CE 573: Artificial Intelligence

Autumn 2010

Lecture 3: A* Search
10/7/2010

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Based on slides from Dan Klein

Multiple slides from Stuart Russell or Andrew Moore

Announcements

- Projects:
  - Project 1 (Search) is out, due next Friday Oct 15th
  - You don't need to submit answers the project's discussion questions
  - Can talk to each other, but must write own solutions

Today

- A* Search
- Heuristic Design
- Graph search

Recap: Search

- Search problem:
  - States (configurations of the world)
  - Successor function: a function from states to lists of (state, action, cost) triples; drawn as a graph
  - Start state and goal test
- Search tree:
  - Nodes: represent plans for reaching states
  - Plans have costs (sum of action costs)
- Search Algorithm:
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)

Example: Pancake Problem

Action: Flip over the top *n* pancakes

Cost: Number of pancakes flipped

Example: Pancake Problem

BOUNDS FOR SORTING BY PREFIX REVERSAL

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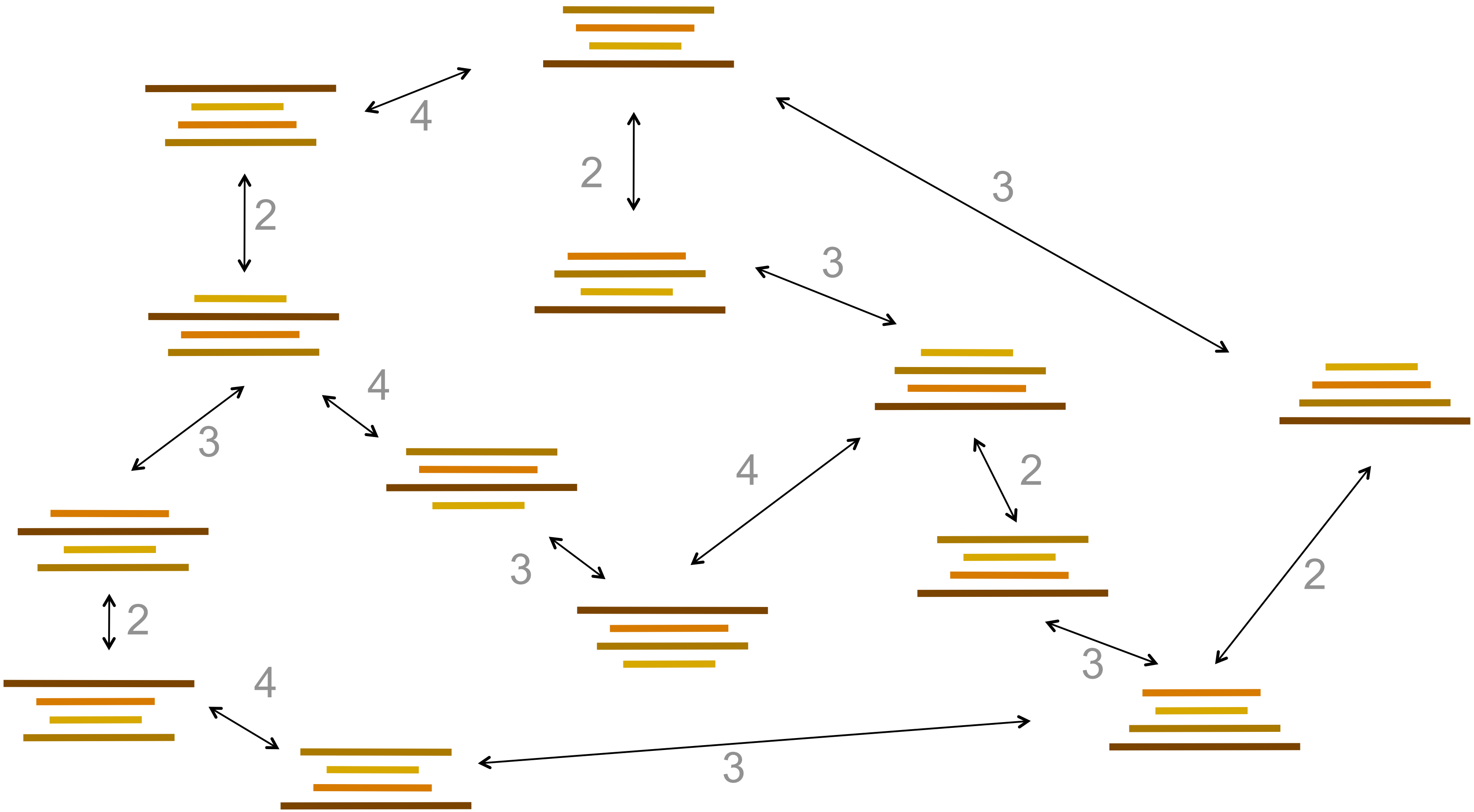
Received 18 January 1978
Revised 28 August 1978

For a permutation σ of the integers from 1 to n, let $f(\sigma)$ be the smallest number of prefix reversals that will transform σ to the identity permutation, and let $f(n)$ be the largest such $f(\sigma)$ for all σ in (the symmetric group) S_n. We show that $f(n) \leqslant (5n+5)/3$, and that $f(n) \geqslant 17n/16$ for n a multiple of 16. If, furthermore, each integer is required to participate in an even number of reversed prefixes, the corresponding function $g(n)$ is shown to obey $3n/2 - 1 \leqslant g(n) \leqslant 2n+3$.

