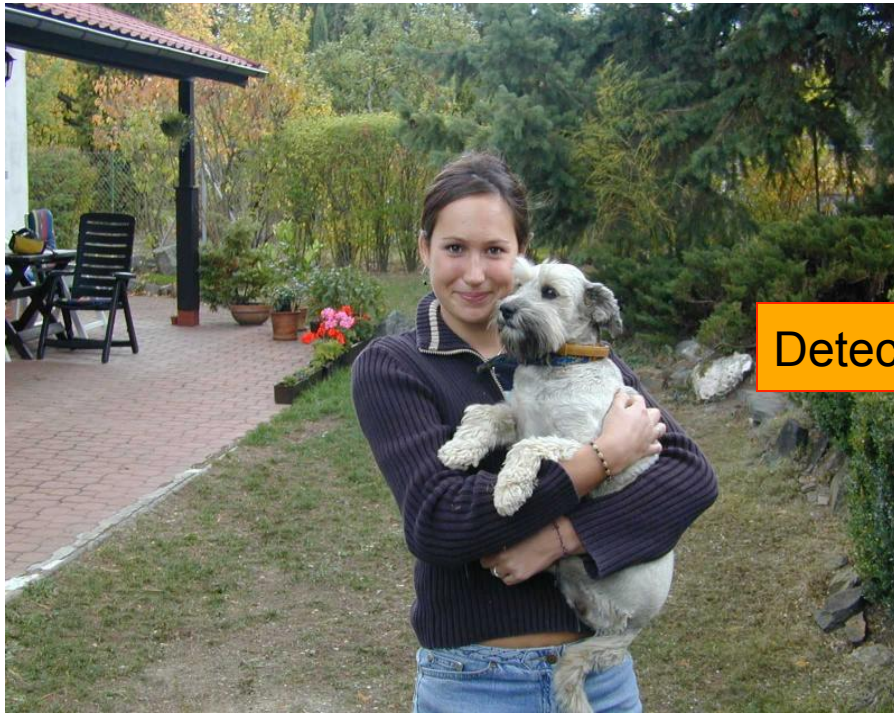


# Face Recognition

CSE 576

# Face recognition: once you've detected and cropped a face, try to recognize it



Detection



Recognition

"Sally"

# Face recognition: overview

- Typical scenario: few examples per face, identify or verify test example
- What's hard: changes in expression, lighting, age, **occlusion**, **viewpoint**
- Basic approaches (all nearest neighbor)
  1. Project into a new subspace
  2. Measure face features

# Typical face recognition scenarios

- Verification: a person is claiming a particular identity; verify whether that is true
  - E.g., security
- Closed-world identification: assign a face to one person from among a known set
- General identification: assign a face to a known person or to “unknown”

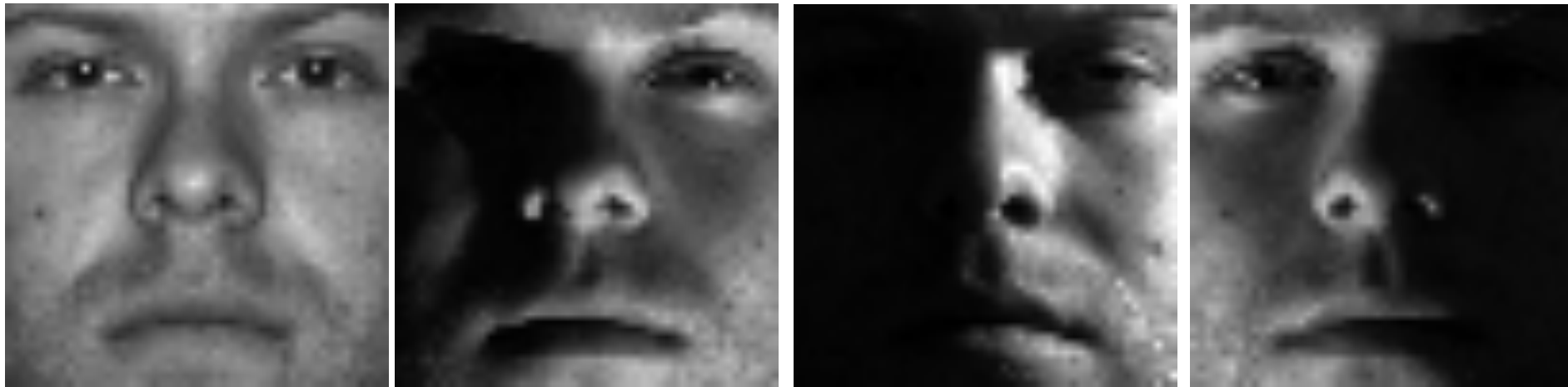
# What makes face recognition hard?

## Expression



# What makes face recognition hard?

## Lighting



# What makes face recognition hard?

Occlusion



# What makes face recognition hard?

## Viewpoint





# Simple idea for face recognition

1. Treat face image as a vector of intensities



2. Recognize face by nearest neighbor in database



$$k = \underset{k}{\operatorname{argmin}} \|\mathbf{y}_k - \mathbf{x}\|$$

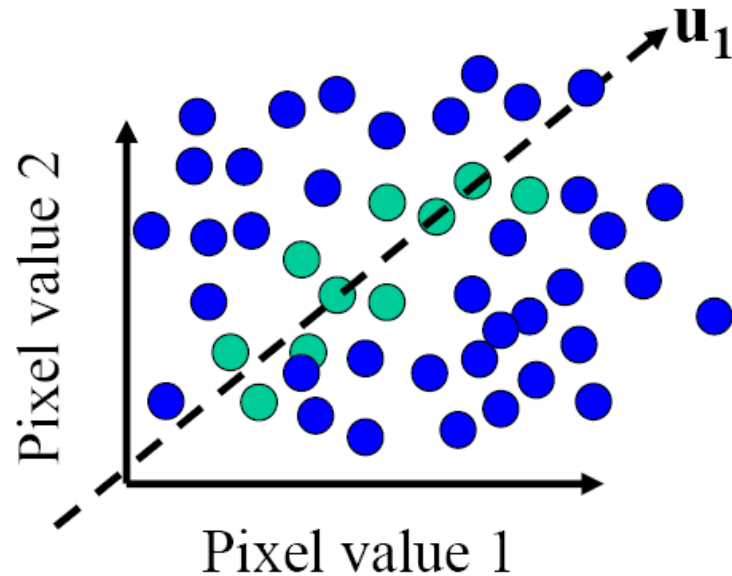
# The space of all face images

- When viewed as vectors of pixel values, face images are extremely high-dimensional
  - 100x100 image = 10,000 dimensions
  - Slow and lots of storage
- But very few 10,000-dimensional vectors are valid face images
- We want to effectively model the subspace of face images



# The space of all face images

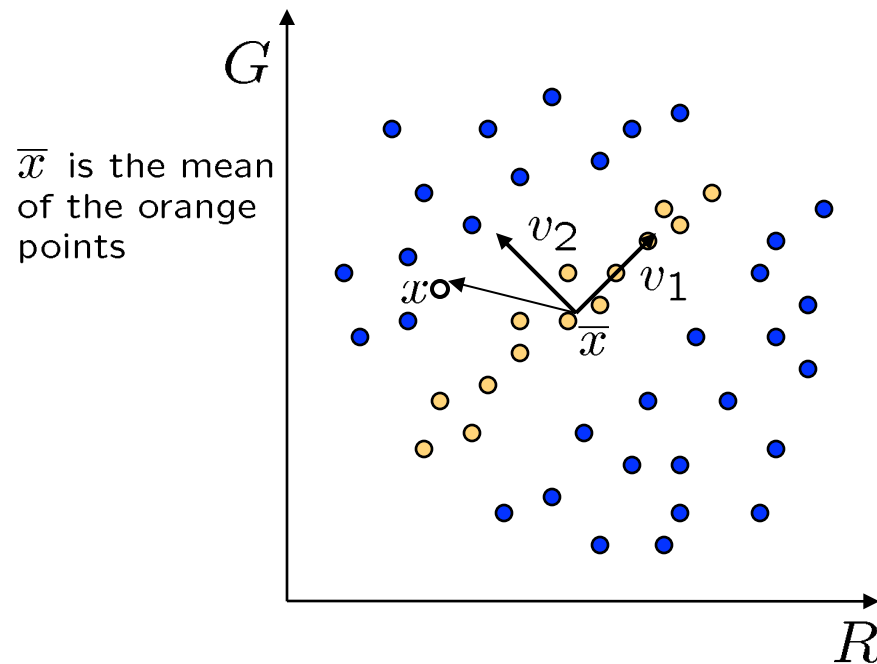
- Idea: construct a low-dimensional linear subspace that best explains the variation in the set of face images



● A face image

● A (non-face) image

# Linear subspaces



Consider the variation along direction  $\mathbf{v}$  among all of the orange points:

$$\text{var}(\mathbf{v}) = \sum_{\text{orange point } \mathbf{x}} \|(\mathbf{x} - \bar{\mathbf{x}})^T \cdot \mathbf{v}\|^2$$

What unit vector  $\mathbf{v}$  minimizes  $\text{var}$ ?

$$\mathbf{v}_2 = \min_{\mathbf{v}} \{\text{var}(\mathbf{v})\}$$

What unit vector  $\mathbf{v}$  maximizes  $\text{var}$ ?

$$\mathbf{v}_1 = \max_{\mathbf{v}} \{\text{var}(\mathbf{v})\}$$

$$\begin{aligned} \text{var}(\mathbf{v}) &= \sum_{\mathbf{x}} \|(\mathbf{x} - \bar{\mathbf{x}})^T \cdot \mathbf{v}\|^2 \\ &= \sum_{\mathbf{x}} \mathbf{v}^T (\mathbf{x} - \bar{\mathbf{x}}) (\mathbf{x} - \bar{\mathbf{x}})^T \mathbf{v} \\ &= \mathbf{v}^T \left[ \sum_{\mathbf{x}} (\mathbf{x} - \bar{\mathbf{x}}) (\mathbf{x} - \bar{\mathbf{x}})^T \right] \mathbf{v} \\ &= \mathbf{v}^T \mathbf{A} \mathbf{v} \quad \text{where } \mathbf{A} = \sum_{\mathbf{x}} (\mathbf{x} - \bar{\mathbf{x}}) (\mathbf{x} - \bar{\mathbf{x}})^T \end{aligned}$$

