Fundamentals of Decision Theory
Chapter 16

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(Based on slides of someone from NPS, Maria Fasli)

Decision Theory

- **"an analytic and systematic approach to the study of decision making"**

Good decisions:
- based on reasoning
- consider all available data and possible alternatives
- employ a quantitative approach

Bad decisions:
- not based on reasoning
- do not consider all available data and possible alternatives
- do not employ a quantitative approach

  - **A good decision may occasionally result in an unexpected outcome; it is still a good decision if made properly**
  - **A bad decision may occasionally result in a good outcome if you are lucky; it is still a bad decision**

Steps in Decision Theory

1. List the possible alternatives (actions/decisions)
2. Identify the possible outcomes
3. List the payoff or profit or reward
4. Select one of the decision theory models
5. Apply the model and make your decision

Example
The Thompson Lumber Company

- Problem.
  - The Thompson Lumber Co. must decide whether or not to expand its product line by manufacturing and marketing a new product, backyard storage sheds
- Step 1: List the possible alternatives

  alternative: "a course of action or strategy that may be chosen by the decision maker"
  - (1) Construct a large plant to manufacture the sheds
  - (2) Construct a small plant
  - (3) Do nothing

The Thompson Lumber Company

- Step 2: Identify the states of nature
  - (1) The market for storage sheds could be favorable
    - high demand
  - (2) The market for storage sheds could be unfavorable
    - low demand

state of nature: "an outcome over which the decision maker has little or no control"
e.g., lottery, coin-toss, whether it will rain today

The Thompson Lumber Company

- Step 3: List the possible rewards
  - A reward for all possible combinations of alternatives and states of nature
  - *Conditional values*: "reward depends upon the alternative and the state of nature"
    - with a favorable market:
      - a large plant produces a net profit of $200,000
      - a small plant produces a net profit of $100,000
      - no plant produces a net profit of $0
    - with an unfavorable market:
      - a large plant produces a net loss of $180,000
      - a small plant produces a net loss of $20,000
      - no plant produces a net profit of $0

Reward tables

- A means of organizing a decision situation, including the rewards from different situations given the possible states of nature

	States of Nature	
Actions	a	b
1	Reward 1a	Reward 1b
2	Reward 2a	Reward 2b

  - Each decision, 1 or 2, results in an outcome, or reward, for the particular state of nature that occurs in the future
  - May be possible to assign probabilities to the states of nature to aid in selecting the best outcome

The Thompson Lumber Company

	States of Nature	
Actions		

The Thompson Lumber Company

	States of Nature	
Actions	Favorable Market	Unfavorable Market
Large plant	$200,000	-$180,000
Small plant	$100,000	-$20,000
No plant	$0	$0

The Thompson Lumber Company

- Steps 4/5: Select an appropriate model and apply it
  - Model selection depends on the operating environment and degree of uncertainty

Decision Making Environments

- Decision making under certainty
- Decision making under uncertainty
  - Non-deterministic uncertainty
  - Probabilistic uncertainty (risk)

Decision Making Under Certainty

- Decision makers know with certainty the consequences of every decision alternative
  - Always choose the alternative that results in the best possible outcome

Non-deterministic Uncertainty

	States of Nature	
Actions	Favorable Market	Unfavorable Market
Large plant	$200,000	-$180,000
Small plant	$100,000	-$20,000
No plant	$0	$0

- What should we do?

Maximax Criterion

"Go for the Gold"

- Select the decision that results in the maximum of the maximum rewards
- A very optimistic decision criterion
  - Decision maker assumes that the most favorable state of nature for each action will occur
- Most risk prone agent

Maximax

	States of Nature		Maximum
Decision	Favorable	Unfavorable	in Row
Large plant	$200,000	-$180,000	$200,000
Small plant	$100,000	-$20,000	$100,000
No plant	$0	$0	$0

- **Thompson Lumber Co. assumes that the most favorable state of nature occurs for each decision alternative**
- **Select the maximum reward for each decision**
  - **All three maximums occur if a favorable economy prevails (a tie in case of no plant)**
- **Select the maximum of the maximums**
  - **Maximum is $200,000; corresponding decision is to build the large plant**
  - **Potential loss of $180,000 is completely ignored**

Maximin Criterion

"Best of the Worst"

- Select the decision that results in the maximum of the minimum rewards
- A very pessimistic decision criterion
  - Decision maker assumes that the minimum reward occurs for each decision alternative
  - Select the maximum of these minimum rewards
- Most risk averse agent

Maximin

	States of Nature		Minimum
Decision	Favorable	Unfavorable	in Row
Large plant	$200,000	-$180,000	-$180,000
Small plant	$100,000	-$20,000	-$20,000
No plant	$0	$0	$0

- **Thompson Lumber Co. assumes that the least favorable state of nature occurs for each decision alternative**
- **Select the minimum reward for each decision**
  - **All three minimums occur if an unfavorable economy prevails (a tie in case of no plant)**
- **Select the maximum of the minimums**
  - **Maximum is $0; corresponding decision is to do nothing**
  - **A conservative decision; largest possible gain, $0, is much less than maximax**

