CSE 573: Artificial Intelligence
Autumn 2012

Particle Filters
for
Hidden Markov Models

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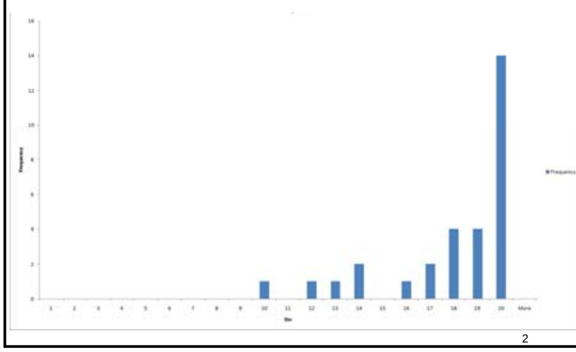
Many slides adapted from Dan Klein, Stuart Russell, Andrew Moore & Luke Zettlemoyer

Homework 2



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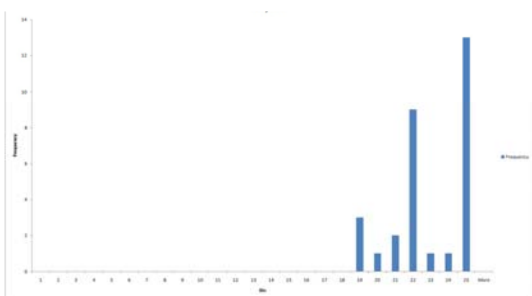
Homework 3



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Logistics

- Mon 11/5 – Resubmit / regrade HW2, HW3
- Mon 11/12 – HW4 due
- Wed 11/14 – project groups & idea
  - 1-1 meetings to follow
  - See course webpage for ideas
  - Plus a new one:
    - Infinite number of card decks
    - 6 decks
    - Add state variable

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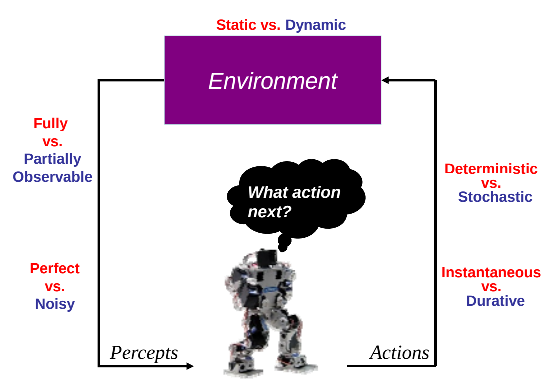
Outline

- Overview
- Probability review
  - Random Variables and Events
  - Joint / Marginal / Conditional Distributions
  - Product Rule, Chain Rule, Bayes' Rule
- Probabilistic inference
  - Enumeration of Joint Distribution
  - Bayesian Networks – Preview
- Probabilistic sequence models (and inference)
  - Markov Chains
  - Hidden Markov Models
  - Particle Filters

Agent



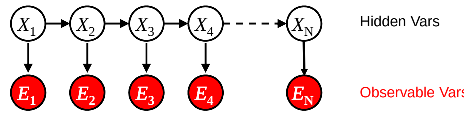
Simple Bayes Net

Defines a joint probability distribution:

$P(X_1, E_1) =$???

$= P(X_1)\, P(E_1|X_1)$

Hidden Markov Model



Defines a joint probability distribution:

$$P(X_1, \ldots, X_n, E_1, \ldots, E_n) =$$

$$P(X_1)P(E_1|X_1)\prod_{t=2}^{N} P(X_t|X_{t-1})P(E_t|X_t)$$

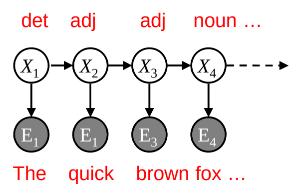
HMM Computations

- Given
  - joint $P(X_{1:n}, E_{1:n})$
  - evidence $E_{1:n} = e_{1:n}$
- Inference problems include:
  - Filtering, find $P(X_t|e_{1:n})$ for current time n
  - Smoothing, find $P(X_t|e_{1:n})$ for time $t < n$
  - Most probable explanation, find