Solution:  $\mathbf{v}_1$  is eigenvector of  $\mathbf{A}$  with largest eigenvalue

$\mathbf{v}_2$  is eigenvector of  $\mathbf{A}$  with smallest eigenvalue

# Principal component analysis (PCA)

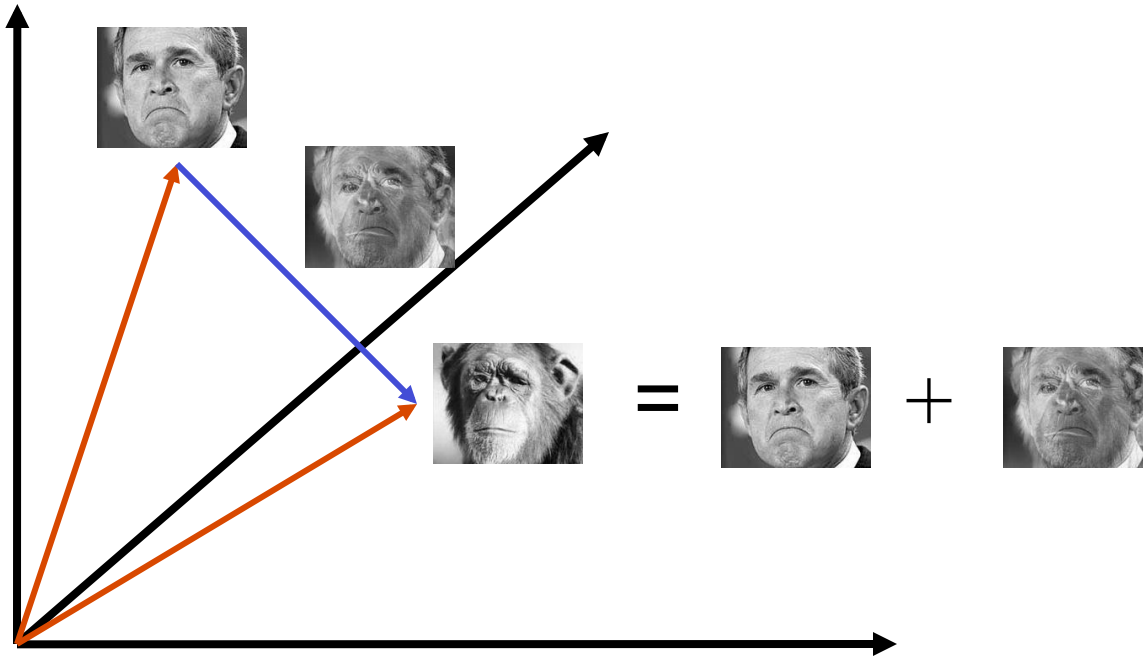
- Suppose each data point is N-dimensional
  - Same procedure applies:

$$\begin{aligned} \text{var}(\mathbf{v}) &= \sum_{\mathbf{x}} \|(\mathbf{x} - \bar{\mathbf{x}})^T \cdot \mathbf{v}\|^2 \\ &= \mathbf{v}^T \mathbf{A} \mathbf{v} \quad \text{where } \mathbf{A} = \sum_{\mathbf{x}} (\mathbf{x} - \bar{\mathbf{x}})(\mathbf{x} - \bar{\mathbf{x}})^T \end{aligned}$$

- The eigenvectors of  $\mathbf{A}$  define a new coordinate system
  - eigenvector with largest eigenvalue captures the most variation among training vectors  $\mathbf{x}$
  - eigenvector with smallest eigenvalue has least variation
- We can compress the data by only using the top few eigenvectors
  - corresponds to choosing a “linear subspace”
    - represent points on a line, plane, or “hyper-plane”
  - these eigenvectors are known as the principal components

# The space of faces

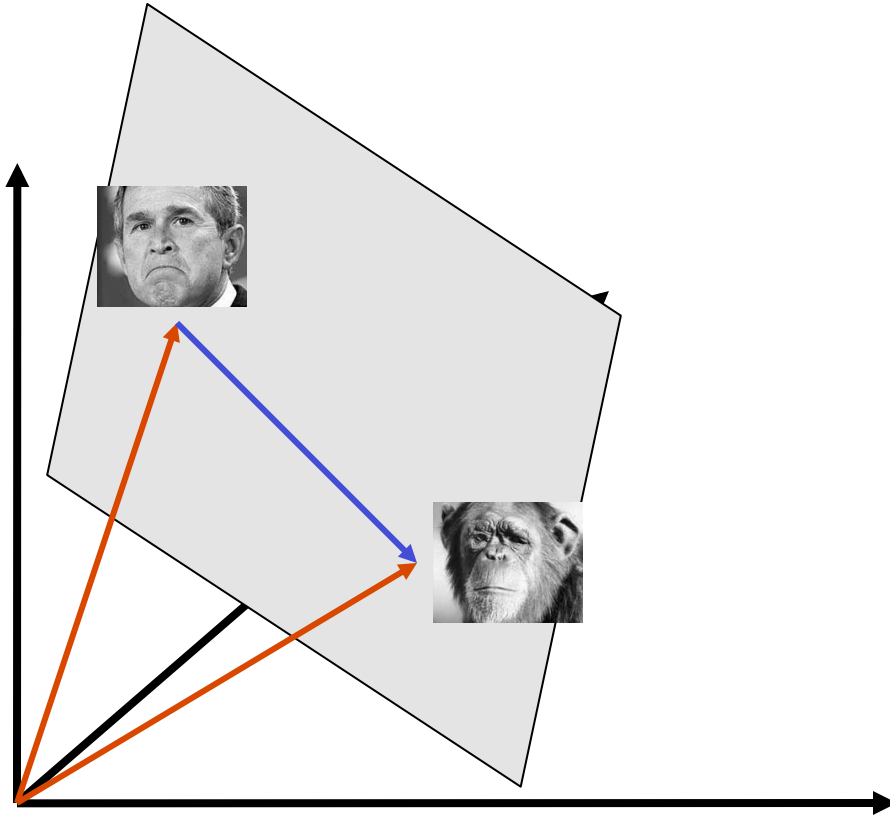
---



- An image is a point in a high dimensional space
  - An  $N \times M$  image is a point in  $\mathbb{R}^{NM}$
  - We can define vectors in this space as we did in the 2D case

# Dimensionality reduction

---

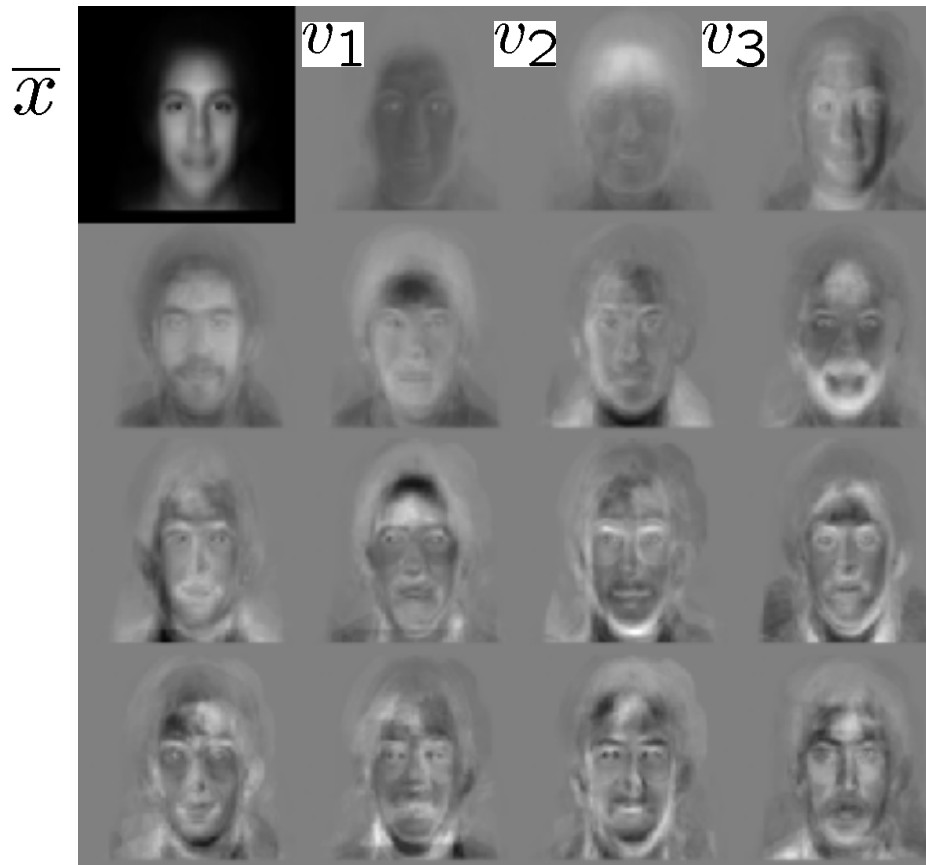


- The set of faces is a “subspace” of the set of images
  - Suppose it is  $K$  dimensional
  - We can find the best subspace using PCA
  - This is like fitting a “hyper-plane” to the set of faces
    - spanned by vectors  $v_1, v_2, \dots, v_K$
    - any face  $x \approx \bar{x} + a_1v_1 + a_2v_2 + \dots + a_kv_k$

# Eigenfaces

---

- PCA extracts the eigenvectors of  $A$ 
  - Gives a set of vectors  $v_1, v_2, v_3, \dots$
  - Each one of these vectors is a direction in face space
    - what do these look like?