Equal Likelihood Criterion

- Assumes that all states of nature are equally likely to occur
  - Maximax criterion assumed the most favorable state of nature occurs for each decision
  - Maximin criterion assumed the least favorable state of nature occurs for each decision
- Calculate the *average reward* for each alternative and select the alternative with the maximum number
  - *Average reward*: the sum of all rewards divided by the number of states of nature
- Select the decision that gives the highest average reward

Equal Likelihood

Decision	States of Nature: Favorable	States of Nature: Unfavorable	Row Average
Large plant	$200,000	-$180,000	$10,000
Small plant	$100,000	-$20,000	$40,000
No plant	$0	$0	$0

Row Averages

$$\text{Large Plant} = \frac{\$200{,}000 - \$180{,}000}{2} = \$10{,}000$$

$$\text{Small Plant} = \frac{\$100{,}000 - \$20{,}000}{2} = \$40{,}000$$

$$\text{Do Nothing} = \frac{\$0 + \$0}{2} = \$0$$

- Select the decision with the highest weighted value
  - Maximum is $40,000; corresponding decision is to build the small plant

Criterion of Realism

- Also known as the weighted average or Hurwicz criterion
  - A compromise between an optimistic and pessimistic decision
- A coefficient of realism, α, is selected by the decision maker to indicate optimism or pessimism about the future

$$0 \leq \alpha \leq 1$$

When α is close to 1, the decision maker is optimistic.
When α is close to 0, the decision maker is pessimistic.

- *Criterion of realism* = α(row maximum) + (1-α)(row minimum)
  - A weighted average where maximum and minimum rewards are weighted by α and (1 - α) respectively

Criterion of Realism

- Assume a coefficient of realism equal to 0.8

Decision	States of Nature: Favorable	States of Nature: Unfavorable	Criterion of Realism
Large plant	$200,000	-$180,000	$124,000
Small plant	$100,000	-$20,000	$76,000
No plant	$0	$0	$0

Weighted Averages

Large Plant = (0.8)($200,000) + (0.2)(-$180,000) = $124,000

Small Plant = **(0.8)($100,000) + (0.2)(-$20,000) = $76,000**

Do Nothing = **(0.8)($0) + (0.2)($0) = $0**

Select the decision with the highest weighted value

Maximum is $124,000; corresponding decision is to build the large plant

Minimax Regret

- Regret/Opportunity Loss: "the difference between the optimal reward and the actual reward received"
- Choose the alternative that minimizes the maximum regret associated with each alternative
  - Start by determining the maximum regret for each alternative
  - Pick the alternative with the minimum number

Regret Table

- If I knew the future, how much I'd regret my decision…
- Regret for any state of nature is calculated by subtracting each outcome in the column from the best outcome in the same column

Minimax Regret

Decision	Favorable Payoff	Favorable Regret	Unfavorable Payoff	Unfavorable Regret	Row Maximum
Large plant	$200,000	$0	-$180,000	$180,000	$180,000
Small plant	$100,000	$100,000	-$20,000	$20,000	$100,000
No plant	$0	$200,000	$0	$0	$200,000
Best payoff	$200,000		$0		

- Select the alternative with the lowest maximum regret

Minimum is $100,000; corresponding decision is to build a small plant

Summary of Results

Criterion	Decision
Maximax	**Build a large plant**
Maximin	**Do nothing**
Equal likelihood	**Build a small plant**
Realism	**Build a large plant**
Minimax regret	**Build a small plant**

Decision Making Environments

- Decision making under certainty
- Decision making under uncertainty
  - Non-deterministic uncertainty
  - Probabilistic uncertainty (risk)

Probabilistic Uncertainty

- Decision makers know the probability of occurrence for each possible outcome
  - Attempt to maximize the expected reward
- Criteria for decision models in this environment:
  - Maximization of expected reward
  - Minimization of expected regret
    - Minimize expected regret = maximizing expected reward!

Expected Reward (Q)

- called Expected Monetary Value (EMV) in DT literature
- "the probability weighted sum of possible rewards for each alternative"
  - Requires a reward table with conditional rewards and probability assessments for all states of nature

Q(action a) = (reward of 1st state of nature)
X (probability of 1st state of nature)
+ (reward of 2nd state of nature)
X (probability of 2nd state of nature)
+ . . . + (reward of last state of nature)
X (probability of last state of nature)

The Thompson Lumber Company

- Suppose that the probability of a favorable market is exactly the same as the probability of an unfavorable market. Which alternative would give the greatest Q?

	States of Nature		
Decision	**Favorable Mkt p = 0.5**	**Unfavorable Mkt p = 0.5**	**EMV**
Large plant	**$200,000**	**-$180,000**	**$10,000**
Small plant	**$100,000**	**-$20,000**	**$40,000**
No plant	**$0**	**$0**	**$0**

Q(large plant) = **(0.5)($200,000) + (0.5)(-$180,000) = $10,000**

Q(small plant) = **(0.5)($100,000) + (0.5)(-$-20,000) = $40,000**

Q(no plant) = **(0.5)($0) + (0.5)($0) = $0**

Build the small plant

Expected Value of Perfect Information (EVPI)

- It may be possible to purchase additional information about future events and thus make a better decision
  - Thompson Lumber Co. could hire an economist to analyze the economy in order to more accurately determine which economic condition will occur in the future
    - How valuable would this information be?

