Robust Visual Motion Analysis:
Piecewise-Smooth Optical Flow

Ming Ye
Electrical Engineering, University of Washington
mingye@u.washington.edu

What Is Visual Motion

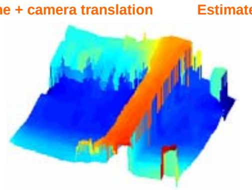
- **2D image velocity**
  - 3D motion projection
  - Temporal correspondence
  - Image deformation

- **Optical flow**
  - An image of 2D velocity
  - Each pixel $V_{s=(x,y)} = (u_s, v_s)$
    where u_s and v_s are the displacements in x and y.
  - $(x,y,t) \Leftrightarrow (x+u,y+v,t+1)$

Structure From Motion

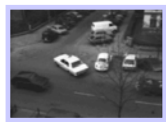
Rigid scene + camera translation

Estimated horizontal motion



Depth map

Scene Dynamics Understanding



Estimated horizontal motion

Brighter pixels => larger speeds.

- Surveillance
- Event analysis
- Video compression

Motion smoothness

Motion boundaries are smooth.

Target Detection and Tracking

A tiny airplane --- only observable by its distinct motion

Tracking results

Optical Flow Estimation:Basics

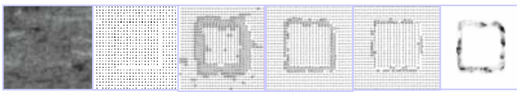
- **Template matching**

- **Assumptions:**
  - Brightness conservation
  - Flow smoothness
- **Difficulties:**
  - Aperture problem (local information insufficient)
  - Outliers (motion boundaries, abrupt image noise)

red square: homogenous area (extreme case, motion completely ambiguous)
green square: directionally homogenous (motion parallel to the edge ambiguous)
yellow square: good template (little ambiguity) In Slide Show, you'll see the content in the 2 yellow squares matching
blue square: motion discontinuity

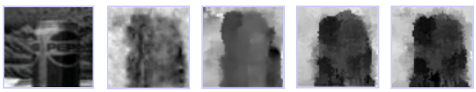
Results from Prior Methods:

LS = Least Squares, LS-R = Robust Least Squares, R = new robust method



Sampled by2: True LS-LS LS-R R-R Confidence

LS = Least Squares, LS-R = Robust Least Squares, R = new robust method



Horizontal flow: M-OFC LS-LMedS LS-R R-R

M-OFC = solving the optical flow constraint using the M-Estimator
LMedS = Least Median of Squares

Estimating Piecewise-Smooth Optical Flow with Global Matching and Graduated Optimization

A Bayesian Approach

Problem Statement

Assuming only brightness conservation and piecewise-smooth motion, find the optical flow to best describe the intensity change in three frames.

Approach: Matching-Based Global Optimization

- **Step 1. Robust local gradient-based method for high-quality initial flow estimate.**
- **Step 2. Global gradient-based method to improve the flow-field coherence.**
- **Step 3. Global matching that minimizes energy by a greedy approach.**

Global Energy Design

V is the optical flow field.

V_S is the optical flow at pixel s.

- **Global energy** $E = \sum_{\text{all sites s}} E_B(V_s) + E_S(V_s)$

E_B is the brightness conservation.

- **Matching error** $E_B(V_s) = \rho(e_W(V_s), \sigma_{B_s})$
  - Warping error $e_W(V_s) = \min(|I^-(V_s) - I_s|, |I^+(V_s) - I_s|)$

I^- and I^+ are prev & next frame; $I^-(V_S)$ is the warped intensity in prev frame.

- **Smoothness error**

$$E_S(V_i) = \frac{1}{8} \sum_{n \in N_s^8} \rho(|V_s - V_n|, \sigma_{S_s})$$

Error function: $\rho(x, \sigma) = \frac{x^2}{\sigma^2 + x^2}$

E_S is the flow smoothness error in a neighborhood about pixel s.

Step 1: Gradient-Based Local Regression

- **A crude flow estimate is assumed available (and has been compensated for)**
- **A robust gradient-based local regression is used to compute the incremental flow ΔV.**
- **The dominant translational motion in the neighborhood of each pixel is computed by solving a set of flow equations using a least-median-of-squares criterion.**

Step 2: Gradient-Based Global Optimization

• The coherence of ΔV using a gradient-based global optimization method.

• The energy to minimize is given by

$$E(\Delta V) = \sum_{\text{all sites } s} \{\rho(e_B(\Delta V_s), \sigma_{B_s}) + \frac{1}{8}\sum_{n \in N_s^8} \rho(|V_s + \Delta V_s - V_n - \Delta V_n|, \sigma_{S_s})\}$$

where e_B is the residual of the OFC, V_s is the ith vector of the initial flow, and the sigmas are parameters.

Step 3: Global Matching

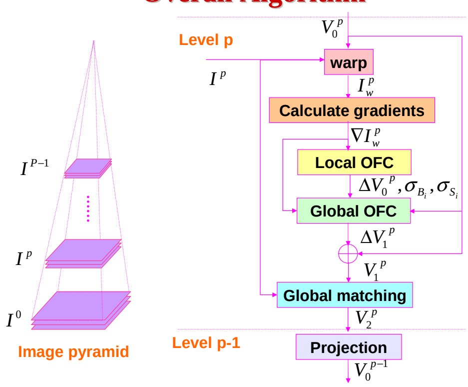
• The new flow estimate still exhibits gross errors at motion boundaries and other places with poor gradient estimates.

• This error is reduced by solving the matching-based formulation equation through greedy propagation.

• The energy is calculated for all pixels.