    $x^*_{1:n} = \text{argmax}_{x_{1:n}} P(x_{1:n}|e_{1:n})$

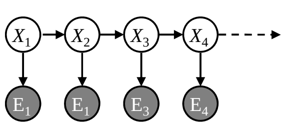
Real HMM Examples

- Part-of-speech (POS) Tagging:
  - Observations are words (thousands of them)
  - States are POS tags (eg, noun, verb, adjective, det…)



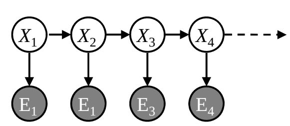
Real HMM Examples

- Speech recognition HMMs:
  - Observations are acoustic signals (continuous valued)
  - States are specific positions in specific words (so, tens of thousands)



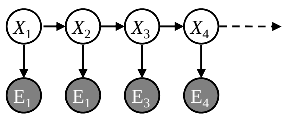
Real HMM Examples

- Machine translation HMMs:
  - Observations are words
  - States are translation options



Real HMM Examples

- Robot tracking:
  - Observations are range readings (continuous)
  - States are positions on a map (continuous)



Ghostbusters HMM

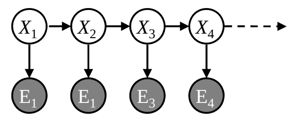
- $P(X_1)$ = uniform
- P(X'|X) = usually move clockwise, but sometimes move in a random direction or stay in place
- P(E|X) = same sensor model as before: red means close, green means far away.

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

$P(X_1)$

1/6	1/6	1/2
0	1/6	0
0	0	0

P(X'|X=<1,2>)

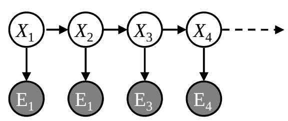


P(E|X)

P(red | 3)	P(orange | 3)	P(yellow | 3)	P(green | 3)
0.05	0.15	0.5	0.3

Conditional Independence

HMMs have two important independence properties:
- Markov hidden process, future depends on past via the present
- Current observation independent of all else given current state

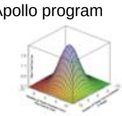


Quiz: does this mean successive observations are independent?
- [No, correlated by the hidden state]

Filtering aka Monitoring, State Estimation

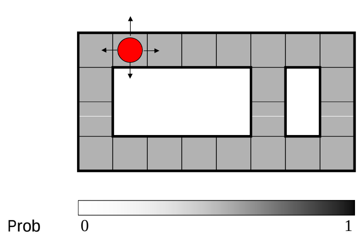
- Filtering is the task of tracking the distribution B(X) (the belief state) over time
- We start with B(X) in an initial setting, usually uniform
- As time passes, or we get observations, we update B(X)

- Aside: the Kalman filter
  - Invented in the 60's for trajectory estimation in the Apollo program
  - State evolves using a linear model, eg $x = x_0 + vt$
  - Observe: value of x with Gaussian noise



Example: Robot Localization

Example from Michael Pfeiffer

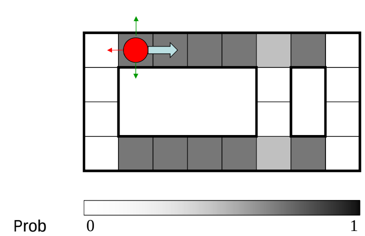


t=0

Sensor model: never more than 1 mistake

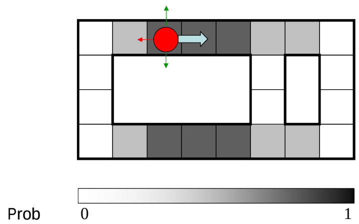
Motion model: may not execute action with small prob.