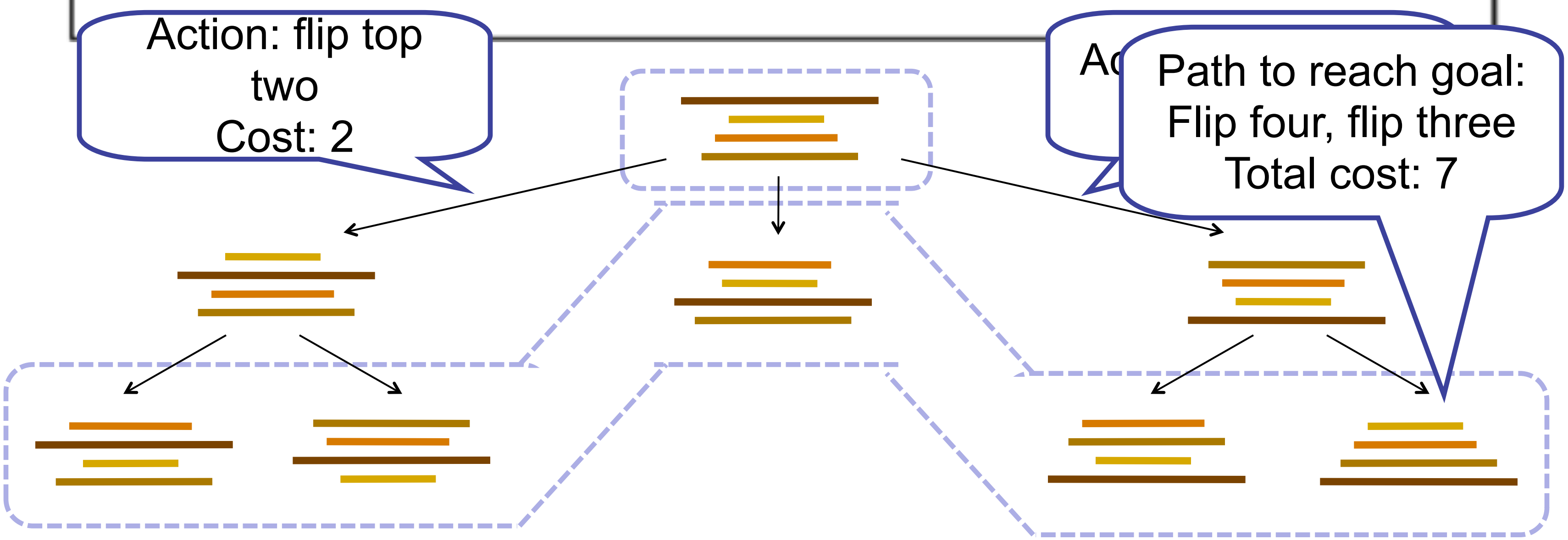
Example: Pancake Problem

State space graph with costs as weights



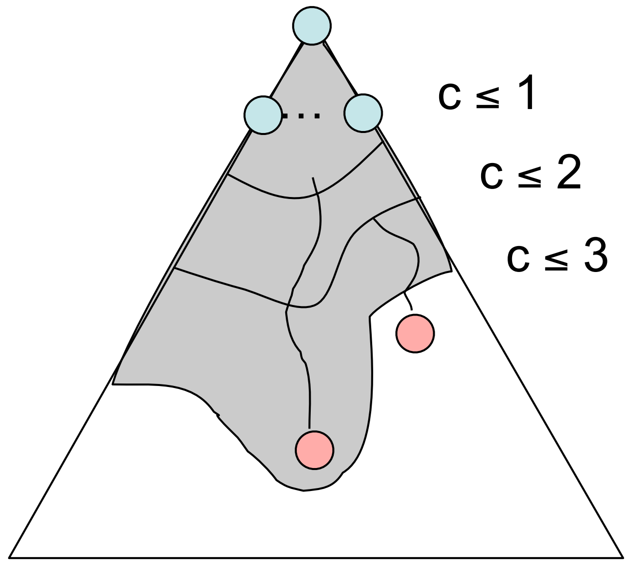
General Tree Search

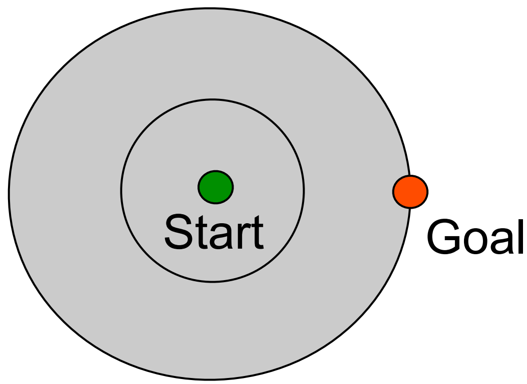
```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```



Uniform Cost Search

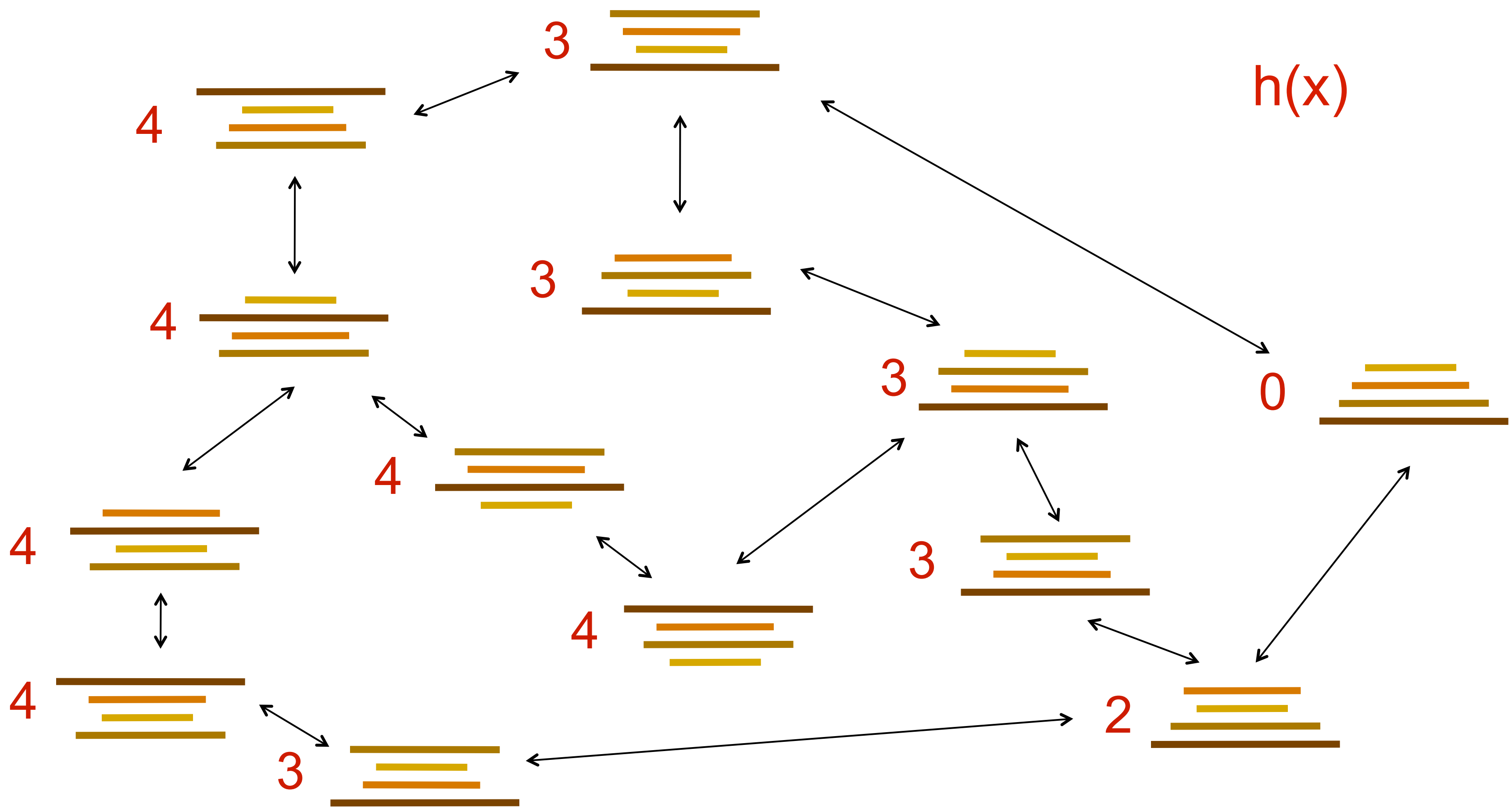
- Strategy: expand lowest path cost
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every "direction"
  - No information about goal location





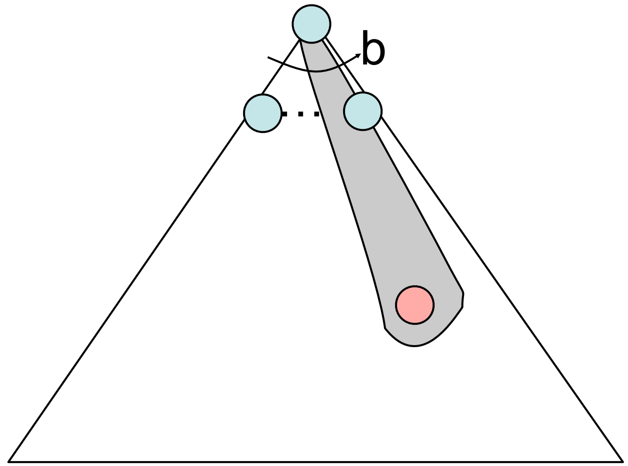
Example: Heuristic Function

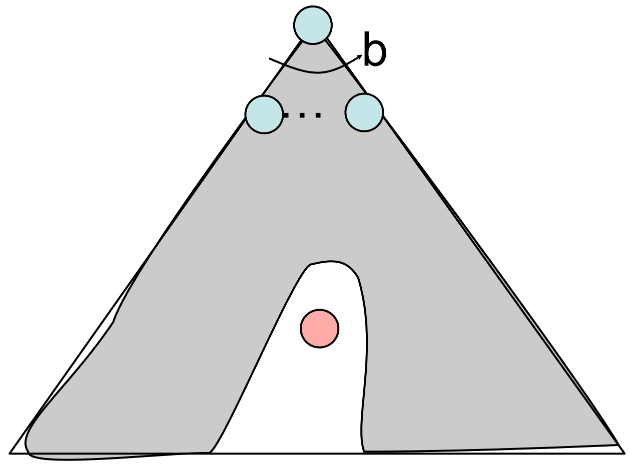
Heuristic: the largest pancake that is still out of place



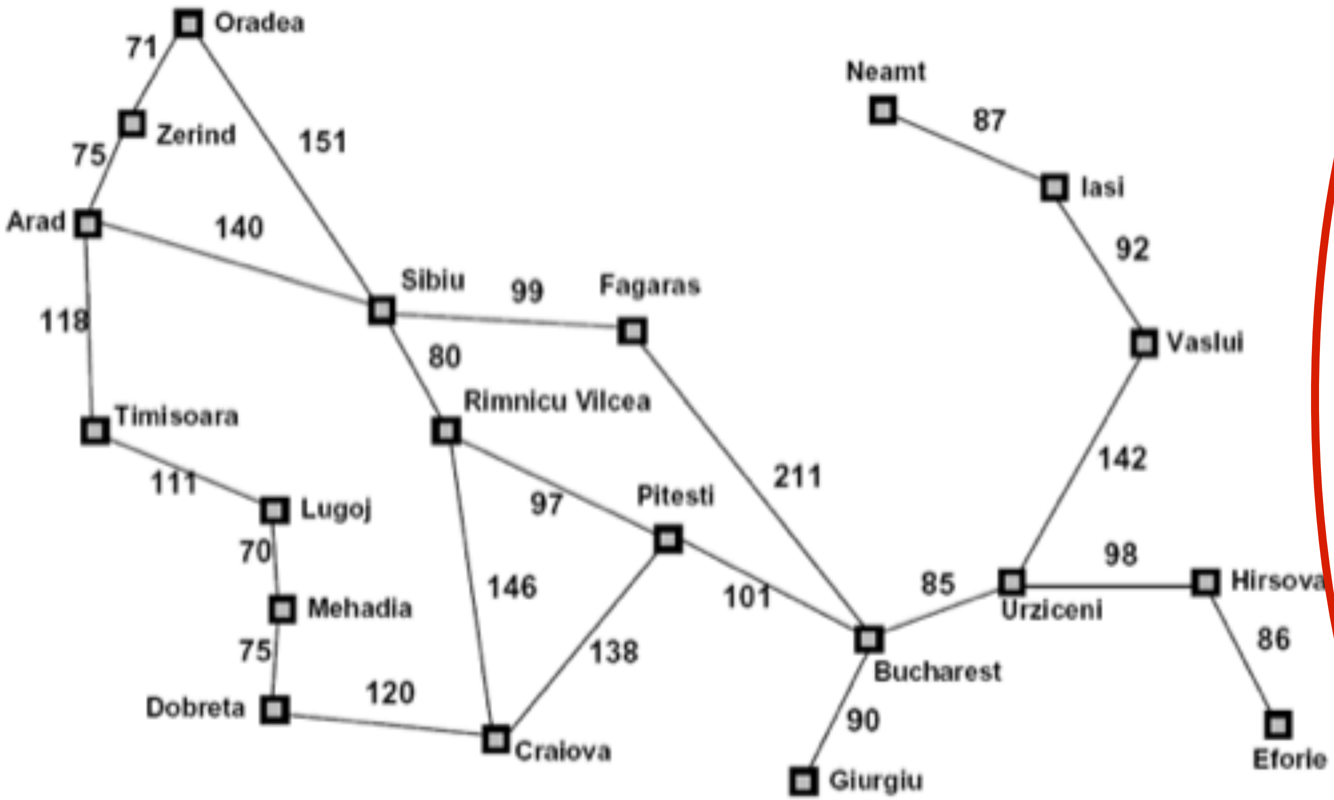
Best First (Greedy)

- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state
- A common case:
  - Best-first takes you straight to the (wrong) goal
- Worst-case: like a badly-guided DFS





Example: Heuristic Function



