0.1 |
| WordList-Big | 100.0 | 97.2 | $\approx 10^{5}$ | 1.0 |
| WordList | 100.0 | 92.6 | $\approx 10^{4}$ | 1.7 |
| WordList-CWDB | 100.0 | 92.3 | $\approx 10^{3}$ | 2.8 |
| ExactMatch | 40.3 | 91.4 | 1.3 | 35.9 |
| Transformation | 32.7 | 79.8 | 1.5 | 8.4 |
| KindOf | 3.7 | 62.9 | 44.7 | 0.8 |
| Blanks-Books | 2.8 | 35.5 | 43.8 | 0.1 |
| Blanks-Geo | 1.8 | 28.1 | 60.3 | 0.1 |
| Blanks-Movies | 6.0 | 71.2 | 35.8 | 3.2 |
| Blanks-Music | 3.4 | 40.4 | 39.9 | 0.4 |
| Blanks-Quotes | 3.9 | 45.8 | 49.6 | 0.1 |
| Movies | 6.3 | 66.4 | 19.0 | 2.2 |
| Writers | 0.1 | 100.0 | 1.2 | 0.1 |
| Compass | 0.4 | 63.6 | 5.9 | 0.0 |
| Geography | 1.8 | 25.3 | 322.0 | 0.0 |
| Myth | 0.1 | 75.0 | 61.0 | 0.0 |
| Music | 0.9 | 11.8 | 49.3 | 0.0 |
| WordNet | 42.8 | 22.6 | 30.0 | 0.9 |
| WordNetSyns | 11.9 | 44.0 | 3.4 | 0.9 |
| RogetSyns | 9.7 | 42.9 | 8.9 | 0.4 |
| MobySyns | 12.0 | 81.6 | 496.0 | 0.4 |
| Encyclopedia | 97.9 | 32.2 | 262.0 | 1.3 |
| LSI-Ency | 94.7 | 43.8 | 995.0 | 1.0 |
| LSI-CWDB | 99.1 | 77.6 | 990.0 | 1.2 |
| PartialMatch | 92.6 | 71.0 | 493.0 | 8.1 |
| Dijkstra1 | 99.7 | 84.8 | 620.0 | 4.6 |
| Dijkstra2 | 99.7 | 82.2 | 996.0 | 8.7 |
| Dijkstra3 | 99.5 | 80.4 | 285.0 | 13.3 |
| Dijkstra4 | 99.5 | 80.8 | 994.0 | 0.1 |

Table 3: Performance on 70 puzzles ( 5374 clues) shows differences in the number of targets returned (Len) and contribution to the overall lists (Best). Also measured but not shown are the implicit modules.
or end of the clue, depluralizing a word in the clue and pluralizing the associated target, and others. The following is a list of several non-trivial examples from the tens of thousands of transformations learned:

> | nice $X \leftrightarrow X$ in france | $X$ starter $\leftrightarrow$ prefix with $X$ |
| :---: | :---: |
| $X$ for short $\leftrightarrow X$ abbr | $X$ city $\leftrightarrow X$ capital |

## Information Retrieval Modules

Crossword clues present an interesting challenge to traditional information retrieval (IR) techniques. While queries of similar length to clues have been studied, the "documents" to be returned are quite different (words or short sequences of words). In addition, the queries themselves are often purposely phrased to be ambiguous, and never share words with the "documents" to be returned. Despite these differences, it seemed natural to try a variety of existing IR techniques over several document collections.
Encyclopedia This module is based on an indexed set of encyclopedia articles. For each query term, we compute a distribution of terms "close" to the
query term in the text. A term is counted $10-k$ times in this distribution for every time it appears at a distance of $k<10$ words away from the query term. A term is also counted once if it appears in an article for which the query term is in the title, or vice versa. Terms of the correct target length are assigned scores proportional to their frequencies in the "close" distribution, divided by their frequency in the corpus. The distribution of scores is normalized to 1 . If a query contains multiple terms, the score distributions are combined linearly according to the log inverse frequency of the query terms in the corpus. If the query contains very common terms such as "as" and "and," they are ignored.
Partial Match Consider the standard vector space model (Salton \& McGill 1983), defined by a vector space with one dimension for every word in the dictionary. A clue is represented as a vector in this space. For each word $w$ a clue contains, it gets a component in dimension $w$ of magnitude $-\log ($ frequency $(w))$.
For a clue $c$, we find all clues in the CWDB that share words with $c$. For each such clue, we give its target a weight based on the dot product of the clue with $c$. The assigned weight is geometrically interpolated between $1 /$ size(dictionary) and 1 based on this dot product.
LSI Latent semantic indexing (LSI) is an extension of the vector space model that uses singular value decomposition to identify correlations between words. LSI has been successfully applied to the problem of synonym selection on a standardized test (Landauer \& Dumais 1997), which is closely related to solving crossword clues. Our LSI modules were trained on CWDB (all clues with the same target were treated as a document) and separately on an online encyclopedia and returned the closest words (by cosine) with each clue.
Dijkstra Modules The Dijkstra modules were inspired by the intuition that related words either co-occur with one another or co-occur with similar words. This suggests a measure of relatedness based on graph distance. From a selected set of text databases, the module builds a weighted directed graph on the set of all terms. For each database $d$ and each pair of terms $(t, u)$ that co-occur in the same document, we place an edge from $t$ to $u$ in the graph with weight,