Example: Robot Localization



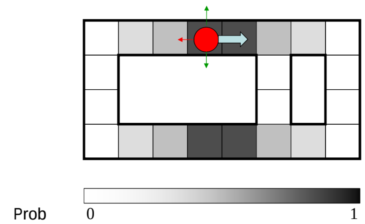
t=1

Example: Robot Localization



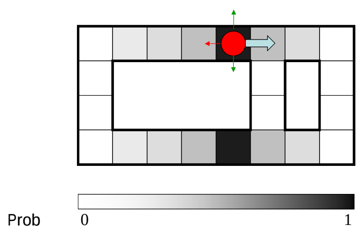
t=2

Example: Robot Localization



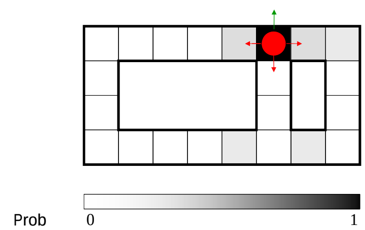
t=3

Example: Robot Localization



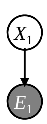
t=4

Example: Robot Localization



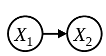
t=5

Inference Recap: Simple Cases



$P(X_1|e_1)$

$$P(x_1|e_1) = P(x_1, e_1)/P(e_1)$$
$$\propto_{X_1} P(x_1, e_1)$$
$$= P(x_1)P(e_1|x_1)$$

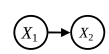


$P(X_2)$

$$P(x_2) = \sum_{x_1} P(x_1, x_2)$$
$$= \sum_{x_1} P(x_1)P(x_2|x_1)$$

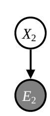
Online Belief Updates

- Every time step, we start with current P(X | evidence)
- We update for time:



$$P(x_t|e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}|e_{1:t-1}) \cdot P(x_t|x_{t-1})$$

- We update for evidence:



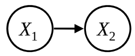
$$P(x_t|e_{1:t}) \propto_X P(x_t|e_{1:t-1}) \cdot P(e_t|x_t)$$

Passage of Time

- Assume we have current belief P(X | evidence to date)

$$B(X_t) = P(X_t|e_{1:t})$$

- Then, after one time step passes:



$$P(X_{t+1}|e_{1:t}) = \sum_{x_t} P(X_{t+1}|x_t)P(x_t|e_{1:t})$$

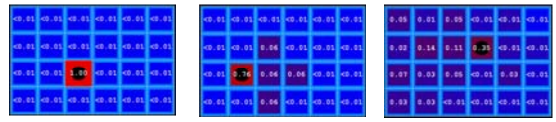
- Or, compactly:

$$B'(X') = \sum_x P(X'|x)B(x)$$

- Basic idea: beliefs get "pushed" through the transitions
  - With the "B" notation, we have to be careful about what time step t the belief is about, and what evidence it includes

Example: Passage of Time

- As time passes, uncertainty "accumulates"



T = 1 T = 2 T = 5

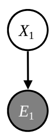
$$B'(X') = \sum_x P(X'|x)B(x)$$

Transition model: ghosts usually go clockwise

Observation

- Assume we have current belief P(X | previous evidence):

$$B'(X_{t+1}) = P(X_{t+1}|e_{1:t})$$



- Then:

$$P(X_{t+1}|e_{1:t+1}) \propto P(e_{t+1}|X_{t+1})P(X_{t+1}|e_{1:t})$$

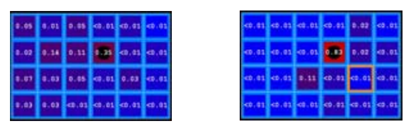
- Or:

$$B(X_{t+1}) \propto P(e|X)B'(X_{t+1})$$

- Basic idea: beliefs reweighted by likelihood of evidence
- Unlike passage of time, we have to renormalize

Example: Observation

- As we get observations, beliefs get reweighted, uncertainty "decreases"