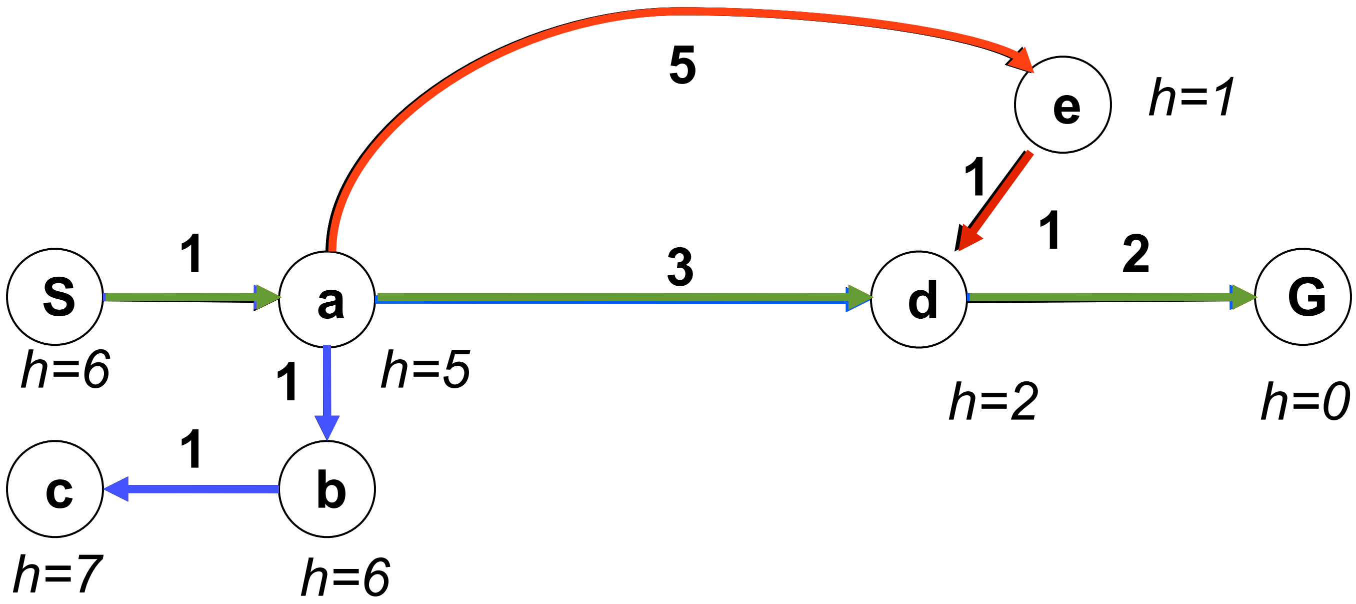
Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

h(x)

Combining UCS and Greedy

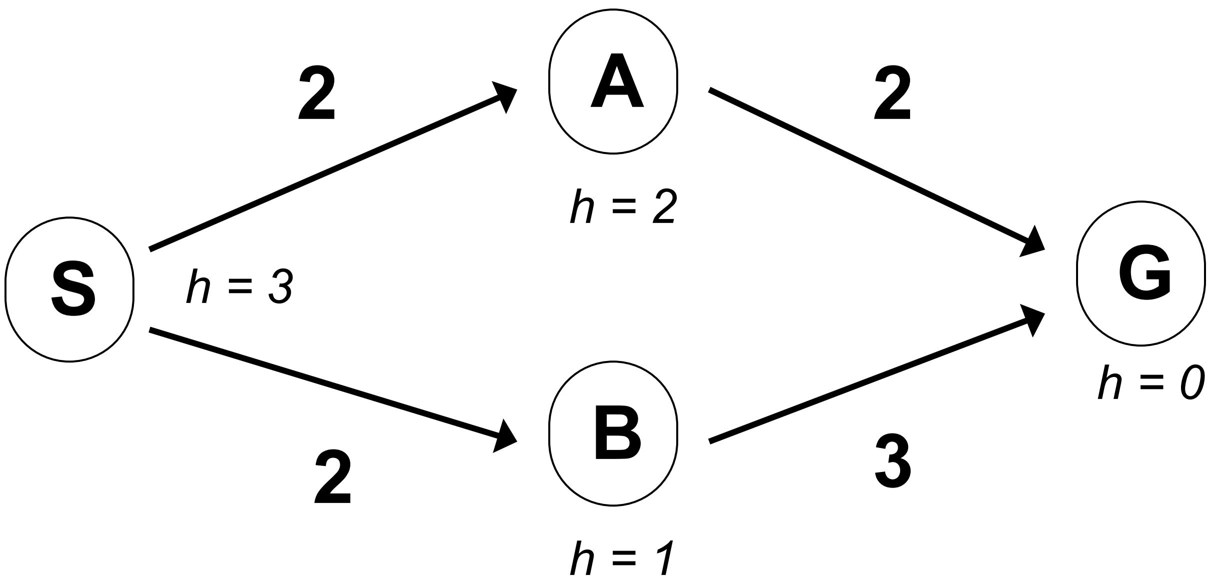
- Uniform-cost orders by path cost, or *backward cost* g(n)
- Best-first orders by goal proximity, or *forward cost* h(n)
- A* Search orders by the sum: f(n) = g(n) + h(n)



Example: Teg Grenager

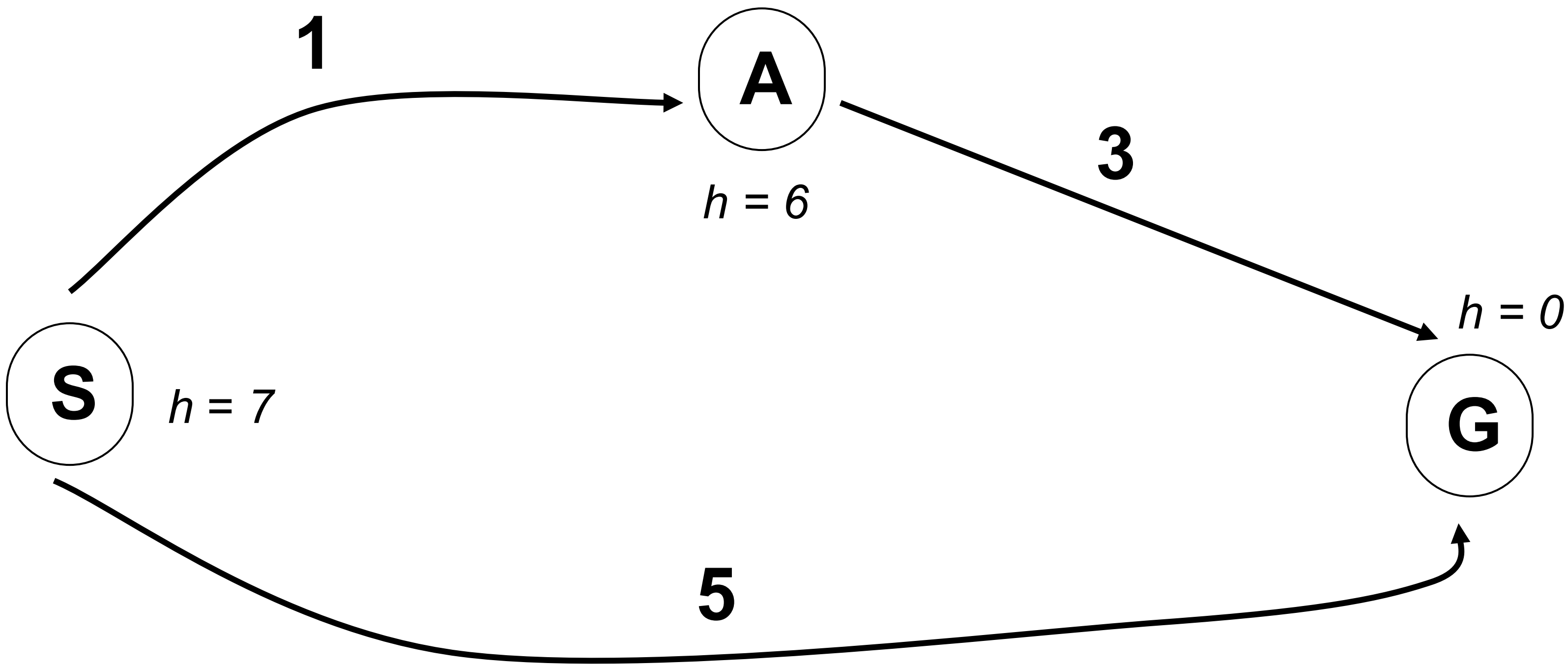
When should A* terminate?

- Should we stop when we enqueue a goal?



- No: only stop when we dequeue a goal

Is A* Optimal?



- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

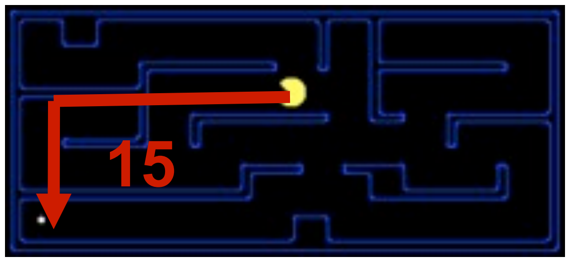
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

- Examples:

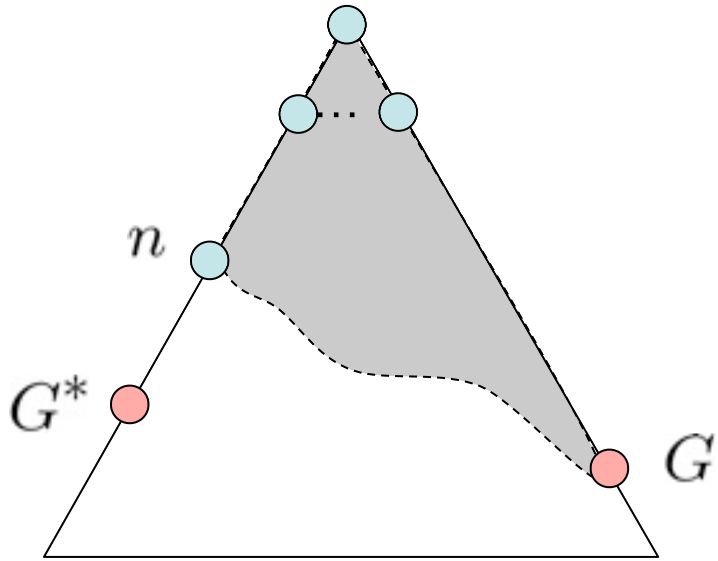


- Coming up with admissible heuristics is most of what's involved in using A* in practice.

Optimality of A*: Blocking

Notation:

- g(n) = cost to node n