# Visualization of eigenfaces

Principal component (eigenvector)  $u_k$



$\mu + 3\sigma_k u_k$



$\mu - 3\sigma_k u_k$



# Projecting onto the eigenfaces

- The eigenfaces  $v_1, \dots, v_K$  span the space of faces

- A face is converted to eigenface coordinates by

$$\mathbf{x} \rightarrow \left( \underbrace{(\mathbf{x} - \bar{\mathbf{x}}) \cdot \mathbf{v}_1}_{a_1}, \underbrace{(\mathbf{x} - \bar{\mathbf{x}}) \cdot \mathbf{v}_2}_{a_2}, \dots, \underbrace{(\mathbf{x} - \bar{\mathbf{x}}) \cdot \mathbf{v}_K}_{a_K} \right)$$

$$\mathbf{x} \approx \bar{\mathbf{x}} + a_1 \mathbf{v}_1 + a_2 \mathbf{v}_2 + \dots + a_K \mathbf{v}_K$$



$\mathbf{x}$



$a_1 \mathbf{v}_1$   $a_2 \mathbf{v}_2$   $a_3 \mathbf{v}_3$   $a_4 \mathbf{v}_4$   $a_5 \mathbf{v}_5$   $a_6 \mathbf{v}_6$   $a_7 \mathbf{v}_7$   $a_8 \mathbf{v}_8$



# Recognition with eigenfaces

---

- Algorithm

1. Process the image database (set of images with labels)

- Run PCA—compute eigenfaces
- Calculate the  $K$  coefficients for each image

2. Given a new image (to be recognized)  $\mathbf{x}$ , calculate  $K$  coefficients

$$\mathbf{x} \rightarrow (a_1, a_2, \dots, a_K)$$

3. Detect if  $\mathbf{x}$  is a face

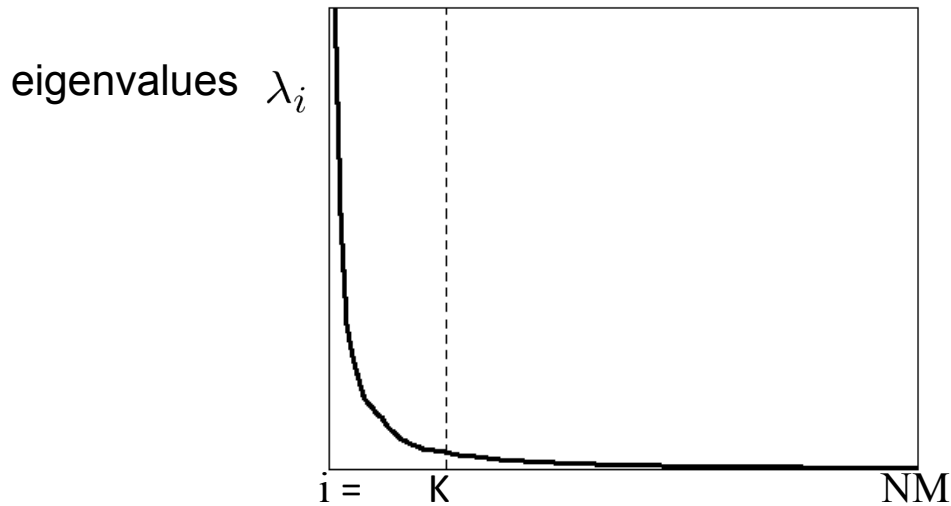
$$\|\mathbf{x} - (\bar{\mathbf{x}} + a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \dots + a_K\mathbf{v}_K)\| < \text{threshold}$$

4. If it is a face, who is it?

- Find closest labeled face in database
- nearest-neighbor in  $K$ -dimensional space

# Choosing the dimension K

---



- How many eigenfaces to use?
- Look at the decay of the eigenvalues
  - the eigenvalue tells you the amount of variance “in the direction” of that eigenface
  - ignore eigenfaces with low variance

# PCA

- General dimensionality reduction technique
- Preserves most of variance with a much more compact representation
  - Lower storage requirements (eigenvectors + a few numbers per face)
  - Faster matching

# Enhancing gender



more

same

**original**

androgynous

more opposite

D. Rowland and D. Perrett,  
[“Manipulating Facial Appearance through Shape and Color,”](#) IEEE CG&A,  
September 1995

Slide credit: A. Efros

# Changing age

- Face becomes “rounder” and “more textured” and “grayer”

- original



- color

D. Rowland and D. Perrett,  
[“Manipulating Facial Appearance through Shape and Color,”](#) IEEE CG&A,  
September 1995

Slide credit: A. Efros

# Which face is more attractive?



<http://www.beautycheck.de>



# Which face is more attractive?



# Which face is more attractive?



0.5(attractive + average)



attractive

# Which face is more attractive?



<http://www.beautycheck.de>

# Which face is more attractive?



# Which face is more attractive?



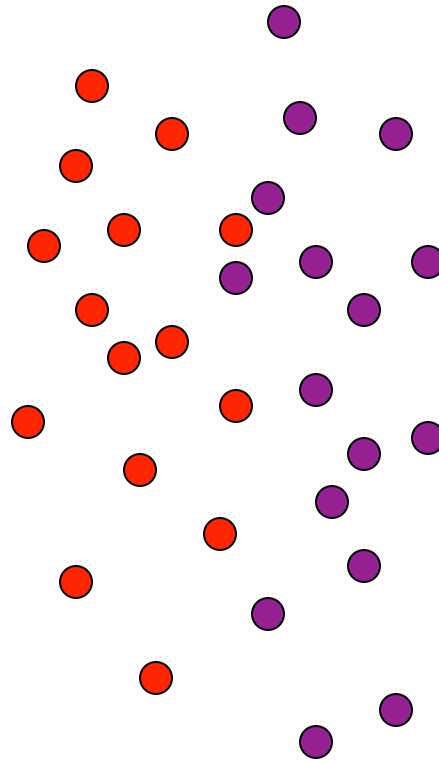
0.5(adult+child)



adult

# Limitations

- The direction of maximum variance is not always good for classification



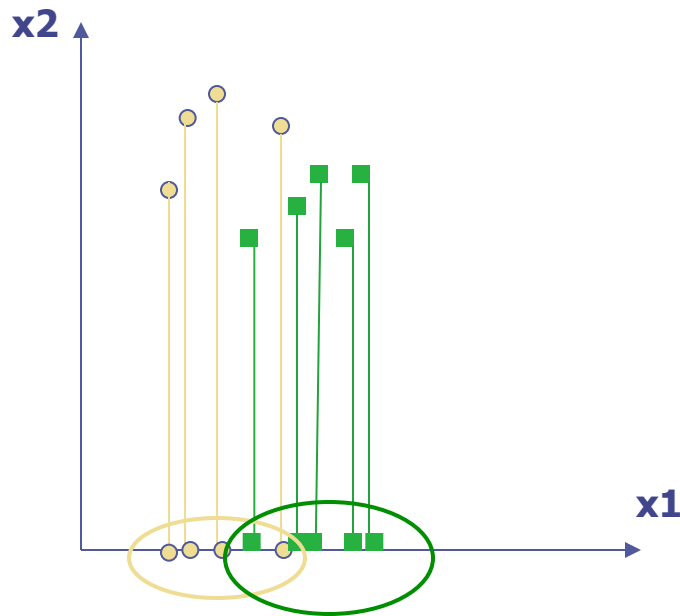
# A more discriminative subspace: FLD

- Fisher Linear Discriminants → “Fisher Faces”
- PCA preserves maximum variance
- FLD preserves discrimination
  - Find projection that maximizes scatter between classes and minimizes scatter within classes

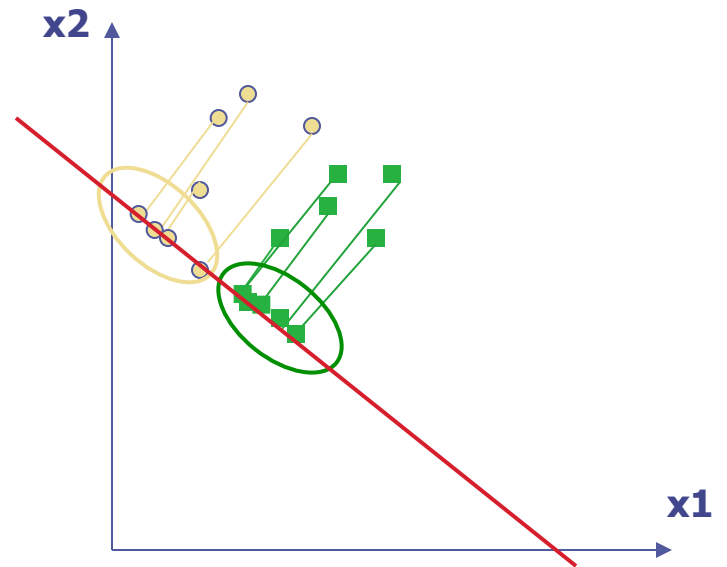
Reference: [Eigenfaces vs. Fisherfaces, Belheumer et al., PAMI 1997](#)

# Illustration of the Projection

- ◆ Using two classes as example:



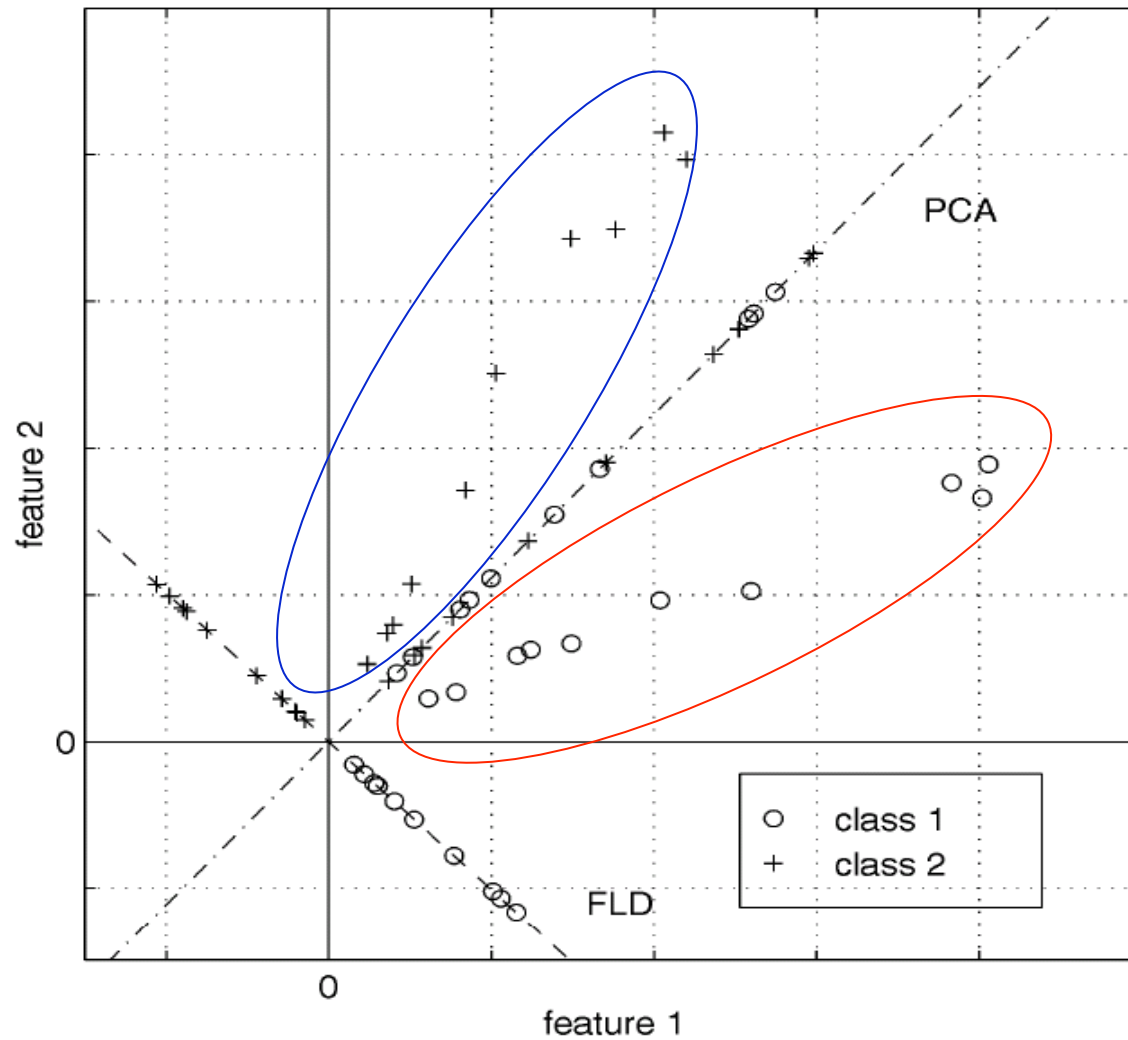
Poor Projection



Good



# Comparing with PCA



# Variables

- N Sample images:

$$\{\mathbf{x}_1, \dots, \mathbf{x}_N\}$$

- c classes:

$$\{\mathcal{X}_1, \dots, \mathcal{X}_c\}$$

- Average of each class:

$$\mu_i = \frac{1}{N_i} \sum_{\mathbf{x}_k \in \mathcal{X}_i} \mathbf{x}_k$$

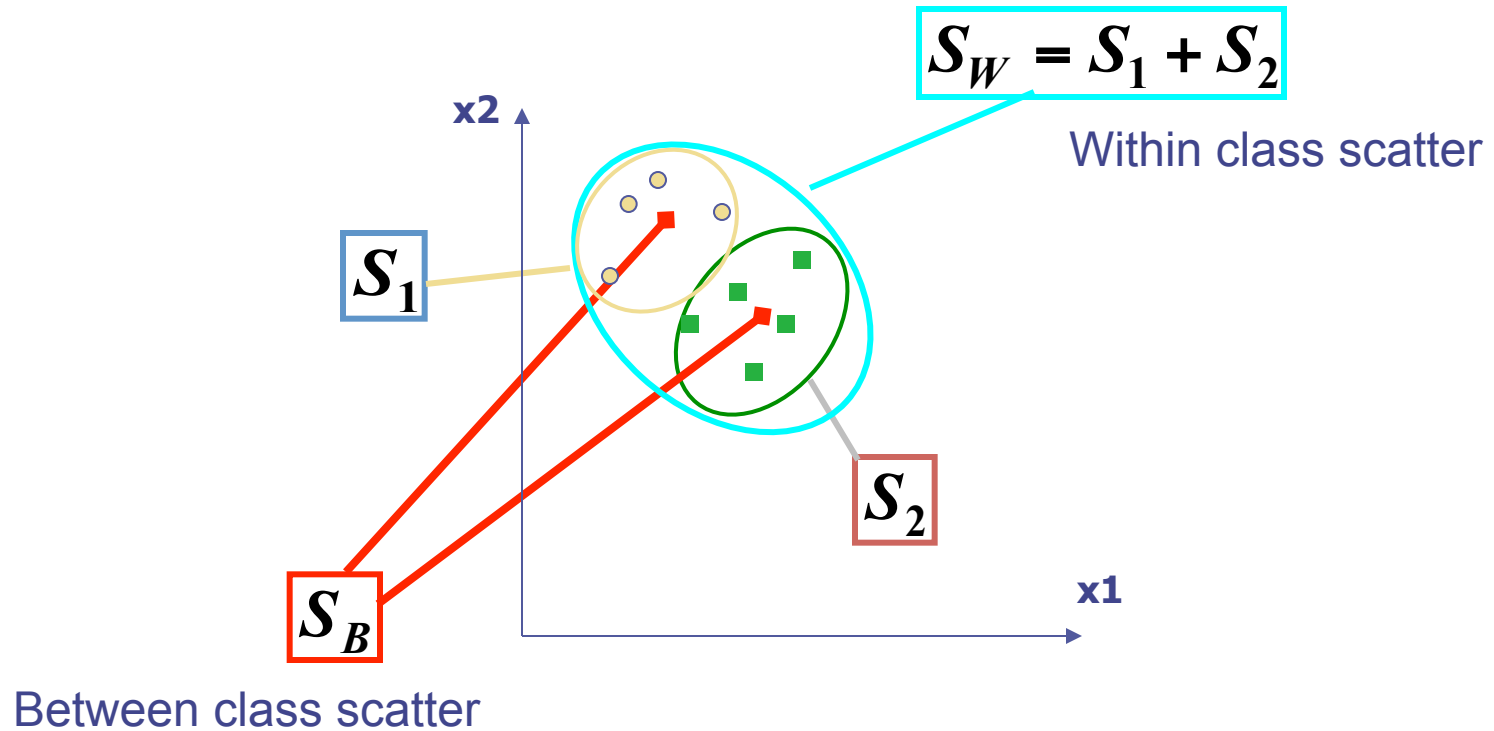
- Average of all data:

$$\mu = \frac{1}{N} \sum_{k=1}^N \mathbf{x}_k$$

# Scatter Matrices

- Scatter of class  $i$ : 
$$S_i = \sum_{x_k \in \mathcal{X}_i} (x_k - \mu_i)(x_k - \mu_i)^T$$
- Within class scatter: 
$$S_W = \sum_{i=1}^c S_i$$
- Between class scatter: 
$$S_B = \sum_{i=1}^c N_i (\mu_i - \mu)(\mu_i - \mu)^T$$

# Illustration



# Mathematical Formulation

- After projection
  - Between class scatter  $y_k = W^T x_k$
  - Within class scatter  $\tilde{S}_B = W^T S_B W$
  - Within class scatter  $\tilde{S}_W = W^T S_W W$

- Objective

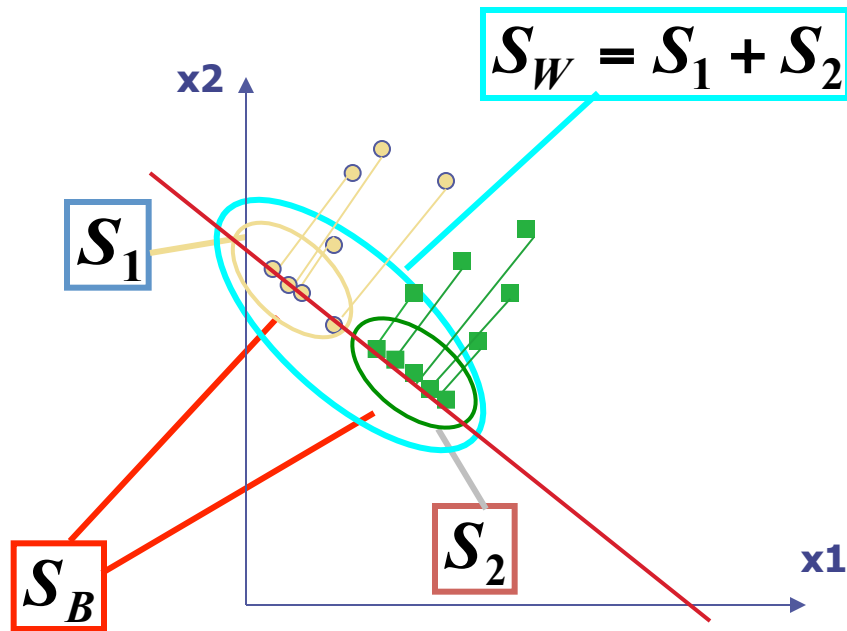
$$W_{opt} = \arg \max_W \frac{|\tilde{S}_B|}{|\tilde{S}_W|} = \arg \max_W \frac{|W^T S_B W|}{|W^T S_W W|}$$

- Solution: Generalized Eigenvectors

$$S_B w_i = \lambda_i S_W w_i \quad i = 1, \dots, m$$

- Rank of  $W_{opt}$  is limited
  - Rank( $S_B$ )  $\leq$  |C|-1
  - Rank( $S_W$ )  $\leq$  N-C