$$
-\log \left(\frac{\# \text { documents in } d \text { containing } t \text { and } u}{\# \text { documents in } d \text { containing } t}\right) .
$$

For a one-word clue $t$, we assign a term $u$ a score of $-\log ($ fraction of documents containing $t)-$ weight(minimum weight path $t \rightarrow u$ ).
We find the highest scoring terms with a shortest-path-like search. For a multi-word clue, we break the clue into individual terms and add the scores as
computed above. The four Dijkstra modules in our system use variants of this technique.
For databases, we used an encyclopedia index, two thesauri, a database of wordforms and the CWDB.

## Database Modules

Movie The Internet Movie Database (www.imdb.com) is an online resource with a wealth of information about all manner of movies and T.V. shows. This module looks for a number of patterns in the clue (e.g. quoted titles as in $\prec ' A l i c e ' ~ s t a r ~ L i n d a ~[5]: ~$ lavin $\succ$, or Boolean operations on names as in <Cary or Lee [5]: grant $\succ$ ), and formulates queries to a local copy of the database in a variety of forms.
Music, Literary, Geography These modules use simple pattern matching of the clue (looking for keywords "city", "author", "band" and others as in $\prec$ Iowa city [4]: ames $\succ$ ) to formulate a query to a topical database. The literary database is culled from both online and encyclopedia resources. The geography database is from the Getty Information Institute, with additional data supplied from online lists.
Synonyms There are four distinct synonym modules, based on three different thesauri. Using the WordNet (Miller et al. 1990) database, one module looks for root forms of words in the clue, and then finds a variety of related words (e.g. $\langle$ Stroller [6]: gocart $\succ$ ). In addition, a type of relevance feedback is used to generate lists of synonyms of synonyms. Finally, if necessary, the forms of the related words are coverted back to the form of the original clue word (number, tense, etc.), for example $\prec$ Contrives [7]: devises $\succ$.

## Syntactic Modules

Fill-in-the-Blanks Over five percent of all clues in CWDB have a blank in them. We searched a variety of databases to find clue patterns with a missing word (music, geography, movies, literary and quotes). For example, given $\prec ' T i m e ~ \ldots-~ M y ~ S i d e ’ ~$ (Stones hit) [4]: ison $\succ$, these modules would search for the pattern time .... my side, allowing any four charaters to fill the blanks, including multiple words. In some of our pretests we also ran these searches over more general sources of text like encyclopedias and archived news feeds, but for efficiency, we left these out of the final runs.
KindOf "Kind of" clues are similar to fill-in-theblank clues in that they involve pattern matching over short phrases. We identified over 50 cues that indicate a clue of this type, for example, "starter for" ( $\langle$ Starter for saxon [5]: anglo $\succ$ ), and "suffix with" ( $\langle$ Suffix with switch or sock [4]: eroo〉).

## Merging Candidate Lists

After each expert module has generated a weighted candidate list, we must somehow merge these into a
unified candidate list with a common weighting scheme for the solver. This problem is similar to the problem facing meta-crawler search engines in that separately weighted return lists must be combined in a sensible way. An advantage of this domain is ready access to precise and abundant training data.

For a given clue, each expert module $m$ returns a weighted set of candidates and a numerical level of confidence that the correct target is in this set. For each expert module $m$, we set three real parameters: $\operatorname{scale}(m)$, length-scale $(m)$ and $\operatorname{spread}(m)$. For each clue, we reweight the candidate set by raising each weight to the power spread $(m)$, then normalizing their sum to 1 . We multiply the confidence level by the product of scale $(m)$ and length-scale $(m)^{\text {targetlength }}$. To compute our combined probability distribution over candidates, we linearly combine the modified candidate sets of all the modules weighted by their modified confidence levels, and normalize the sum to 1 .

The scale, length-scale and spread parameters give the merger control over how the information returned by an expert module is incorporated into the final candidate list. We set these parameters using a naive hillclimbing technique.

The objective function for optimization is the average log probability assigned to the correct target. This corresponds to maximizing the average log probability assigned by the solver to the correct puzzle fill-in, since in our model the probability of a puzzle solution is proportional to the product of the prior probabilities on the answers in each of the slots. The optimal value we achieve on the 70 puzzle training set is $\log \left(\frac{1}{33.56}\right)$.

