Structure From Motion

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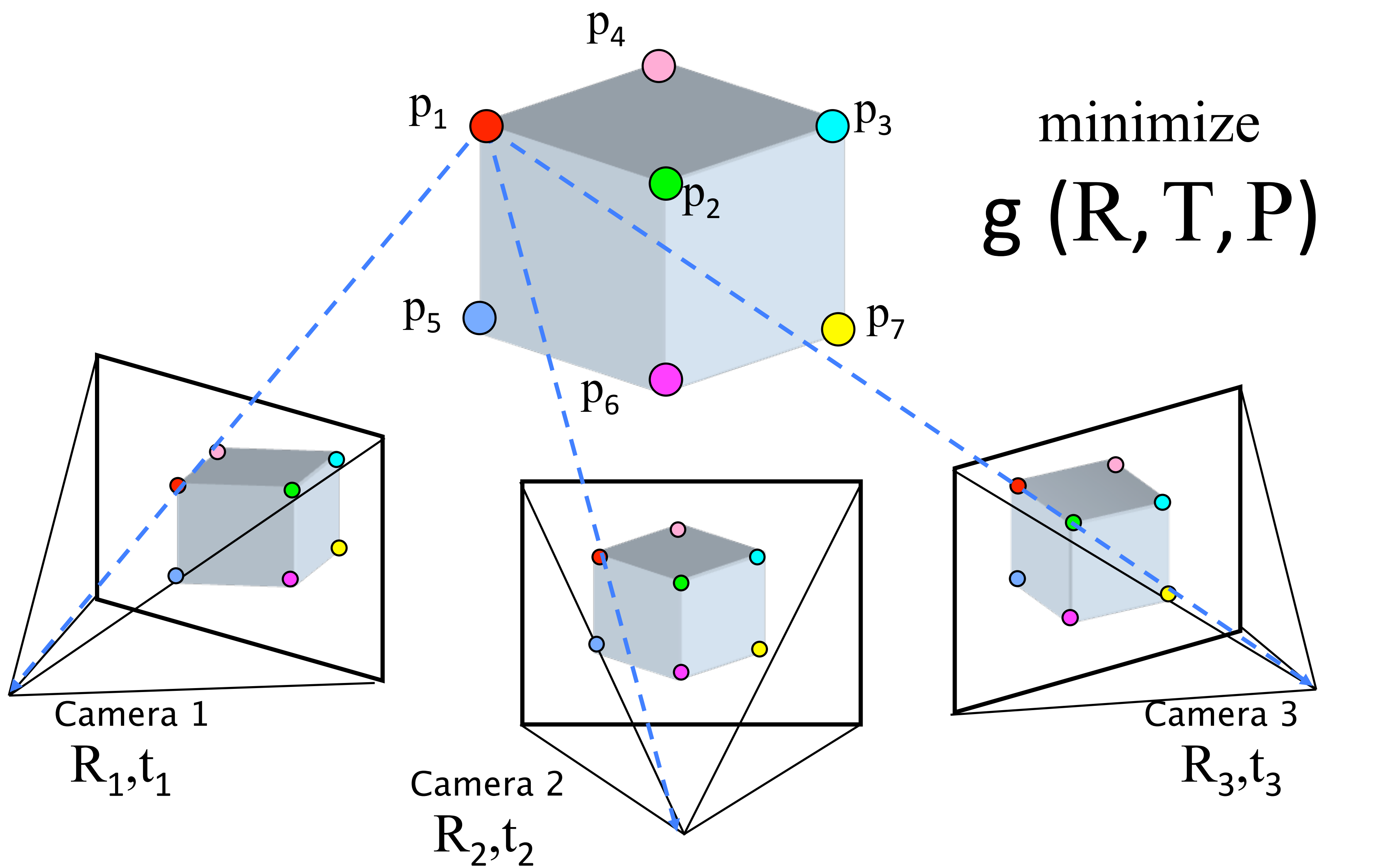
CSE 576

Several slides from Steve Seitz, Rick Szeliski, Martial Hebert, and Noha Snavely

Structure from motion

- aka "bundle adjustment" (texts: [Zisserman](); [Faugeras]())



SfM objective function

- Given point **x** and rotation and translation **R**, **t**

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \mathbf{R}\mathbf{x} + \mathbf{t} \qquad u' = \frac{fx'}{z'} \quad v' = \frac{fy'}{z'} \qquad \begin{bmatrix} u' \\ v' \end{bmatrix} = \mathbf{P}(\mathbf{x}, \mathbf{R}, \mathbf{t})$$