# Illustration



# Recognition with FLD

- Use PCA to reduce dimensions to N-C

$$W_{pca} = \text{pca}(X)$$

- Compute within-class and between-class scatter matrices for PCA coefficients

$$S_i = \sum_{x_k \in \mathcal{X}_i} (x_k - \mu_i)(x_k - \mu_i)^T \quad S_W = \sum_{i=1}^c S_i \quad S_B = \sum_{i=1}^c N_i (\mu_i - \mu)(\mu_i - \mu)^T$$

- Solve generalized eigenvector problem

$$W_{fld} = \arg \max_W \frac{|W^T S_B W|}{|W^T S_W W|} \quad S_B w_i = \lambda_i S_W w_i \quad i = 1, \dots, m$$

- Project to FLD subspace (c-1 dimensions)

$$\hat{x} = W_{opt}^T x$$

- Classify by nearest neighbor

Note: x in step 2 refers to PCA coef; x in step 4 refers to original data

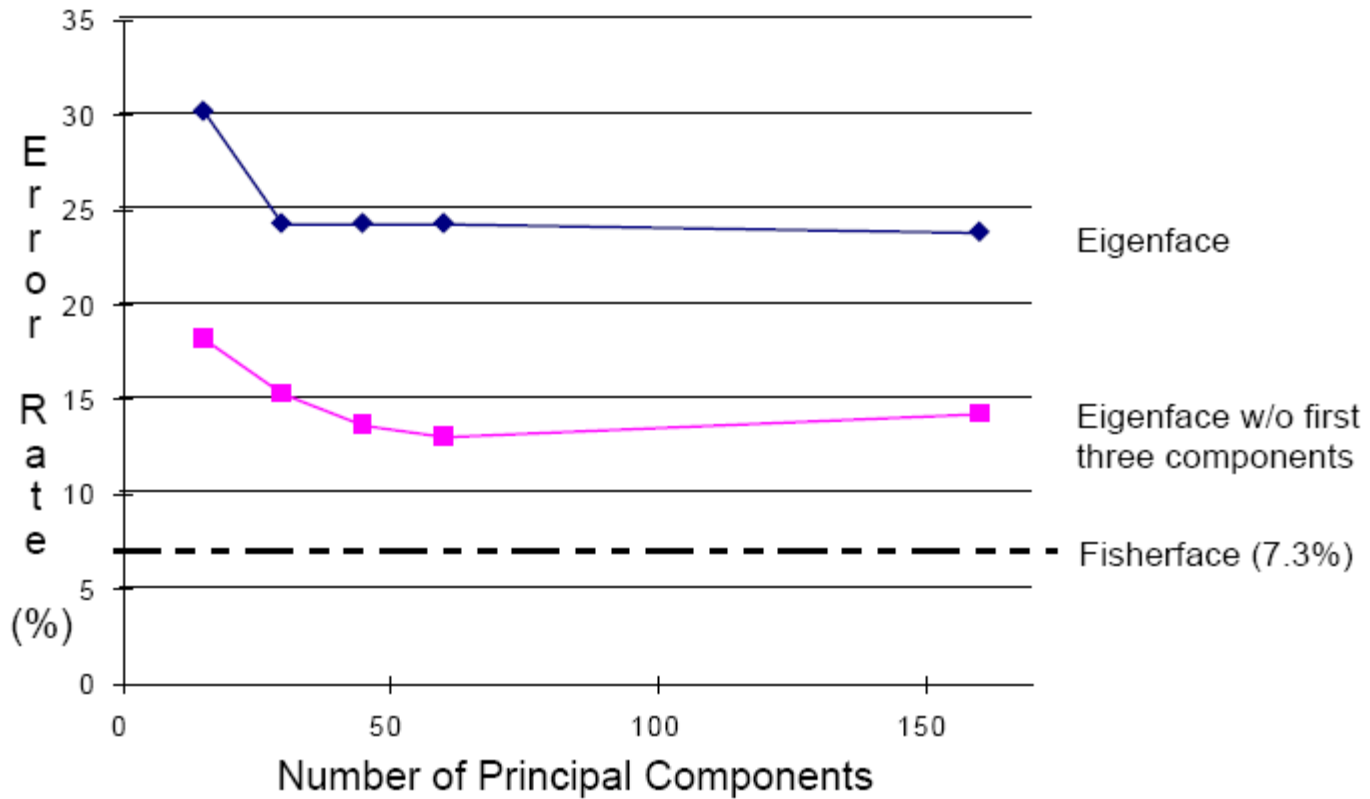
# Results: Eigenface vs. Fisherface

- Input: 160 images of 16 people
- Train: 159 images
- Test: 1 image
- Variation in Facial Expression, Eyewear, and Lighting



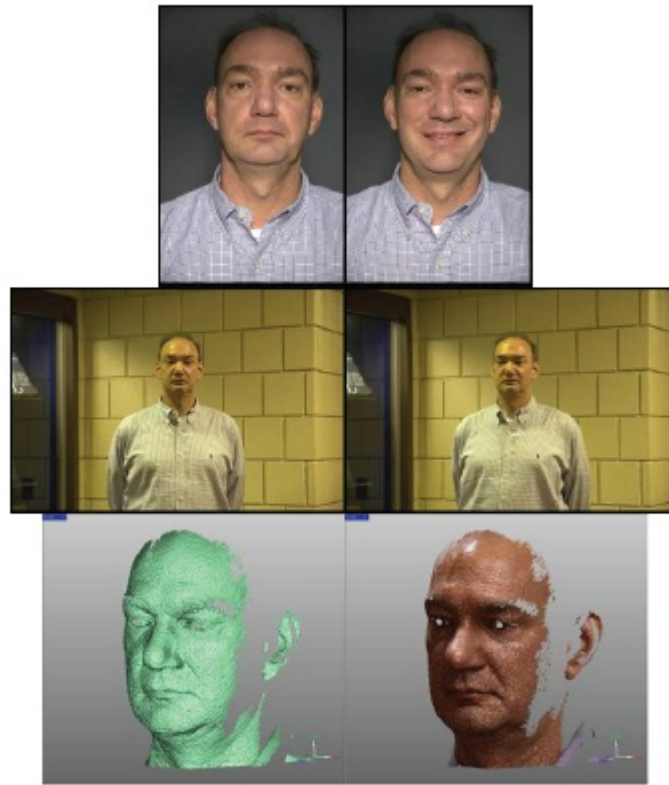


# Eigenfaces vs. Fisherfaces



# Large scale comparison of methods

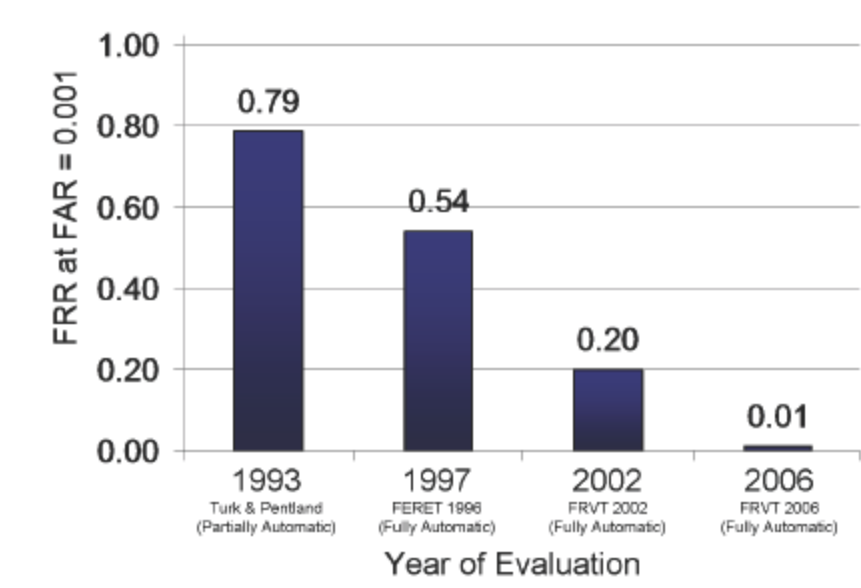
- [FRVT 2006 Report](#)