## Grid Filling

After realizing how much repetition occurs in crosswords, in both targets and clues, and therefore how well the CWDB covers the domain, one might wonder whether this coverage is enough to constrain the domain to such an extent that there is not much for the grid-filling algorithm to do. We did not find this to be the case. Simplistic grid filling yielded only mediocre results. As a measure of the task left to the grid-filling algorithm, on the first iteration of solving, using just the weighted candidate lists from the modules, only $40.9 \%$ of targets are in the top of the candidate list for their slot. However, the grid-filling algorithm is able to raise this to $89.4 \%{ }^{4}$
The algorithm employed by Proverb (Shazeer, Littman, \& Keim 1999) models grid filling as an optimization problem. In particular, the across and down letter intersections establish constraints on how the grid can be filled, and crossword-puzzle filling is often cited as a constraint satisfaction problem. However, in our case, we don't just want to find any satisfying set of candidates for the slots; we want the "best" fit. We can define "best" in several different ways, but

[^3]in these tests we attempted to maximize the expected overlap with the creator's solution, in terms of words correct. Other definitions of "best" include maximizing the probability of getting the entire puzzle correct, or maximizing expected letter overlap. The decision to use expected word overlap is motivated by the scoring system used in human tournaments (see below). Since finding the optimal solution to this problem is intractable, we employ a variety of efficient approximations.

## Implicit Distribution Modules

Our probability measure assigns probability zero to a target that is suggested by no module and probability zero to all solutions containing that target. Therefore, we need to assign non-zero probability to all letter sequences. Clearly, there are too many to actually list explicitly. We augmented the solver to reason with probability distributions over candidate lists that are implicitly represented. These Implicit Distribution Modules generate additional candidates once the solver can give them more information about letter probability distributions over the slot.

The most important of these is a letter Bigram module, which "generates" all possible letter sequences of the given length by returning a letter bigram distribution over all possible strings, learned from the CWDB. Because the bigram probabilities are used throughout the solution process, this module is actually tightly integrated into the solver itself.
Note in Figure 3 that there are some clues for which no module except Bigram is returning the target. In a pretest run on 70 puzzles, the clue-target with the lowest probability was $\prec$ Honolulu wear [14]: hawaiianmuumuu $\succ$. This target never occurs in the CWDB, although both muumuu and hawaiian occur multiple times, and it gets a particularly low probability because of the many unlikely letter pairs in the target. Once the grid-filling process is underway, we have probability distributions for each letter in these longer targets and this can limit our search for candidates.

To address longer, multiword targets, we created free-standing implicit distribution modules. Each implicit distribution module takes a letter probability distribution for each letter of the slot (computed within the solver), and returns weighted candidate lists. These lists are then added to the previous candidate lists, and the grid-filling algorithm continues. This process of getting new candidates can happen several times during the solution process.
Tetragram The tetragram module suggests candidates based on a letter tetragram model, built from the WordList-Big. We hoped this would provide a better model for word boundaries than the bigram model mentioned above, since this list contains many multiword terms.

Segmenter The segmenter calculates the $n$ most probable word sequences with respect to both the letter probabilities and word probabilities from several sources ( $n=10$ currently) using dynamic programming. The base word probabilities are unigram word probabilities from the CWDB. In addition, the Dijkstra module (described above) suggests the best 1000 words (with weights) given the current clue. These weights and the unigram probabilities are then combined for a new distribution of word probabilities.
For example, consider the clue $\prec$ Tall footwear for rappers? [11]: hiphopboots $\succ$. Given a letter distribution and a combined word distribution, the segmenter returned the following top ten during: tiptopboots, hiphoproots, hiphopbooks, hiphoptoots, hiphopboots, hiphoproofs, riptaproots, hippopboots, hiptaproots, hiptapboots. Note that the reweighting done by the Dijkstra module by examining the clue raises the probabilites of related words like boots.

## Results

To evaluate Proverb's performance, we ran it on a large collection of daily puzzles, and on the most recent human tournament puzzles.

## Daily Puzzles

We tested the system on puzzles from seven daily sources, listed in Table 1. The TV Guide puzzles go back to 1996, but the other sources were all from between August and December of 1998. We selected 70 puzzles, 10 from each source, as training puzzles for the system. The reweighting process described above was trained on the 5374 clues from these 70 puzzles. Additional debugging and modification of the modules was done after evaluation on these training puzzles.
Having fixed the modules and reweighting parameters, we then ran the system on the 370 puzzles in the final pool. The system acheived an average $95.3 \%$ words correct, $98.1 \%$ letters correct, and $46.2 \%$ puzzles completely correct ( $94.1 \%, 97.6 \%$, and $37.6 \%$ without the implicit distribution modules).
In Figure 4, we plot the scores on each of the 370 daily puzzles attempted by Proverb, grouped by the source. In addition, we split the NYT puzzles into two groups: Monday through Wednesday (MTW), and Thursday through Sunday (TFSS). As noted earlier, there is an effort made at the NYT to make puzzles increasingly difficult as the week progresses, and with respect to Proverb's performance they have succeeded. ${ }^{5}$

[^4]

Figure 4: Proverb performance on a variety of daily crossword puzzles.