- Minimize sum of squared reprojection errors:

$$g(\mathbf{X}, \mathbf{R}, \mathbf{T}) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_{ij} \cdot \left\| \underbrace{\mathbf{P}(\mathbf{x}_i, \mathbf{R}_j, \mathbf{t}_j)}_{\text{predicted image location}} - \underbrace{\begin{bmatrix} u_{i,j} \\ v_{i,j} \end{bmatrix}}_{\text{observed image location}} \right\|^2$$

Solving structure from motion

- Minimizing g is difficult:
  - g is non-linear due to rotations, perspective division
  - lots of parameters: 3 for each 3D point, 6 for each camera
  - difficult to initialize
  - gauge ambiguity: error is invariant to a similarity transform (translation, rotation, uniform scale)
- Many techniques use non-linear least-squares optimization (bundle adjustment)
  - Levenberg-Marquardt is a popular algorithm
  - http://en.wikipedia.org/wiki/Levenberg-Marquardt_algorithm
- Good code online
  - Bundler: http://phototour.cs.washington.edu/bundler/
  - Multicore: http://grail.cs.washington.edu/projects/mcba/

Suppose we know 3D points and affine camera parameters …

then, we can compute the observed 2d positions of each point

$$\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_m \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \cdots & \mathbf{X}_n \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{x}}_{11} & \hat{\mathbf{x}}_{12} & \cdots & \hat{\mathbf{x}}_{1n} \\ \hat{\mathbf{x}}_{21} & \hat{\mathbf{x}}_{22} & \cdots & \hat{\mathbf{x}}_{2n} \\ & & \ddots & \\ \hat{\mathbf{x}}_{m1} & \hat{\mathbf{x}}_{m2} & \cdots & \hat{\mathbf{x}}_{mn} \end{bmatrix}$$

Camera Parameters (2mx3)

3D Points (3xn)

2D Image Points (2mxn)

What if we instead observe corresponding 2d image points?

Can we recover the camera parameters and 3d points?

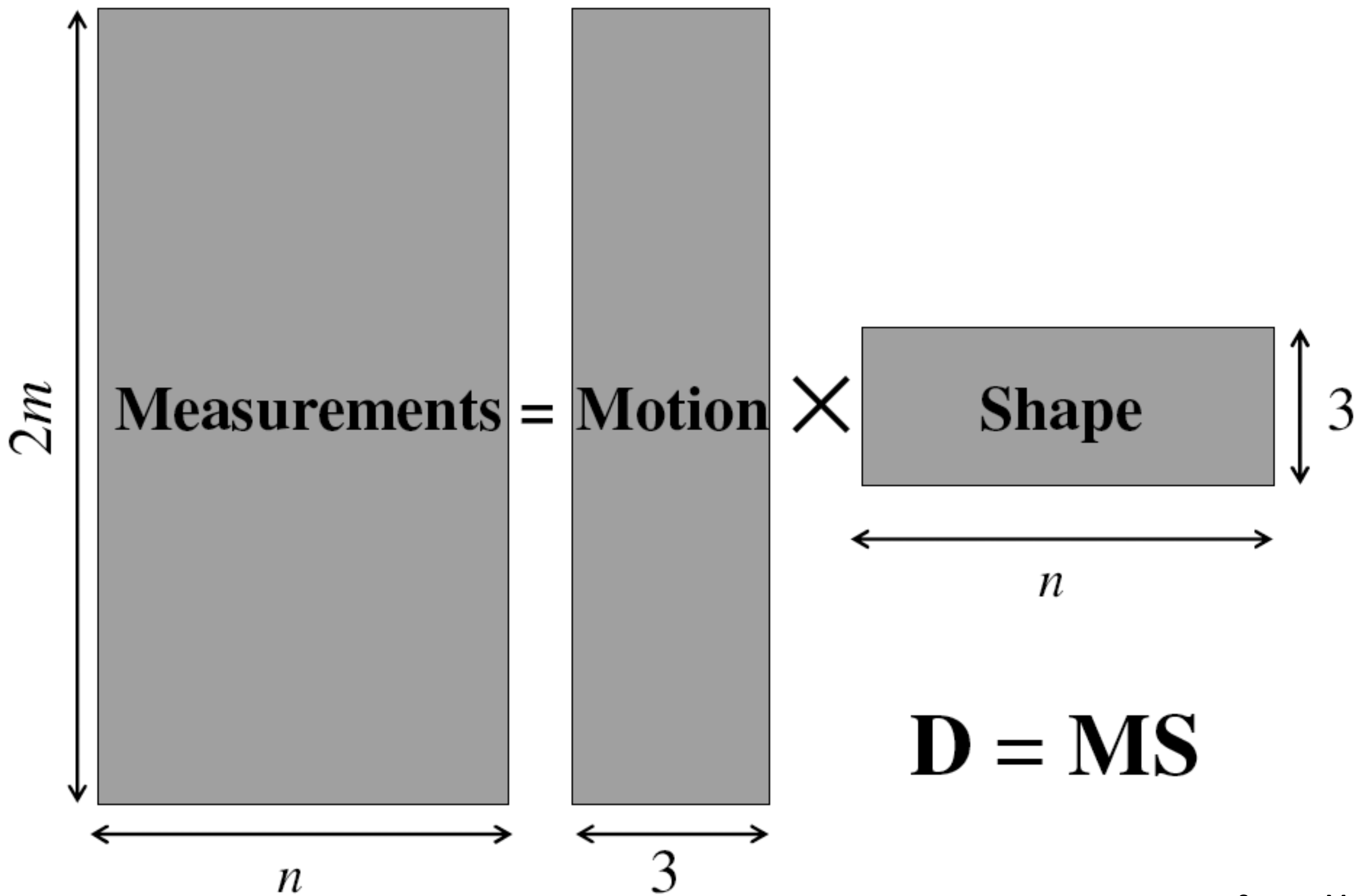
cameras (2 *m*)