# FVRT Challenge

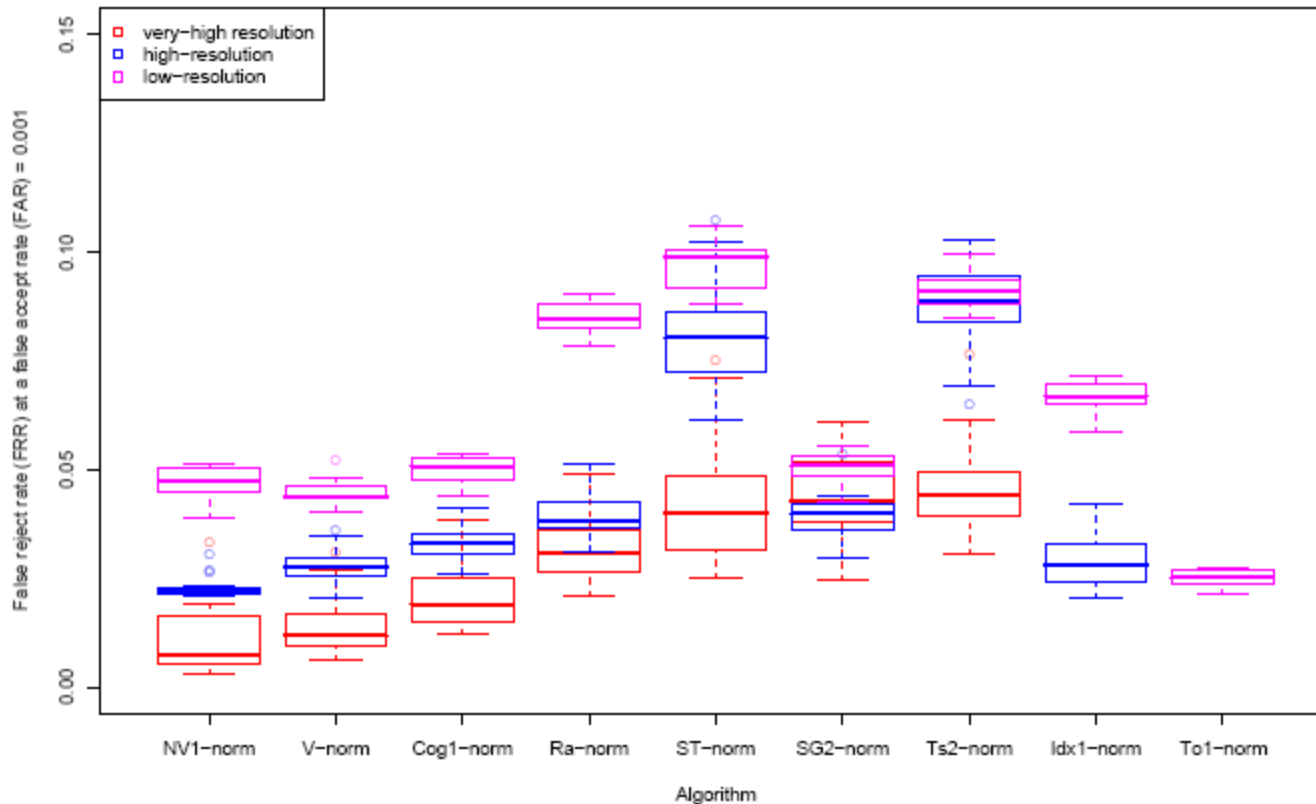
- Frontal faces
  - FVRT2006 evaluation

False  
Rejection  
Rate at False  
Acceptance  
Rate = 0.001



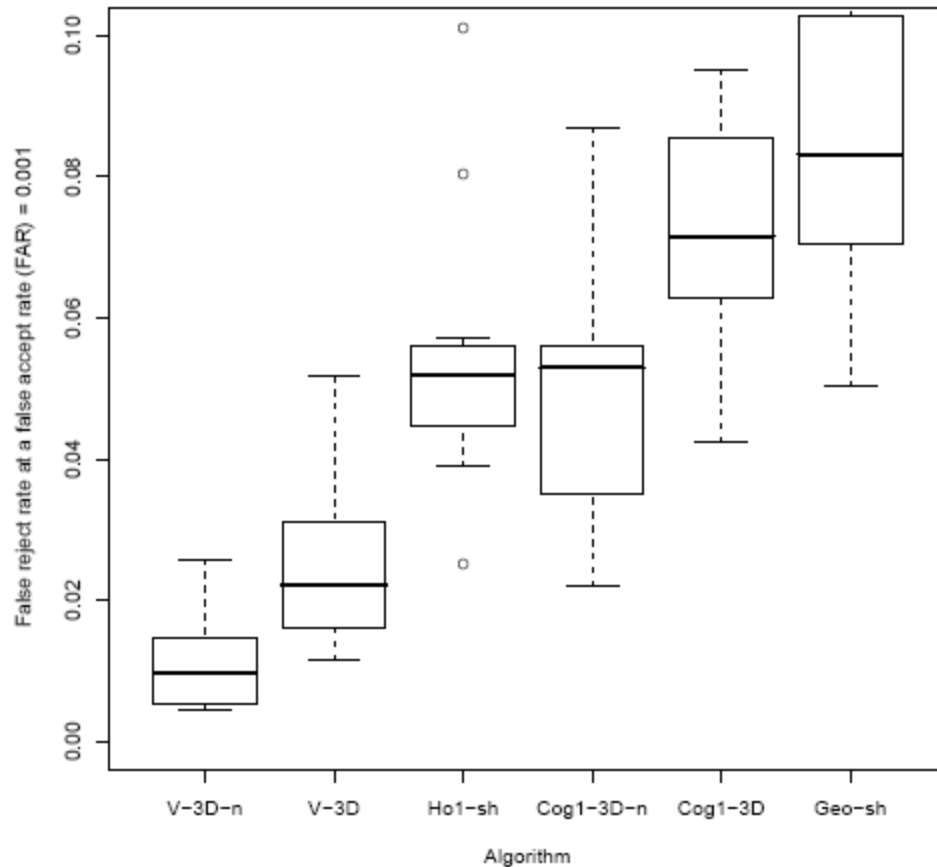
# FVRT Challenge

- Frontal faces
  - FVRT2006 evaluation: controlled illumination



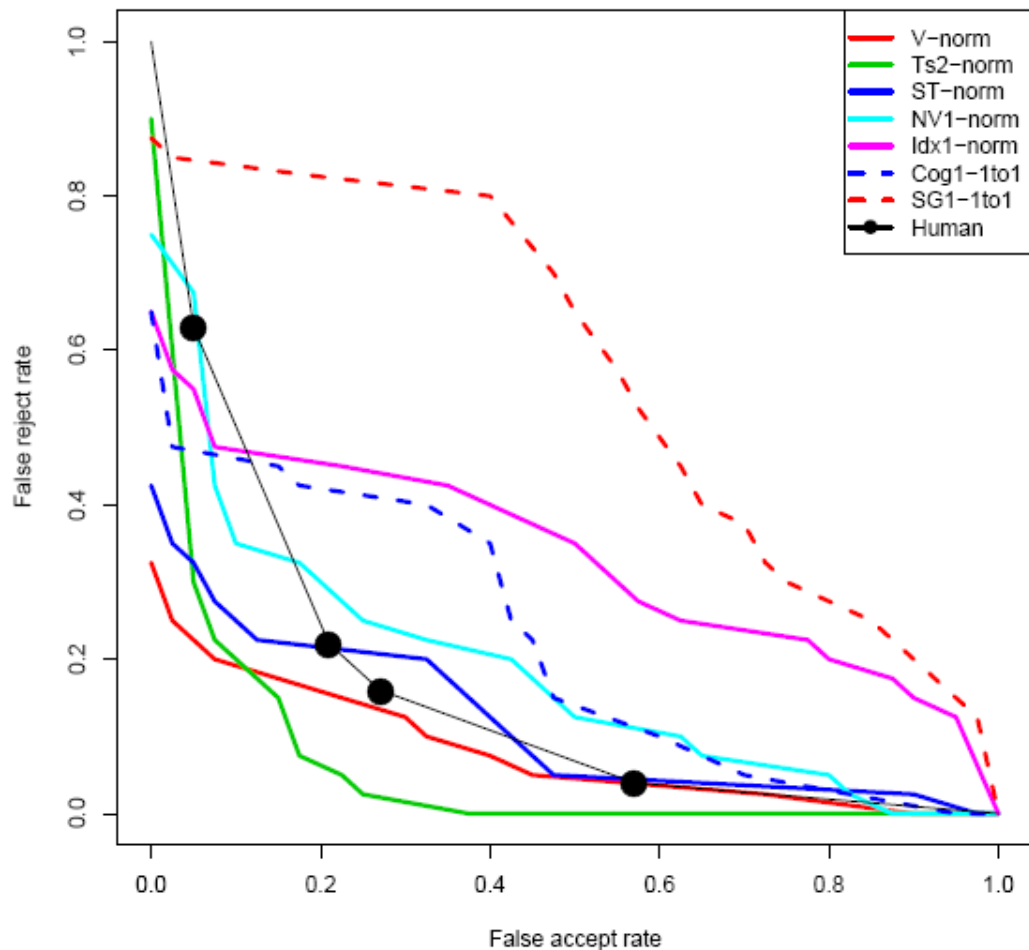
# FVRT Challenge

- Frontal faces
  - FVRT2006 evaluation: uncontrolled illumination



# FVRT Challenge

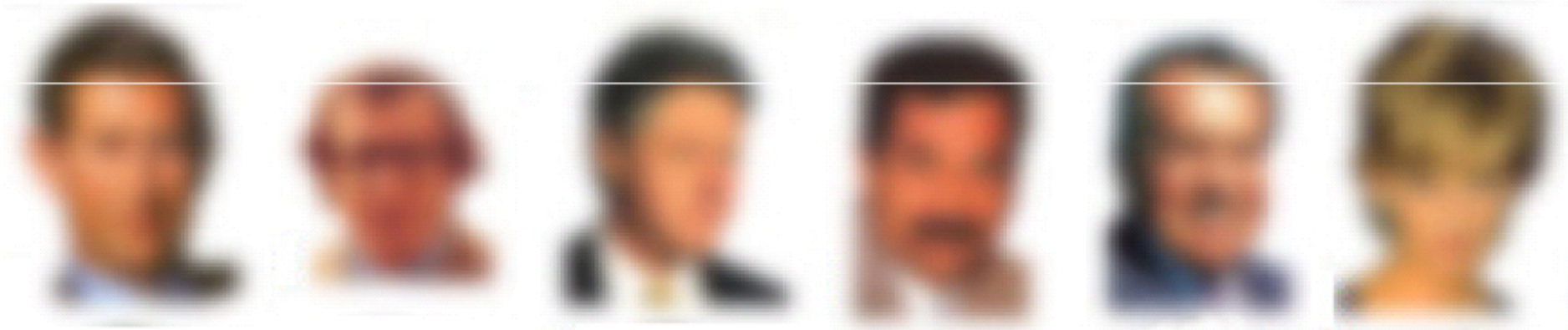
- Frontal faces
  - FVRT2006 evaluation: computers win!