- h(n) = estimated cost from n to the nearest goal (heuristic)
- f(n) = g(n) + h(n) = estimated total cost via n
- G*: a lowest cost goal node
- G: another goal node

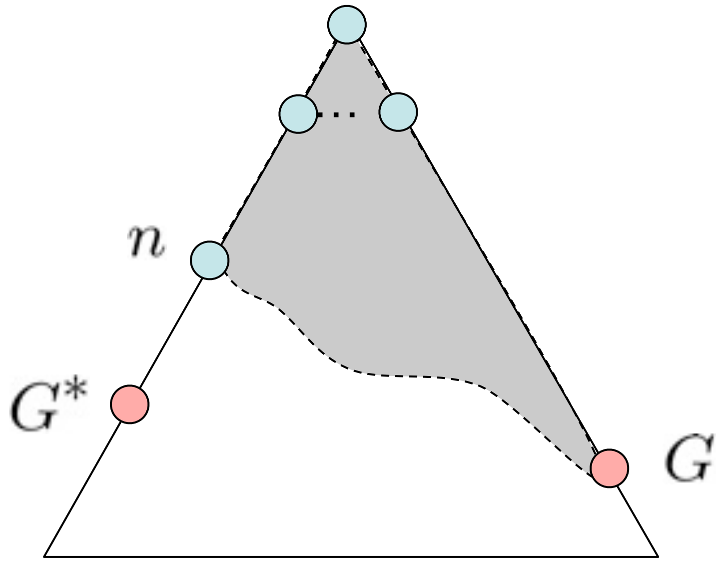


Optimality of A*: Blocking

Proof:

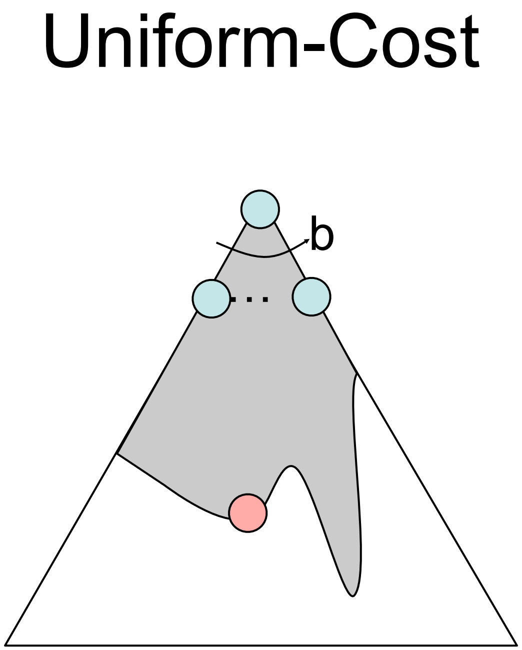
- What could go wrong?
- We'd have to have to pop a suboptimal goal G off the fringe before G*

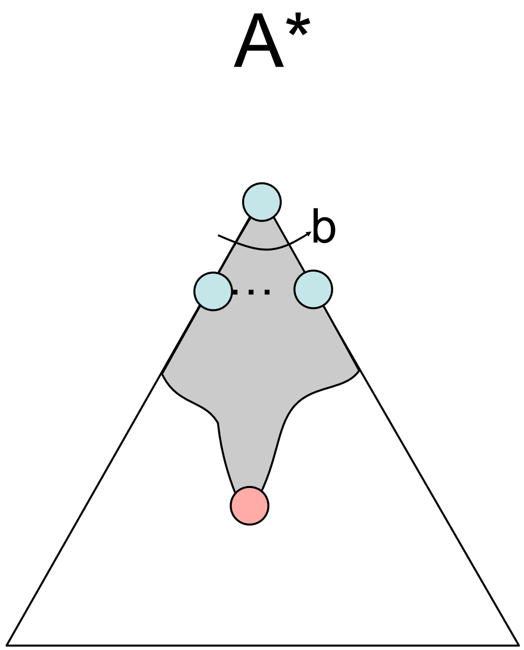
- This can't happen:
  - For all nodes *n* on the best path to G*
    - f(n) < f(G)
  - So, G* will be popped before G



$$f(n) = g(n) + h(n)$$
$$g(n) + h(n) \leq g(G^*)$$
$$g(G^*) < g(G)$$
$$g(G) = f(G)$$
$$f(n) < f(G)$$

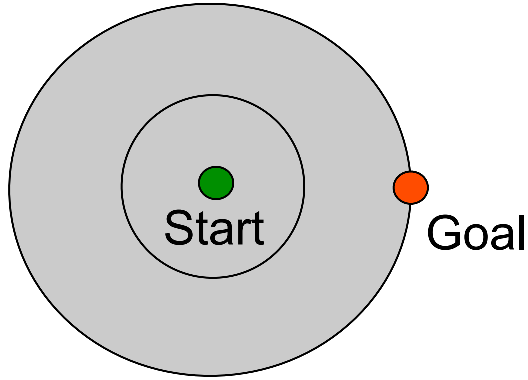
Properties of A*



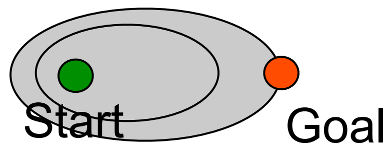


UCS vs A* Contours

- Uniform-cost expanded in all directions

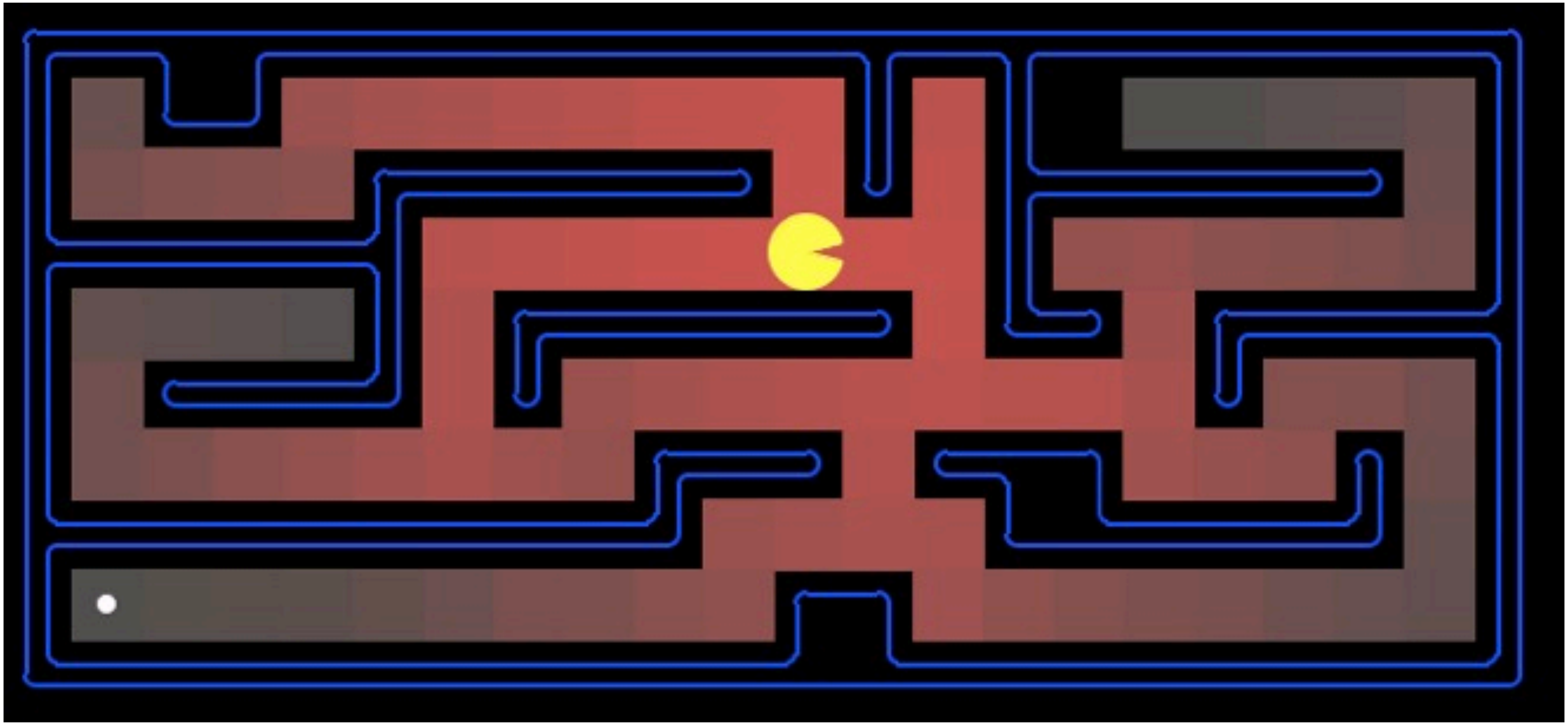


- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



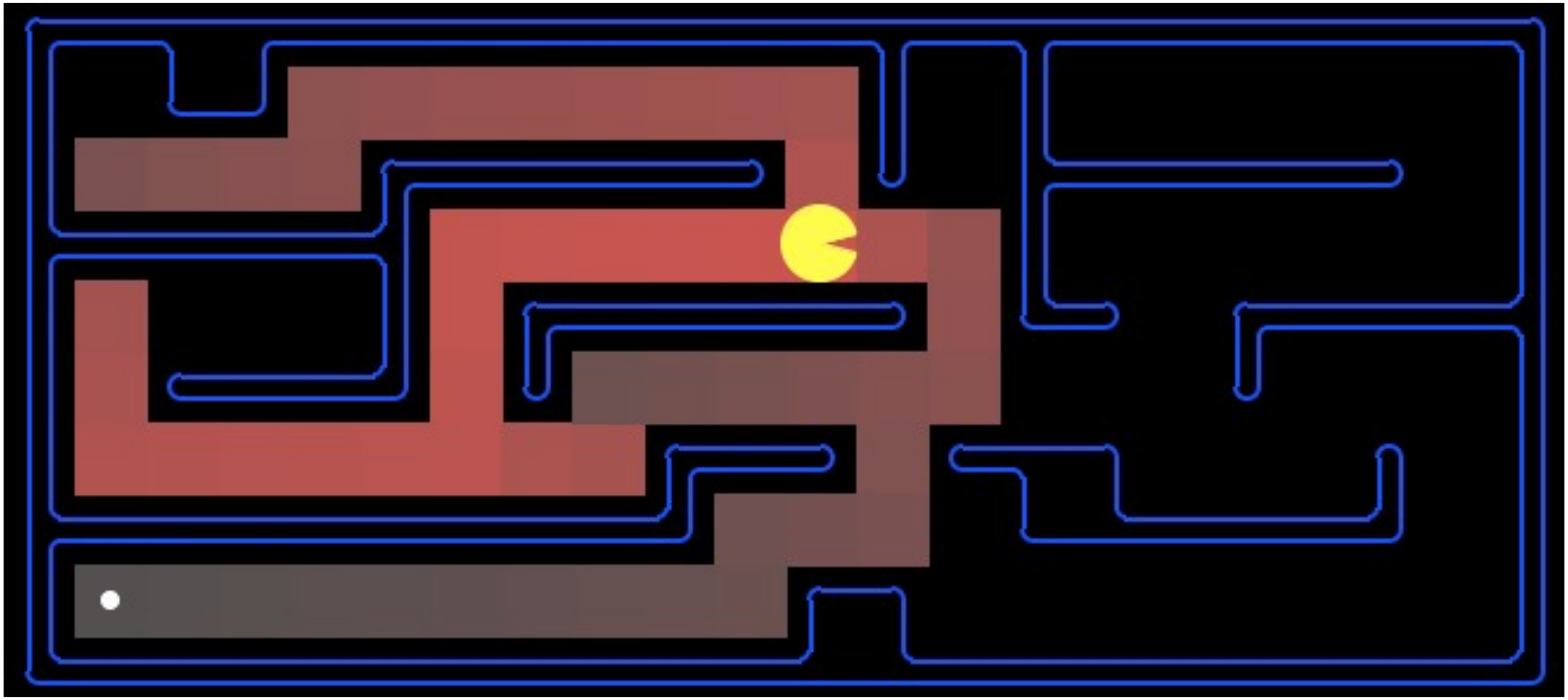