Before observation After observation

$$B(X) \propto P(e|X)B'(X)$$

The Forward Algorithm

- We want to know: $B_t(X) = P(X_t|e_{1:t})$
- We can derive the following updates

$$P(x_t|e_{1:t}) \propto_X P(x_t, e_{1:t})$$

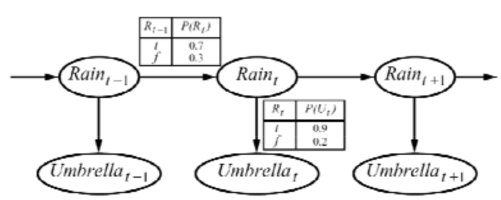
$$= \sum_{x_{t-1}} P(x_{t-1}, x_t, e_{1:t})$$

$$= \sum_{x_{t-1}} P(x_{t-1}, e_{1:t-1})P(x_t|x_{t-1})P(e_t|x_t)$$

$$= P(e_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1})P(x_{t-1}, e_{1:t-1})$$

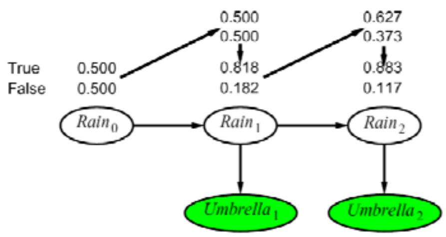
- To get $B_t(X)$ compute each entry and normalize

Example: Run the Filter

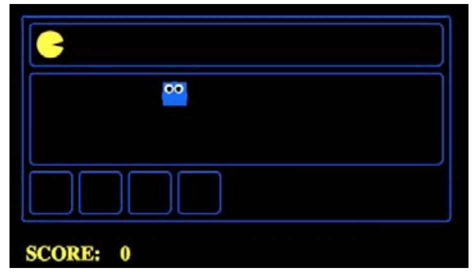


- An HMM is defined by:
  - Initial distribution: $P(X_1)$
  - Transitions: $P(X_t|X_{t-1})$
  - Emissions: $P(E|X)$

Example HMM



Example Pac-man



Summary: Filtering

- Filtering is the inference process of finding a distribution over X_T given e_1 through e_T : P(X_T | $e_{1:t}$)
- We first compute P(X_1 | e_1): $P(x_1|e_1) \propto P(x_1) \cdot P(e_1|x_1)$
- For each t from 2 to T, we have P(X_{t-1} | $e_{1:t-1}$)
- **Elapse time:** compute P(X_t | $e_{1:t-1}$)

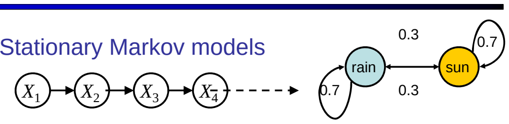
$$P(x_t|e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}|e_{1:t-1}) \cdot P(x_t|x_{t-1})$$

- **Observe:** compute P(X_t | $e_{1:t-1}$, e_t) = P(X_t | $e_{1:t}$)

$$P(x_t|e_{1:t}) \propto P(x_t|e_{1:t-1}) \cdot P(e_t|x_t)$$

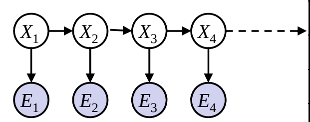
Recap: Reasoning Over Time

- Stationary Markov models



$P(X_1)$ $P(X|X_{-1})$ $P(E|X)$

- Hidden Markov models



X	E	P
rain	umbrella	0.9
rain	no umbrella	0.1
sun	umbrella	0.2
sun	no umbrella	0.8

Add a slide

- Next slide (intro to particle filtering) is confusing because the state spaec is so small – show a huge grid, where it's clear what advantage one gets.
- Maybe also introduce parametric representations (kalman filter) here

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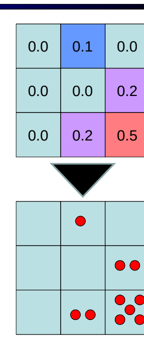
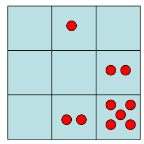
Particle Filtering