# Face recognition by humans

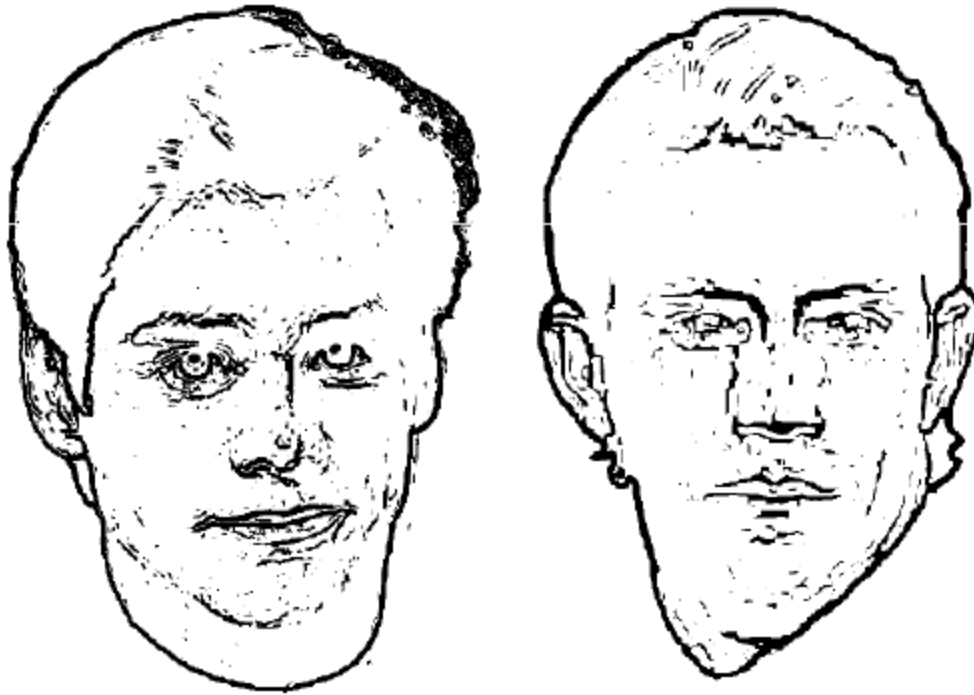
[Face recognition by humans: 20 results](#) (2005)

- ▶ Humans can recognize faces in extremely low resolution images.

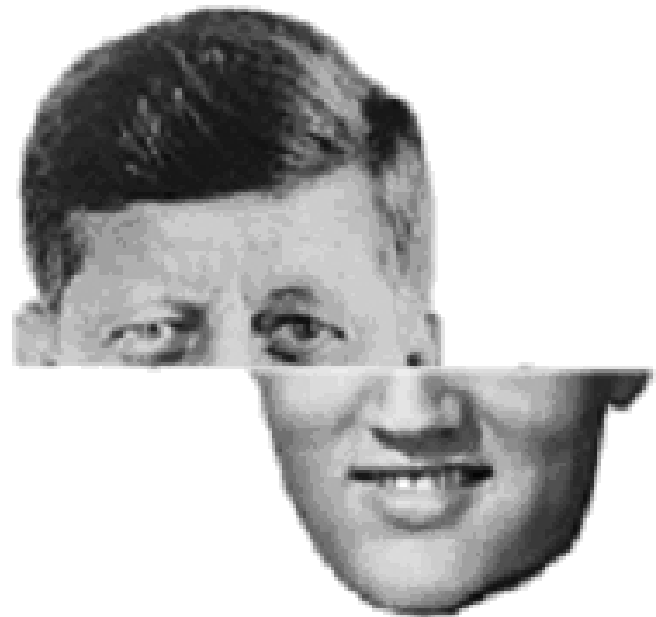
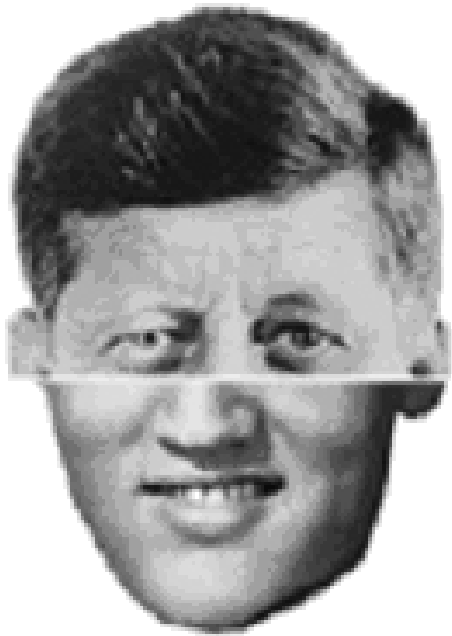




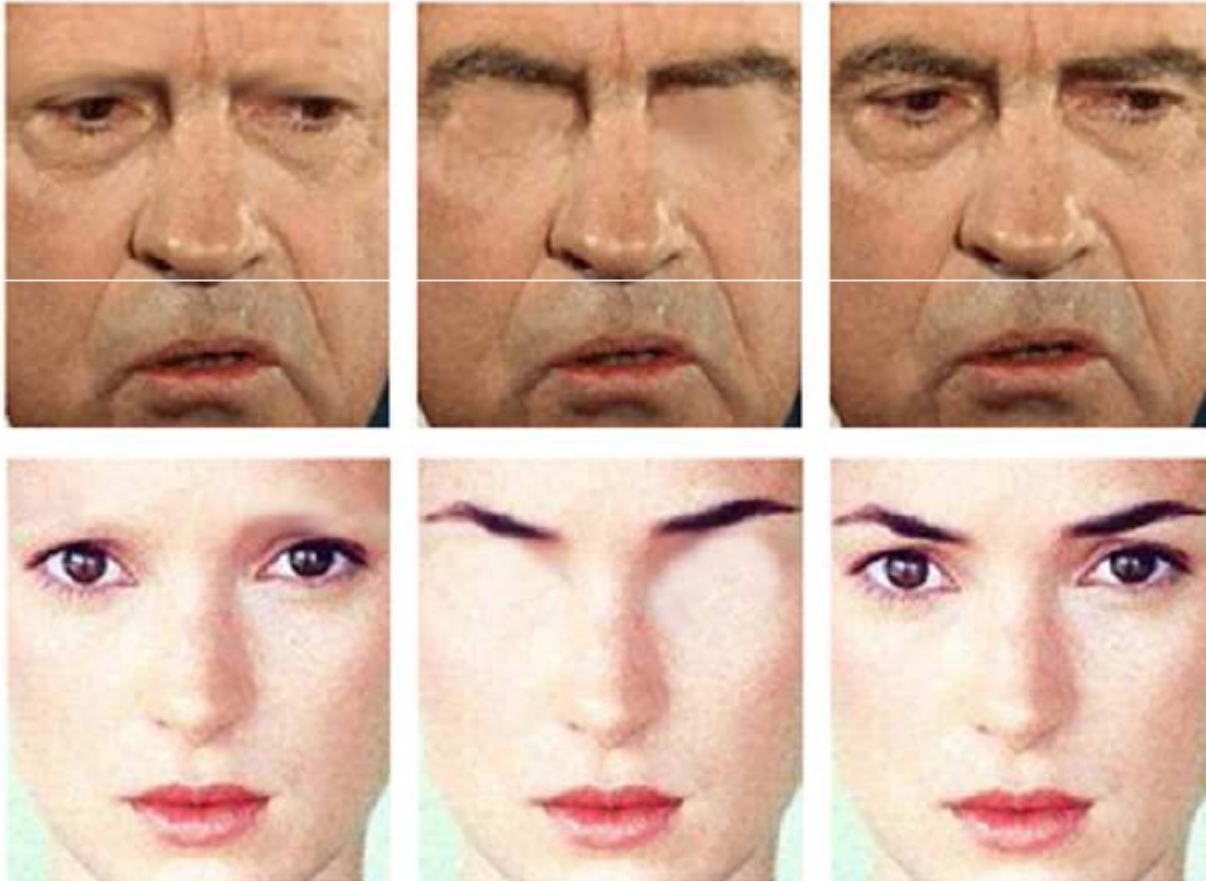
- ▶ High-frequency information by itself does not lead to good face recognition performance



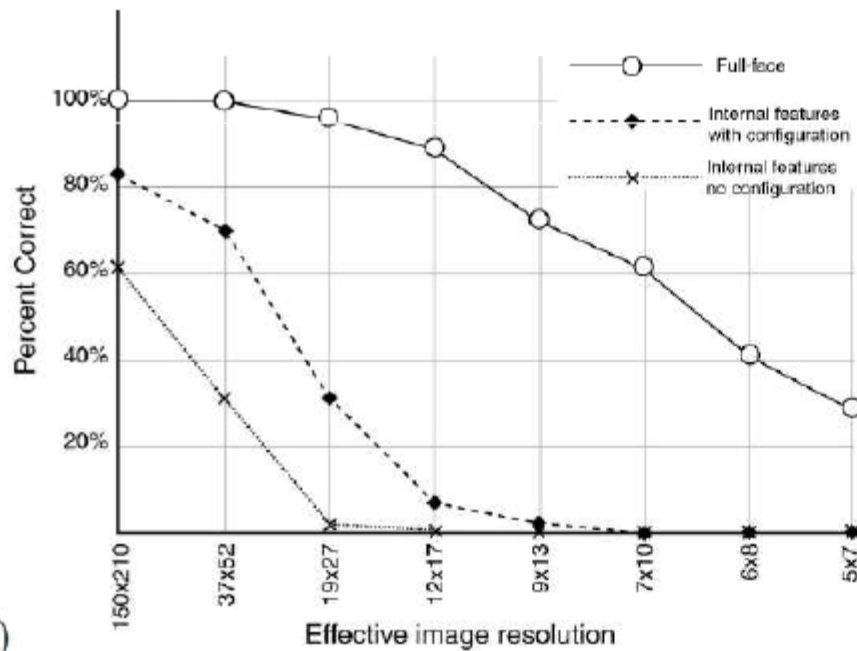
# Result 4: Facial features are processed holistically