EVPI Computation

- Look first at the decisions under each state of nature
  - If information was available that perfectly predicted which state of nature was going to occur, the best decision for that state of nature could be made
    - *expected value with perfect information* (EV w/ PI): "the expected or average return if we have perfect information before a decision has to be made"

EVPI Computation

- Perfect information changes environment from decision making under risk to decision making with certainty
  - Build the large plant if you know for sure that a favorable market will prevail
  - Do nothing if you know for sure that an unfavorable market will prevail

	States of Nature	
Decision	**Favorable p = 0.5**	**Unfavorable p = 0.5**
Large plant	**$200,000**	**-$180,000**
Small plant	**$100,000**	**-$20,000**
No plant	**$0**	**$0**

EVPI Computation

- Even though perfect information enables Thompson Lumber Co. to make the correct investment decision, each state of nature occurs only a certain portion of the time
  - A favorable market occurs 50% of the time and an unfavorable market occurs 50% of the time
  - EV w/ PI calculated by choosing the best alternative for each state of nature and multiplying its reward times the probability of occurrence of the state of nature

EVPI Computation

EV w/ PI = **(best reward for 1st state of nature)**
X (probability of 1st state of nature)
+ (best reward for 2nd state of nature)
X (probability of 2nd state of nature)

EV w/ PI = **($200,000)(0.5) + ($0)(0.5) = $100,000**

	States of Nature	
Decision	**Favorable p = 0.5**	**Unfavorable p = 0.5**
Large plant	**$200,000**	**-$180,000**
Small plant	**$100,000**	**-$20,000**
No plant	**$0**	**$0**

EVPI Computation

- Thompson Lumber Co. would be foolish to pay more for this information than the extra profit that would be gained from having it
  - *EVPI*: "the maximum amount a decision maker would pay for additional information resulting in a decision better than one made *without perfect information* "
    - EVPI is the expected outcome with perfect information minus the expected outcome without perfect information

EVPI = EV w/ PI - Q

EVPI = $100,000 - $40,000 = $60,000

Using EVPI

- EVPI of $60,000 is the <u>maximum</u> amount that Thompson Lumber Co. should pay to purchase perfect information from a source such as an economist
  - "Perfect" information is extremely rare
  - An investor typically would be willing to pay some amount less than $60,000, depending on how reliable the information is perceived to be

Is Expected Value sufficient?

- Lottery 1
  – returns $0 always
- Lottery 2
  – return $100 and -$100 with prob 0.5

- Which is better?

Is Expected Value sufficient?

- Lottery 1
  – returns $100 always
- Lottery 2
  – return $10000 (prob 0.01) and $0 with prob 0.99

- Which is better?
  – depends

Is Expected Value sufficient?

- Lottery 1
  – returns $3125 always
- Lottery 2
  – return $4000 (prob 0.75) and -$500 with prob 0.25

- Which is better?

Is Expected Value sufficient?

- Lottery 1
  – returns $0 always
- Lottery 2
  – return $1,000,000 (prob 0.5) and -$1,000,000 with prob 0.5

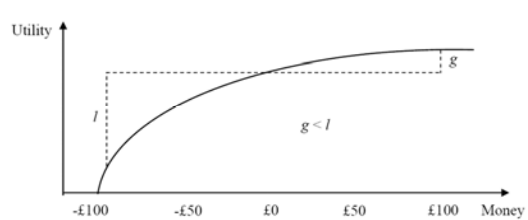
- Which is better?

Utility Theory

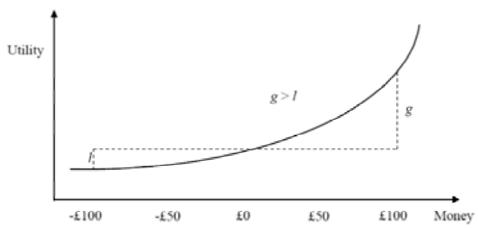
- Adds a layer of utility over rewards
- Risk averse
  – |Utility| of high negative money is much MORE than utility of high positive money
- Risk prone
  – Reverse

- Use expected utility criteria…

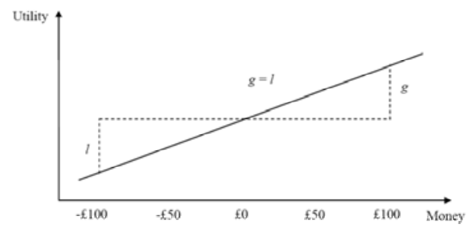
Utility function of risk-averse agent



Utility function of a risk-prone agent



Utility function of a risk-neutral agent



PEAS/Environment

- Performance: utility
- Environment
  - Static – Stochastic – Partially Obs – Discrete – Episodic – Single
- Actuators
  - alternatives
  - ask for perfect information
- Sensor
  - State of nature