Which Algorithm?

- Uniform cost search (UCS):



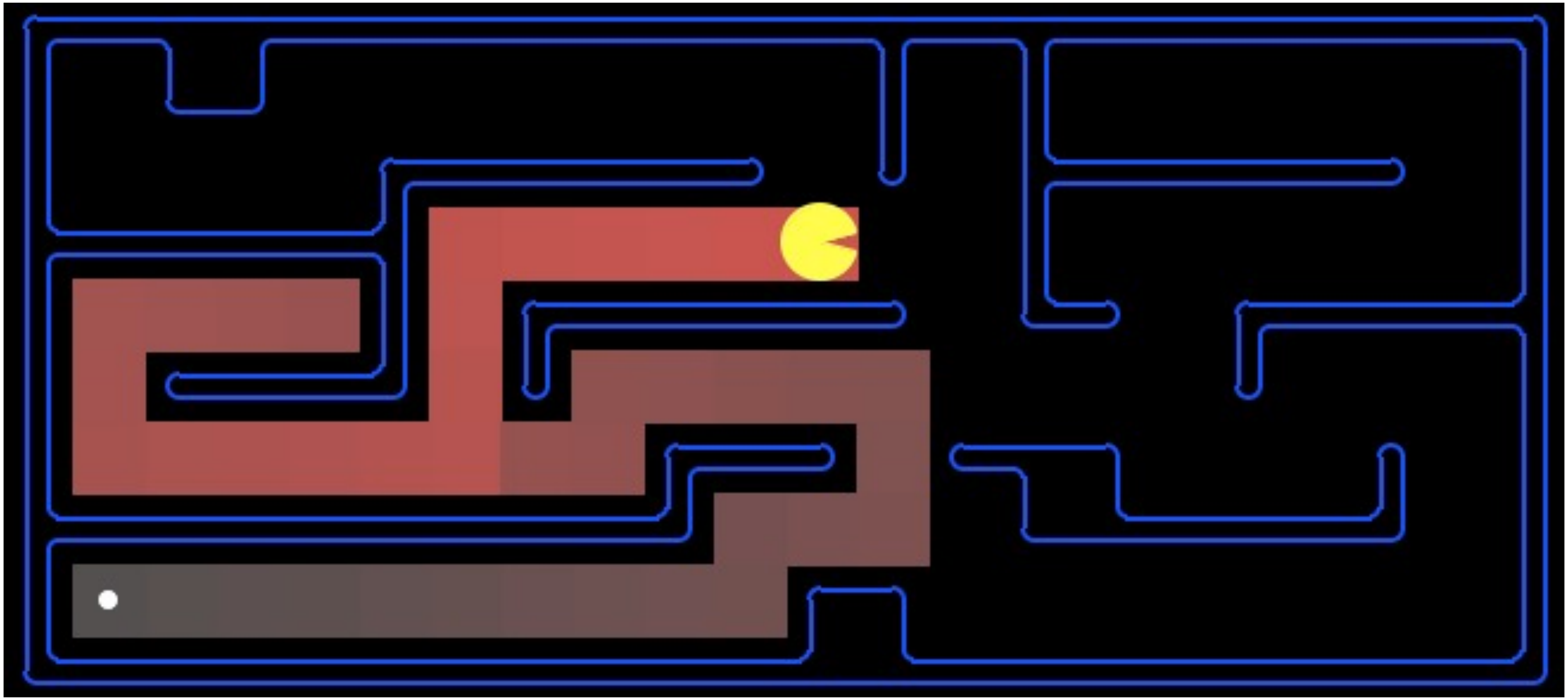
Which Algorithm?

- A*, Manhattan Heuristic:



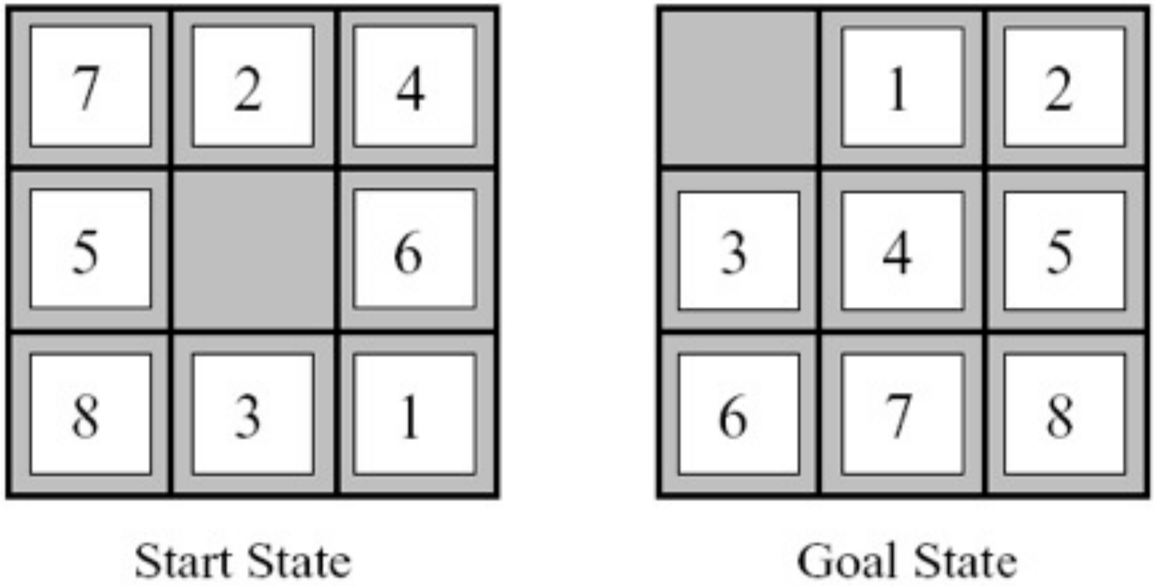
Which Algorithm?

- Best First / Greedy, Manhattan Heuristic:



Creating Heuristics

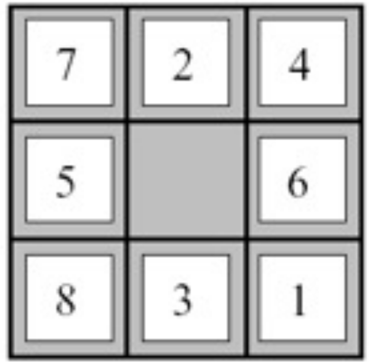
8-puzzle:



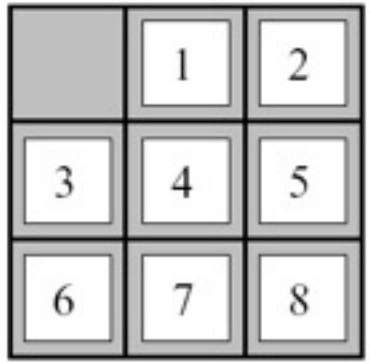
- What are the states?
- How many states?
- What are the actions?
- What states can I reach from the start state?
- What should the costs be?

8 Puzzle I

- Heuristic: Number of tiles misplaced
- h(start) = 8
- Is it admissible?



Start State



Goal State

	Average nodes expanded when optimal path has length…		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

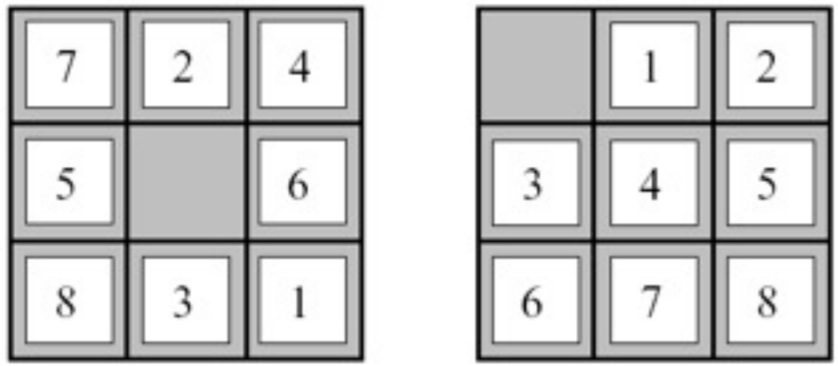
8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total *Manhattan* distance
- h(start) =

  3 + 1 + 2 + …

  = 18

- Admissible?