- Sometimes |X| is too big to use exact inference
  - |X| may be too big to even store B(X)
  - E.g. when X is continuous
  - $|X|^2$ may be too big to do updates
- Solution: approximate inference
  - Track samples of X, not all values
  - Samples are called particles
  - Time per step is linear in the number of samples
  - But: number needed may be large
  - In memory: list of particles, not states
- This is how robot localization works in practice



Representation: Particles

- Our representation of P(X) is now a list of N particles (samples)
  - Generally, N << |X|
  - Storing map from X to counts would defeat the point
- P(x) approximated by number of particles with value x
  - So, many x will have P(x) = 0!
  - More particles, more accuracy
- For now, all particles have a weight of 1



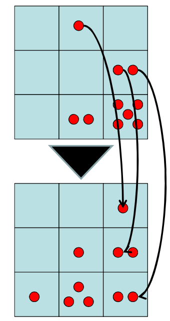
Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(2,1)
(3,3)
(3,3)
(2,1)

Particle Filtering: Elapse Time

- Each particle is moved by sampling its next position from the transition model

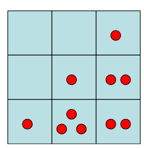
$$x' = \text{sample}(P(X'|x))$$

  - This is like prior sampling – samples' frequencies reflect the transition probs
  - Here, most samples move clockwise, but some move in another direction or stay in place
- This captures the passage of time
  - If we have enough samples, close to the exact values before and after (consistent)



Particle Filtering: Observe

- Slightly trickier:
  - Use P(e|x) to sample observation, and
  - Discard particles which are inconsistent?
    - (Called Rejection Sampling)
  - Problems?



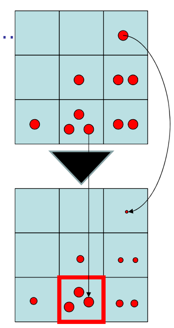
Particle Filtering: Observe

- Instead of sampling the observation…
- Fix It!
  - A kind of **likelihood weighting**
  - Downweight samples based on evidence

$$w(x) = P(e|x)$$

$$B(X) \propto P(e|X)B'(X)$$

  - Note that probabilities don't sum to one: (most have been down-weighted)
    Instead, they sum to an approximation of P(e))
  - ***What to do?!?***

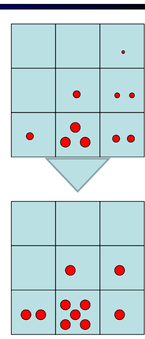


Particle Filtering: Resample

- Rather than tracking weighted samples, we ***resample*** – *why?*
- N times, we choose from our weighted sample distribution (i.e. draw with replacement)
- This is equivalent to renormalizing the distribution
- Now the update is complete for this time step, continue with the next one

Old Particles:
(3,3) w=0.1
(2,1) w=0.9
(2,1) w=0.9
(3,1) w=0.4
(3,2) w=0.3
(2,2) w=0.4
(1,1) w=0.4
(3,1) w=0.4
(2,1) w=0.9
(3,2) w=0.3

New Particles:
(2,1) w=1
(2,1) w=1
(2,1) w=1
(3,2) w=1
(2,2) w=1
(2,1) w=1
(1,1) w=1
(3,1) w=1
(2,1) w=1
(1,1) w=1



Recap: Particle Filtering

At each time step t, we have a set of N particles (aka samples)

- Initialization: Sample from prior
- Three step procedure for moving to time t+1:
  1. Sample transitions: for each each particle x, sample next state
     $$x' = \text{sample}(P(X'|x))$$
  2. Reweight: for each particle, compute its weight given the actual observation e
     $$w(x) = P(e|x)$$
  3. Resample: normalize the weights, and sample N new particles from the resulting distribution over states

Robot Localization

- In robot localization:
  - We know the map, but not the robot's position
  - Observations may be vectors of range finder readings
  - State space and readings are typically continuous (works basically like a very fine grid) and so we cannot store B(X)
  - Particle filtering is a main technique