• Then each pixel is visited, examining whether a trial estimate from the candidates in its neighborhood is better (lower energy). If so, this becomes the new estimate for that pixel. This is repeated iteratively.

Overall Algorithm



Advantages

- **Best of Everything**
  - Local OFC
    - High-quality initial flow estimates
    - Robust local scale estimates
  - Global OFC
    - Improve flow smoothness
  - Global Matching
    - The optimal formulation
    - Correct errors caused by poor gradient quality and hierarchical process
- **Results:** fast convergence, high accuracy, simultaneous motion boundary detection

Experiments

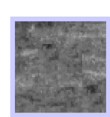
• Experiments were run on several standard test videos.

• Estimates of optical flow were made for the middle frame of every three.

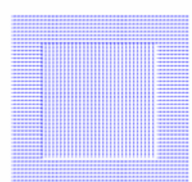
• The results were compared with the Black and Anandan algorithm.

TS: Translating Squares

- Homebrew, ideal setting, test performance upper bound

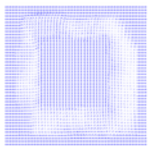


64x64, 1pixel/frame

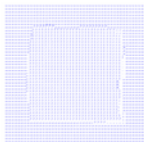


Groundtruth (cropped),
Our estimate looks the same

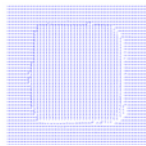
TS: Flow Estimate Plots



LS



BA



S1 (S2 is close)

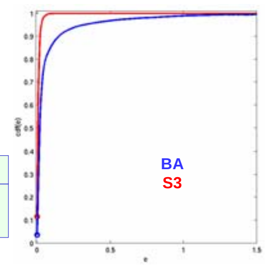
S3 looks the same as the groundtruth.

- S1, S2, S3: results from our Step I, II, III (final)

TT: Translating Tree

150x150 (Barron 94)

	$e_{\angle}(^{\circ})$	$e_{\lvert\bullet\rvert}(\text{pix})$	$\bar{e}(\text{pix})$
BA	2.60	0.128	0.0724
S3	0.248	0.0167	0.00984

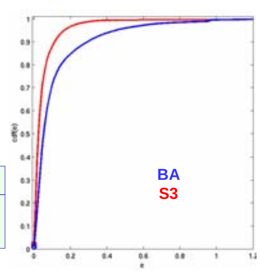


e: error in pixels, cdf: culmulative distribution function for all pixels

DT: Diverging Tree

150x150 (Barron 94)

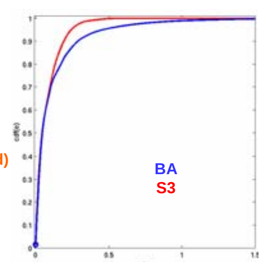
	$e_{\angle}(^{\circ})$	$e_{\lvert\bullet\rvert}(\text{pix})$	$\bar{e}(\text{pix})$
BA	6.36	0.182	0.114
S3	2.60	0.0813	0.0507



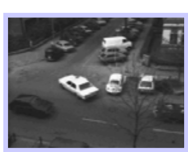
YOS: Yosemite Fly-Through

316x252 (Barron, cloud excluded)

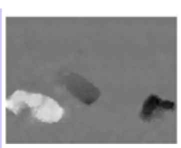
	$e_{\angle}(^{\circ})$	$e_{\lvert\bullet\rvert}(\text{pix})$	$\bar{e}(\text{pix})$
BA	2.71	0.185	0.118
S3	1.92	0.120	0.0776



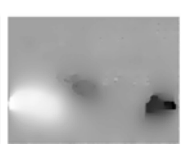
TAXI: Hamburg Taxi



256x190, (Barron 94)
max speed 3.0 pix/frame

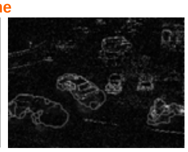


LMS

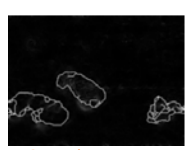


BA

Ours



Error map

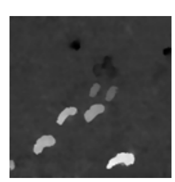


Smoothness error

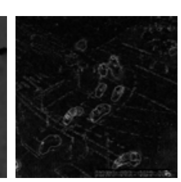
Traffic

512x512
(Nagel)
max speed:
6.0 pix/frame

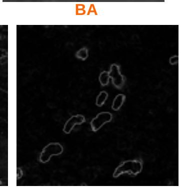
BA



Ours



Error map



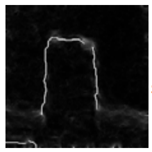
Smoothness error

Pepsi Can

201x201
(Black)
Max speed:
2pix/frame

Ours

BA



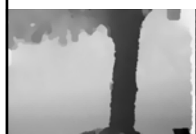
Smoothness
error

FG: Flower Garden

360x240 (Black)
Max speed: 7pix/frame

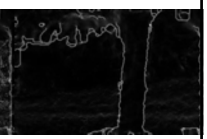
BA

LMS



Ours

Error map



Smoothness error

Contributions (1/2)

- **Formulation**
  - More complete design, minimal parameter tuning
    - Adaptive local scales
    - Strength of two error terms automatically balanced
  - 3-frame matching to avoid visibility problems
- **Solution: 3-step optimization**
  - Robust initial estimates and scales
  - Model parameter self-learning
  - Inherit merits of 3 methods and overcome shortcomings

Contributions (2/2)

- **Results**
  - High accuracy
  - Fast convergence
  - By product: motion boundaries
- **Significance**
  - Foundation for higher-level (model-based) visual motion analysis
  - Methodology applicable to other low-level vision problems