Start State   Goal State

	Average nodes expanded when optimal path has length…		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
MANHATTAN	12	25	73

8 Puzzle III

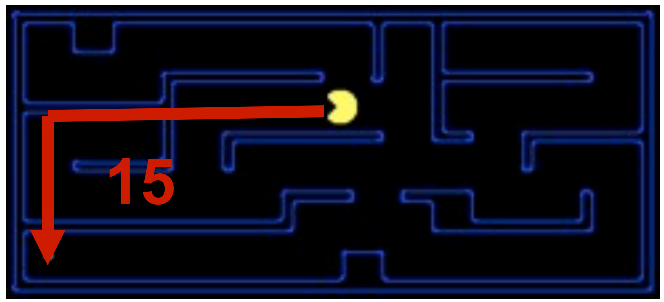
- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What’s wrong with it?

- With A*: a trade-off between quality of estimate and work per node!

Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

- Often, admissible heuristics are solutions to *relaxed problems,* where new actions are available



- Inadmissible heuristics are often useful too (why?)

Trivial Heuristics, Dominance

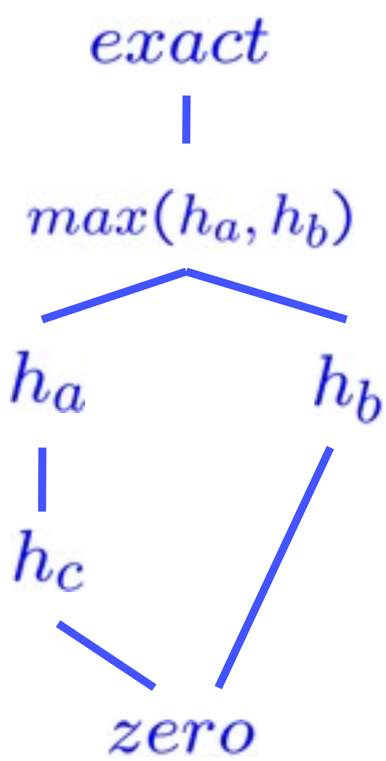
- Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic

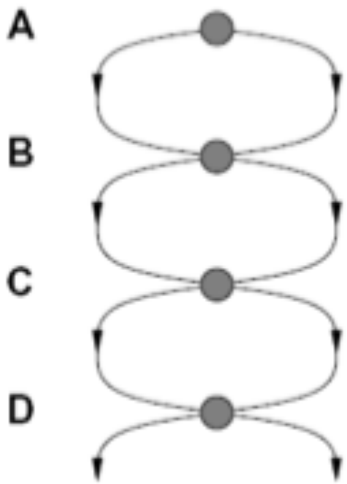


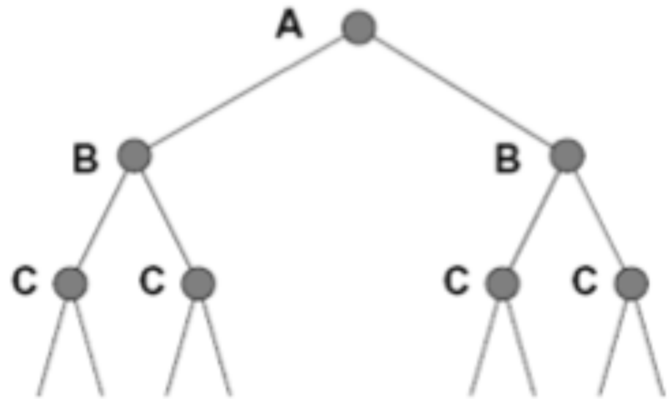
A* Applications

- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …

Tree Search: Extra Work!

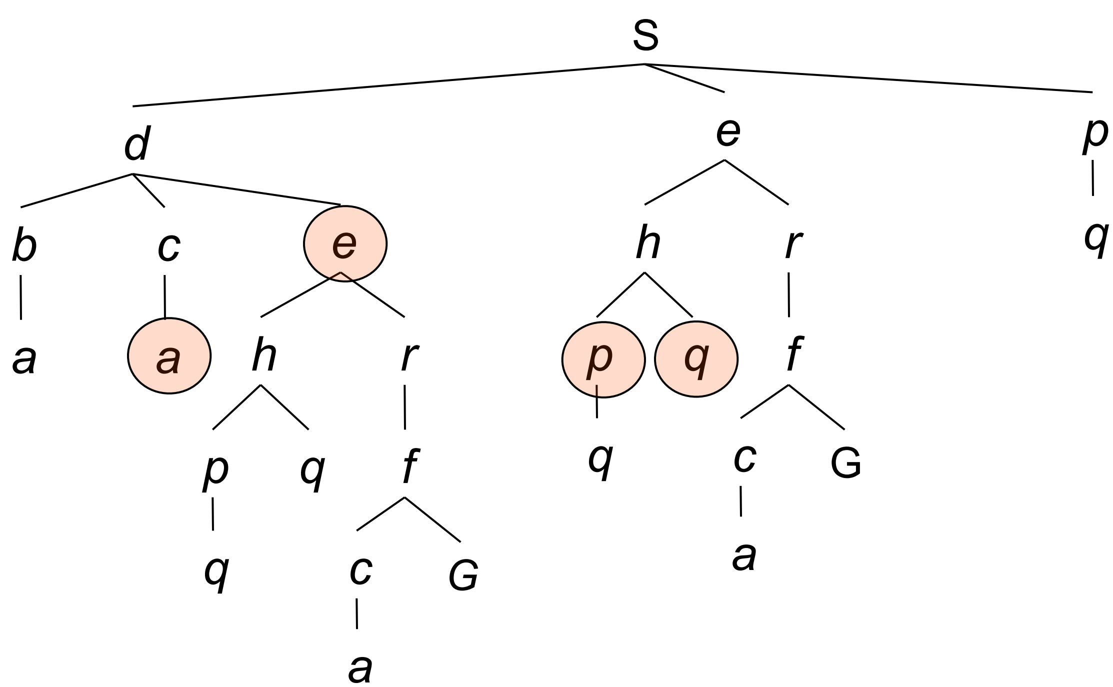