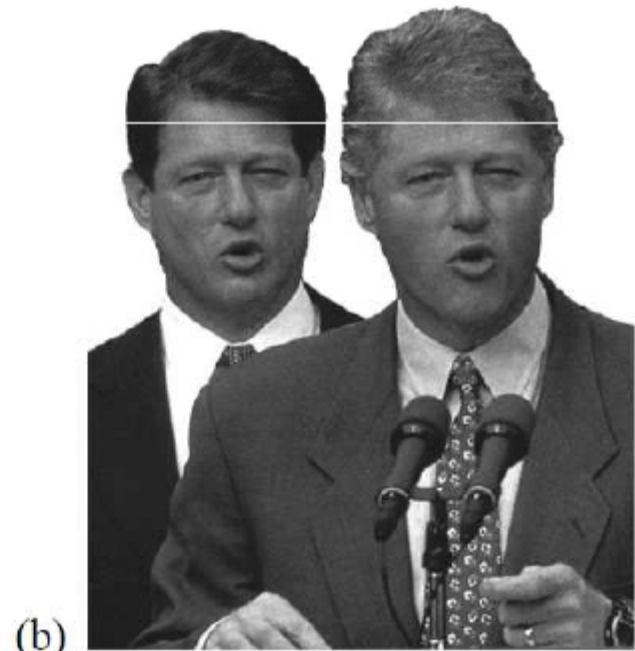
- ▶ Eyebrows are among the most important for recognition



- ▶ Both internal and external facial cues are important and they exhibit non-linear interactions



(a)



(b)

- ▶ The important configural relations appear to be independent across the width and height dimensions



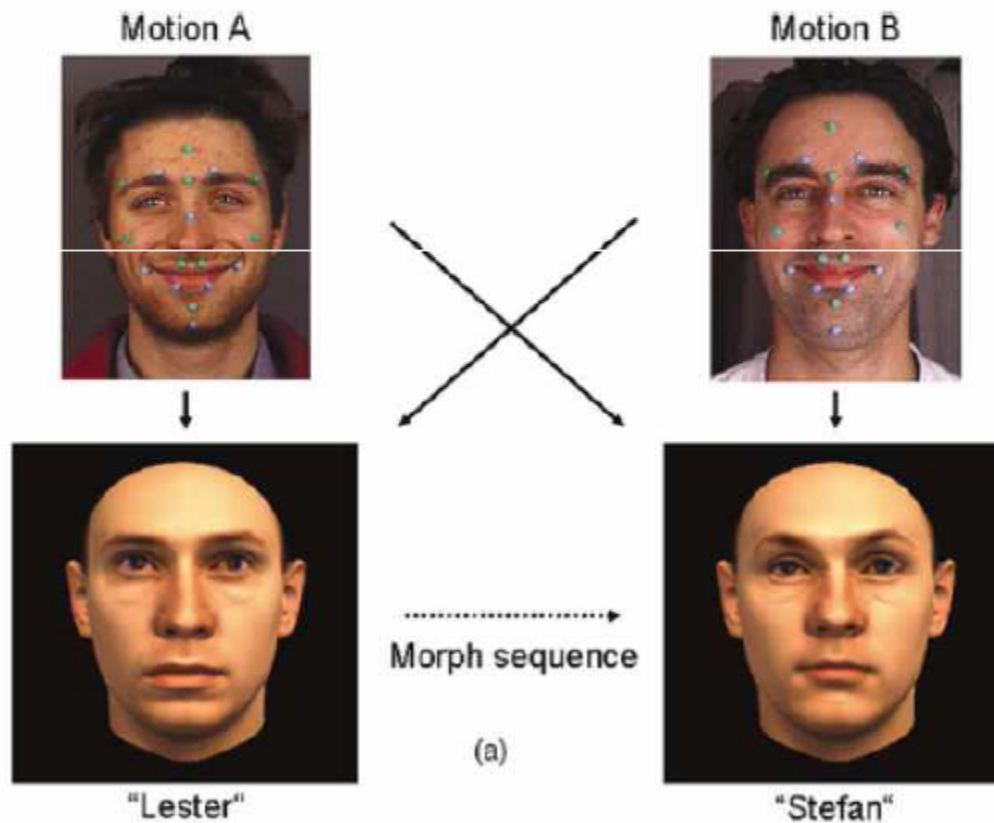
- ▶ Vertical inversion dramatically reduces recognition performance



- ▶ Contrast polarity inversion dramatically impairs recognition performance, possibly due to compromised ability to use pigmentation cues

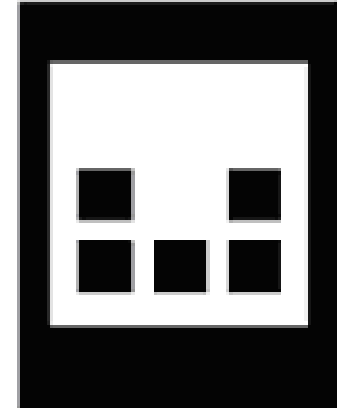
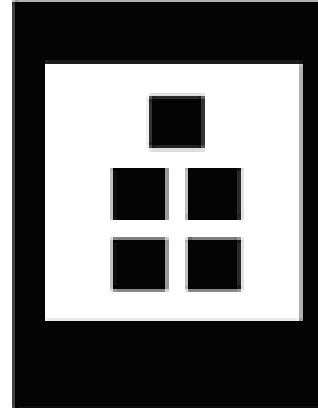
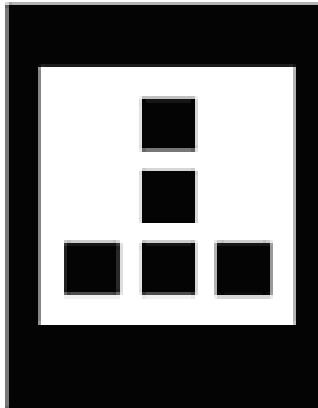
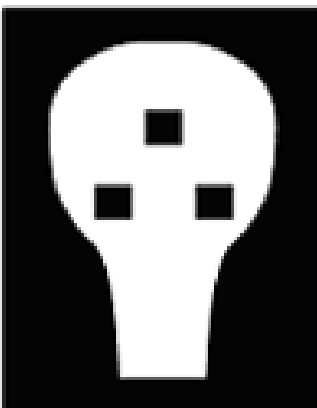
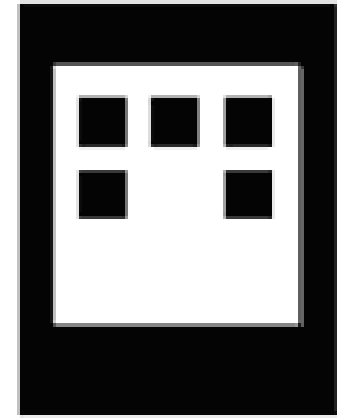
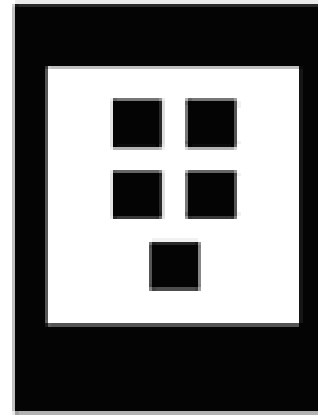
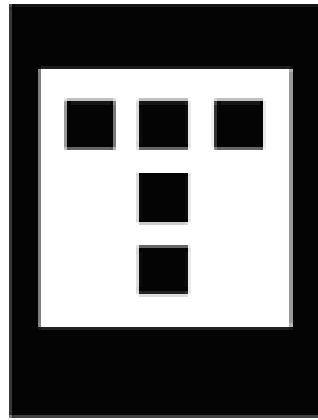
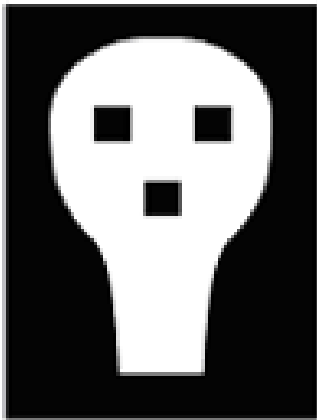


- ▶ Motion of faces appears to facilitate subsequent recognition





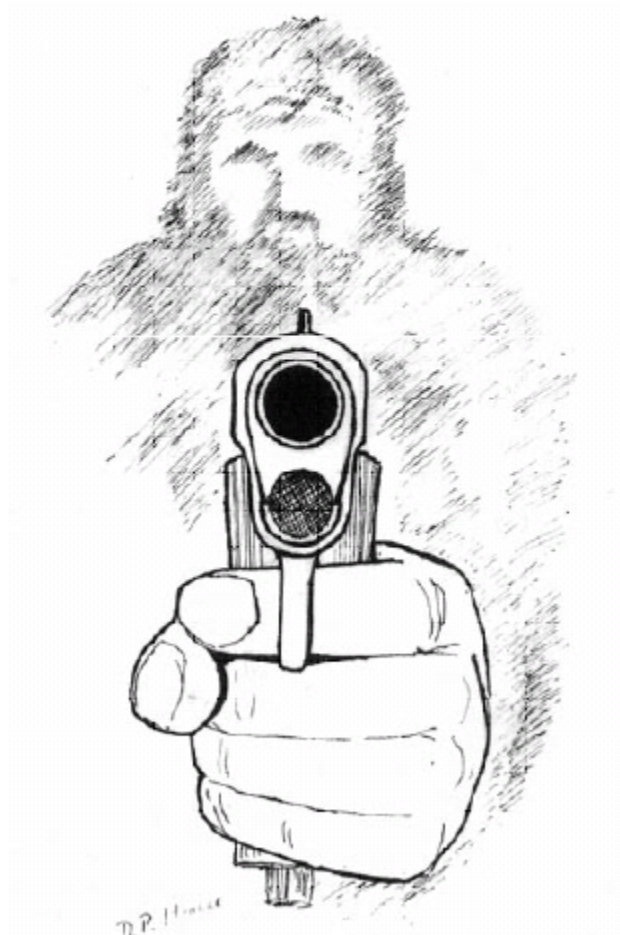
The visual system starts with a rudimentary preference for face- like patterns



# Result 17: Vision progresses from piecemeal to holistic

Age	Correct responses (%)			
	Faces		Houses	
	Upright	Inverted	Upright	Inverted
6	69	64	71	58*†
8	81	67	74	64
10	89	68‡	73	77

- ▶ Human memory for briefly seen faces is rather poor



# Things to remember

- PCA is a generally useful dimensionality reduction technique
  - But not ideal for discrimination
- FLD better for discrimination, though only ideal under Gaussian data assumptions
- Computer face recognition works very well under controlled environments – still room for improvement in general conditions