Robot Localization

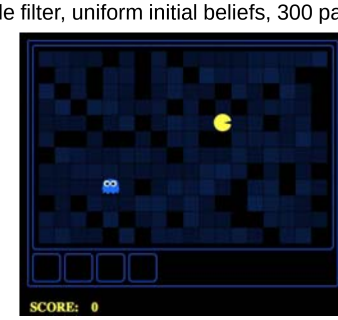
QuickTime™ and a
GIF decompressor
are needed to see this picture.

Which Algorithm?

Exact filter, uniform initial beliefs

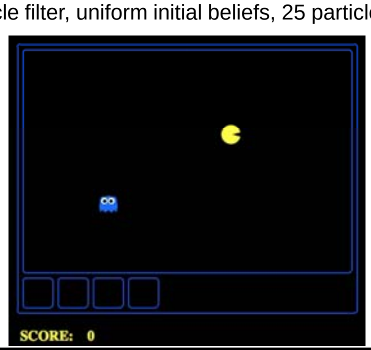
Which Algorithm?

Particle filter, uniform initial beliefs, 300 particles



Which Algorithm?

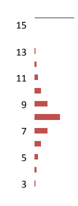
Particle filter, uniform initial beliefs, 25 particles



P4: Ghostbusters

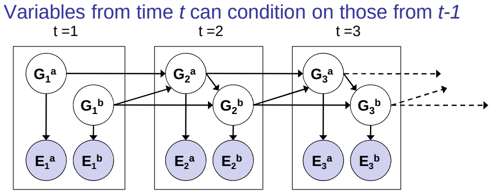
- **Plot:** Pacman's grandfather, Grandpac, learned to hunt ghosts for sport.
- He was blinded by his power, but could hear the ghosts' banging and clanging.
- **Transition Model:** All ghosts move randomly, but are sometimes biased
- **Emission Model:** Pacman knows a "noisy" distance to each ghost

Noisy distance prob
True distance = 8



Dynamic Bayes Nets (DBNs)

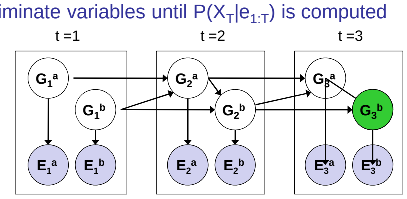
- We want to track multiple variables over time, using multiple sources of evidence
- Idea: Repeat a fixed Bayes net structure at each time
- Variables from time *t* can condition on those from *t-1*



- Discrete valued dynamic Bayes nets are also HMMs

Exact Inference in DBNs

- Variable elimination applies to dynamic Bayes nets
- Procedure: "unroll" the network for T time steps, then eliminate variables until $P(X_T|e_{1:T})$ is computed



- Online belief updates: Eliminate all variables from the previous time step; store factors for current time only

DBN Particle Filters

- A particle is a complete sample for a time step
- **Initialize**: Generate prior samples for the t=1 Bayes net
  - Example particle: $\mathbf{G_1^a}$ = (3,3) $\mathbf{G_1^b}$ = (5,3)
- **Elapse time**: Sample a successor for each particle
  - Example successor: $\mathbf{G_2^a}$ = (2,3) $\mathbf{G_2^b}$ = (6,3)
- **Observe**: Weight each entire sample by the likelihood of the evidence conditioned on the sample
  - Likelihood: $P(\mathbf{E_1^a} | \mathbf{G_1^a}) * P(\mathbf{E_1^b} | \mathbf{G_1^b})$
- **Resample:** Select prior samples (tuples of values) in proportion to their likelihood

SLAM

- SLAM = Simultaneous Localization And Mapping
  - We do not know the map or our location
  - Our belief state is over maps and positions!
  - Main techniques: Kalman filtering (Gaussian HMMs) and particle methods

DP-SLAM, Ron Parr