- Failure to detect repeated states can cause exponentially more work. Why?





Graph Search

- In BFS, for example, we shouldn't bother expanding some nodes (which, and why?)



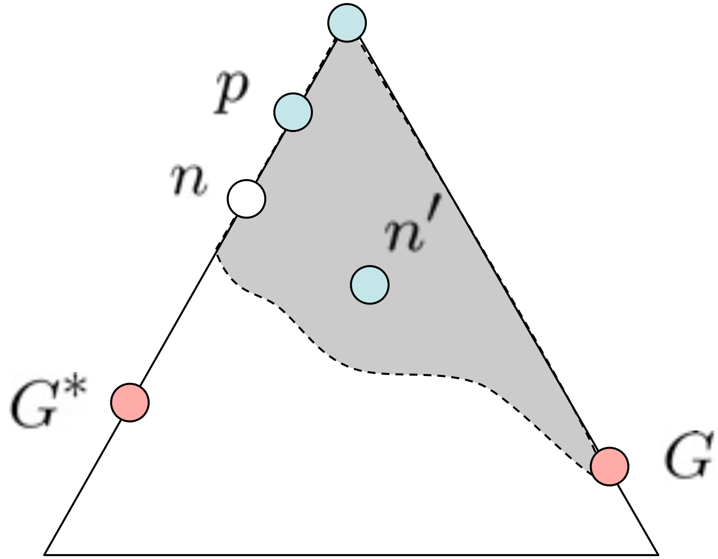
Graph Search

- Idea: never expand a state twice
- How to implement:
  - Tree search + list of expanded states (closed list)
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state is new
- Python trick: store the closed list as a set, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

Optimality of A* Graph Search

Proof:

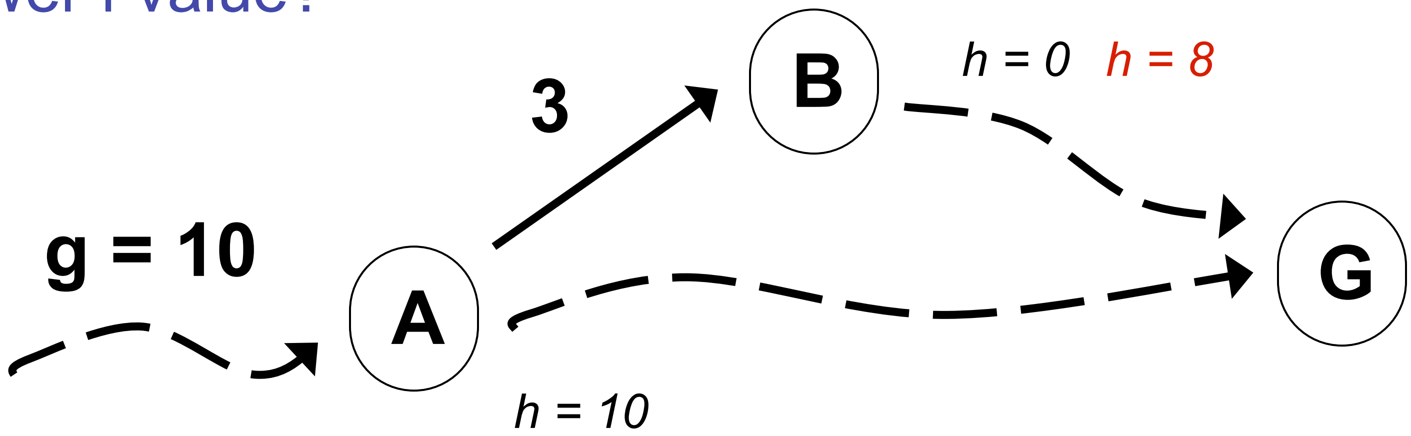
- New possible problem: nodes on path to G* that would have been in queue aren't, because some worse *n'* for the same state as some *n* was dequeued and expanded first (disaster!)



- Let *p* be the ancestor which was on the queue when *n*' was expanded
- Assume *f(p)* < *f(n)*
- *f(n)* < *f(n')* because *n'* is suboptimal
- *p* would have been expanded before *n*'
- Also, *n* would have been expanded before *n*'

Consistency

- Wait, how do we know parents have better f-values than their successors?
- Could we pop some node *n*, and find its child *n'* to have lower f value?



- What can we require to prevent these inversions?
- Consistency: $c(n, a, n') \geq h(n) - h(n')$
- Real cost must always exceed reduction in heuristic

Optimality

- Tree search:
  - A* optimal if heuristic is admissible (and non-negative)
  - UCS is a special case (h = 0)
- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)
- Consistency implies admissibility
- In general, natural admissible heuristics tend to be consistent

Summary: A*

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible heuristics
- Heuristic design is key: often use relaxed problems

To Do:

- Keep up with the readings
- Get started on PS1
  - it is long; start soon
  - due in about a week