## Tournament Puzzles

To better gauge the system's performance against humans, we tested Proverb using puzzles from the 1998 American Crossword Puzzle Tournament (ACPT) (Shortz 1990). The ACPT has been held annually for 20 years, and was attended in 1998 by 251 people. The scoring system for the ACPT requires that a time limit be set for each puzzle. A solver's score is then 10 times the number of words correct, plus a bonus of 150 if the puzzle is completely correct. In addition, the number of incorrect letters is subtracted from the full minutes early the solver finishes. If this number is positive, it is multiplied by 25 and added to the score.

There were seven puzzles in the offical contest, with time limits ranging from 15 to 45 minutes. We used the same version of Proverb described in the previous section. The results over the 1998 puzzles are shown in Table 4. The best human solvers at the competition finished all puzzles correctly, and the winner was determined by finishing time (the champion averaged under seven minutes per puzzle). Thus, while not competitive with the very best human solvers, Proverb would have placed 213 out of 252 ; its score on Puzzle 5 exceeded that of the median human solver at the contest.

The ACPT puzzles are very challenging, and include tricks like multiple letters or words written in a single grid cell, and targets written in the wrong slot. In spite of the fact that Proverb could not produce answers that bend the rules in this way, it still correctly filled in $80 \%$ of the words correctly, on average. The implicit distribution modules ("Proverb(I)") helped improve the word score on these puzzles, but brought down the tournament score because it works more slowly.

| Name | Rank | Total | Avg <br> Time |
| :--- | ---: | ---: | ---: |
| D TP (Maximum) | 1 | 13140 | $1: 00$ |
| TP (Champion) | 1 | 12115 | $6: 51$ |
| JJ (75\%) | 62 | 10025 | - |
| MF (50\%) | 125 | 8575 | - |
| MB (25\%) | 187 | 6985 | - |
| D Proverb-I (24\%) | 190 | 6880 | $1: 00$ |
| Proverb (15\%) | 213 | 6215 | $9: 41$ |
| Proverb-I (15\%) | 215 | 6130 | $15: 07$ |

Table 4: Proverb compared to the 251 elite human contestants at the 1998 championship. Lines preceded by a $\triangleright$ indicate the theoretical score if the solver did every puzzle in under a minute.

## Conclusions

Solving crossword puzzles presents a unique artificial intelligence challenge, demanding from a competitive system broad world knowledge, powerful constraint satisfaction, and speed. Because of the widespread appeal, system designers have a large number of existing puzzles to use to test and tune their systems, and humans with whom to compare.

A successful crossword solver requires many artificial intelligence techniques; in our work, we used ideas from state-space search, probabilistic optimization, constraint satisfaction, information retrieval, machine learning and natural language processing. We found probability theory a potent practical tool for organizing the system and improving performance.

The level of success we acheived would probably not have been possible five years ago, as we depended on extremely fast computers with vast memory and disk storage, and used tremendous amounts of data in machine readable form. Perhaps the time is ripe to use these resources to attack other problems previously deemed too challenging for AI.

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[^0]:    ${ }^{1}$ Crossword Maestro is a commercial solver for Britishstyle crosswords published by Genius 2000 Software. It is intended as a solving aid, and while it appears quite good at thesaurus-type clues, in informal tests it did poorly at grid filling (under $5 \%$ words correct).

[^1]:    ${ }^{2}$ Target appears in fixed-width font; all examples are taken from our clue database. We will note the target length following sample clues in this paper to indicate a complete specification of the clue.

[^2]:    ${ }^{3}$ The five most common targets in the database are era, ore, area, erie and ale. The target erie appears in over $7 \%$ of puzzles. The five most common clues are "Exist," "Greek letter," "Jai ___," "Otherwise," and "Region". The five most common clue-target pairs are $\prec$ Exist [3]: are $\succ$, $\prec$ Jai ___ [4]: alai $\succ$, $\prec$ Otherwise [4]: else $\succ, \prec$ Region [4]: area $\succ$, and $\prec$ Anger [3]: ire $\succ$.

[^3]:    ${ }^{4}$ On average, over the 70 NYT puzzles in the test suite.

[^4]:    ${ }^{5}$ In some of our earlier tests, there appeared to be a finer day-by-day trend from Monday to Saturday, but there is not enough data in this set (10 per day) to see this.