$$\mathbf{D} = \begin{bmatrix} \hat{\mathbf{x}}_{11} & \hat{\mathbf{x}}_{12} & \cdots & \hat{\mathbf{x}}_{1n} \\ \hat{\mathbf{x}}_{21} & \hat{\mathbf{x}}_{22} & \cdots & \hat{\mathbf{x}}_{2n} \\ & & \ddots & \\ \hat{\mathbf{x}}_{m1} & \hat{\mathbf{x}}_{m2} & \cdots & \hat{\mathbf{x}}_{mn} \end{bmatrix} \overset{?}{\Rightarrow} \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_m \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \cdots & \mathbf{X}_n \end{bmatrix}$$

points (*n*)

What rank is the matrix of 2D points?

Factorizing the measurement matrix

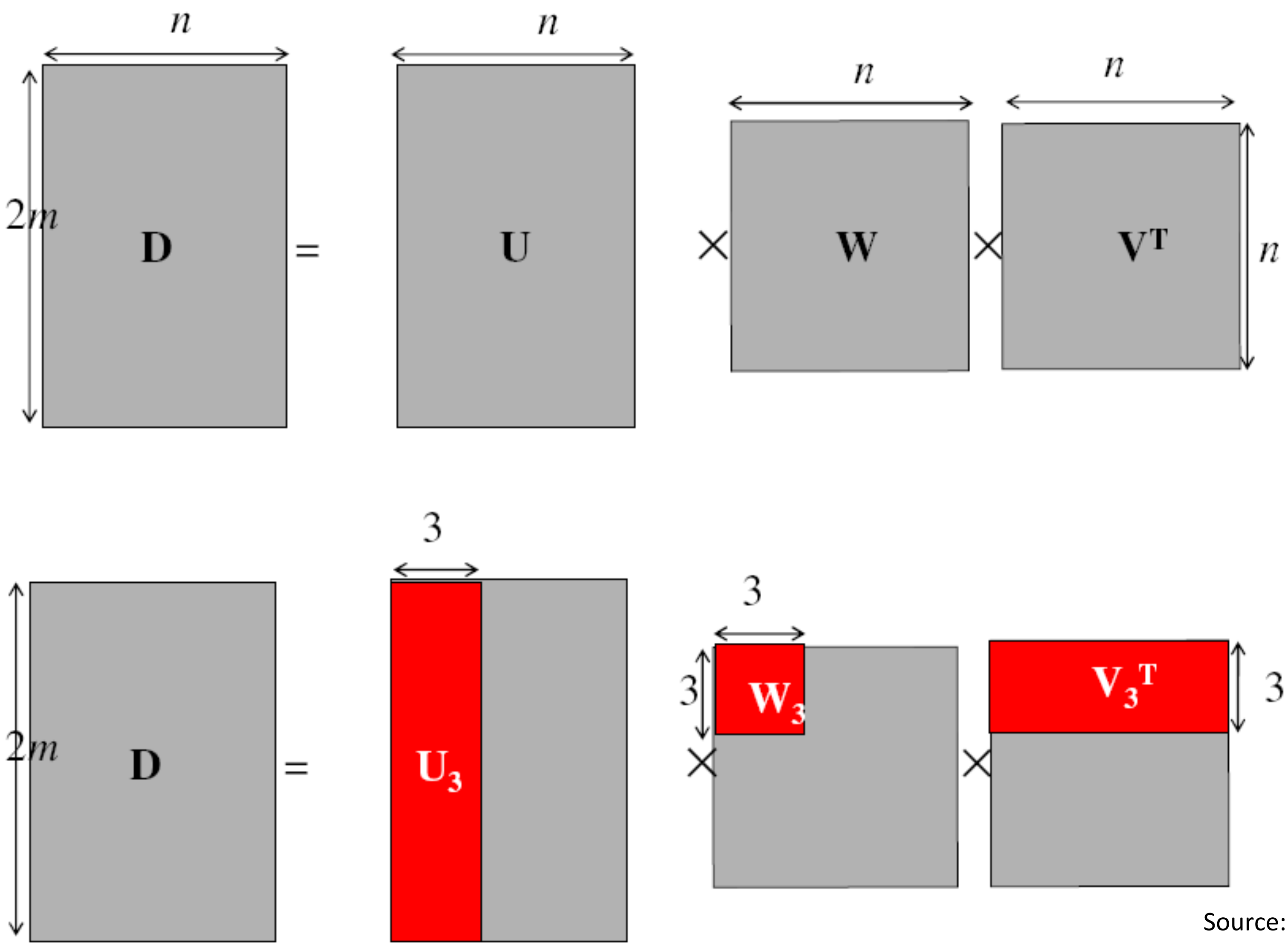


D = MS

Source: M. Hebert

Factorizing the measurement matrix

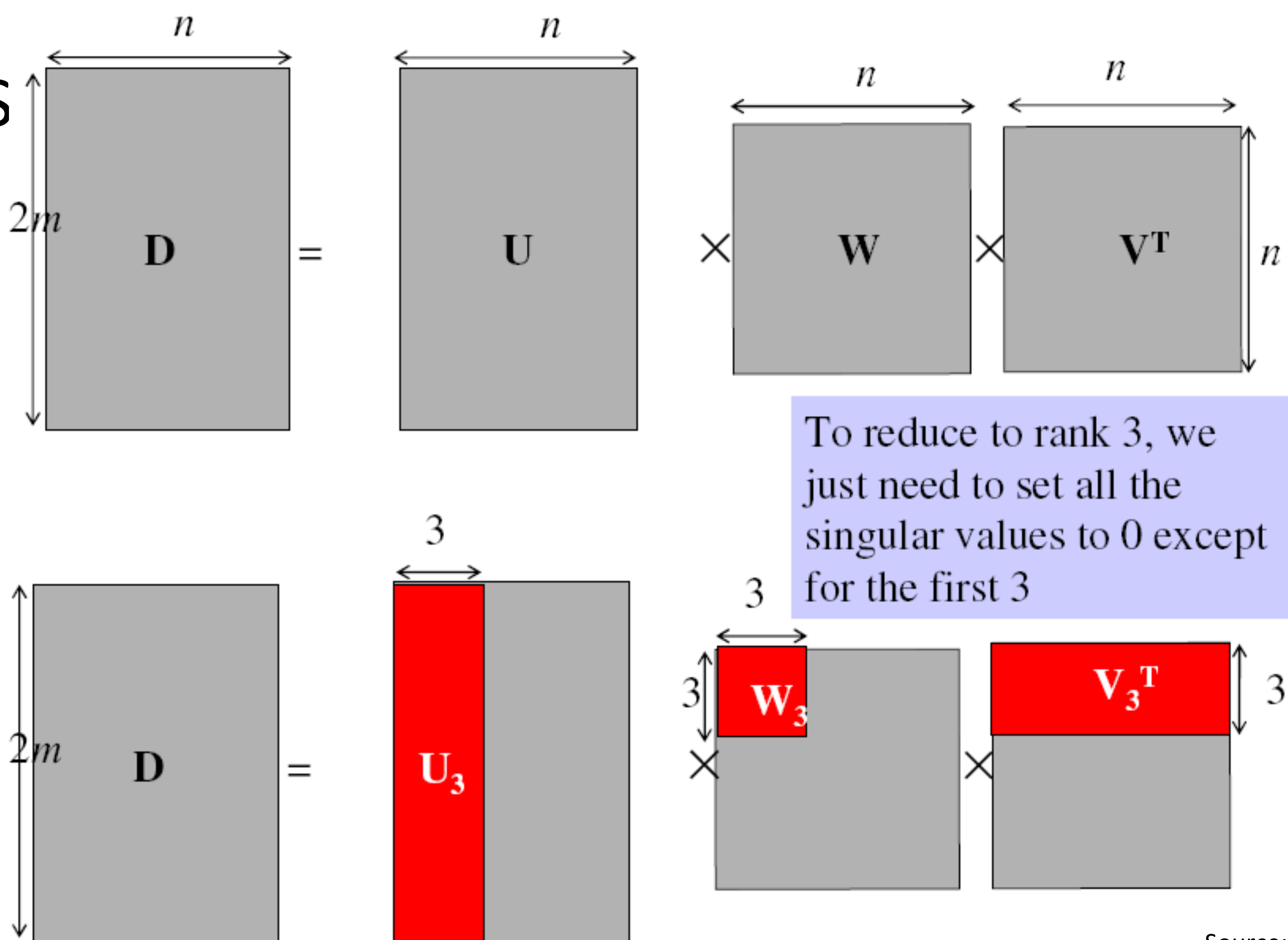
- Singular value decomposition of D:



Source: M. Hebert

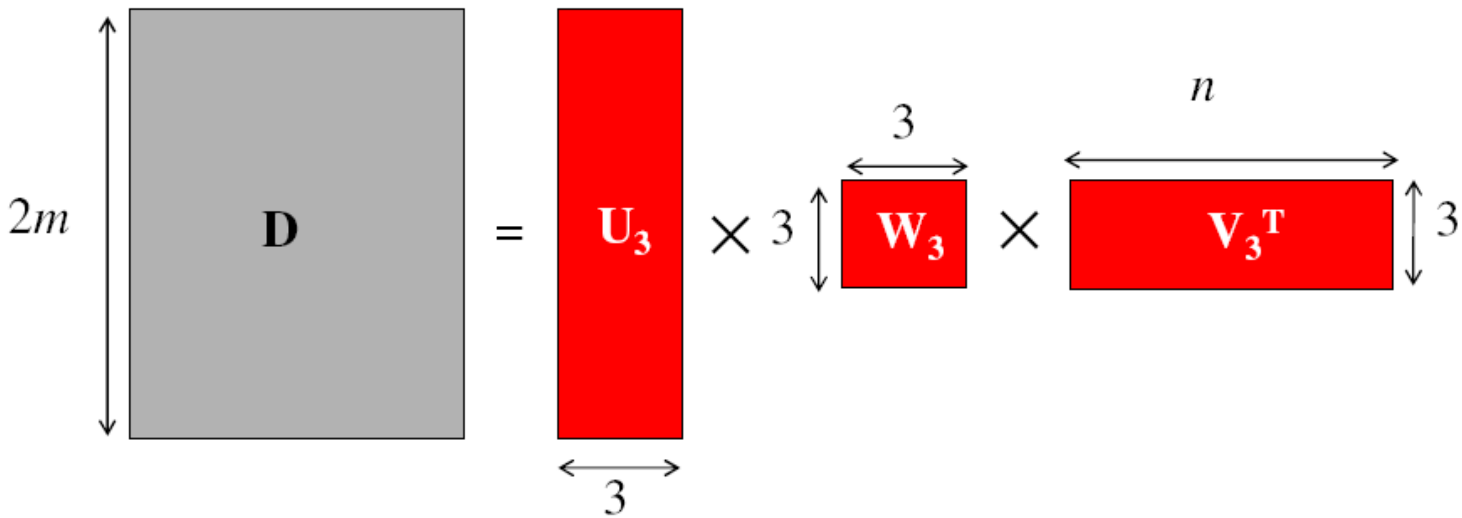
Factorizing the measurement matrix

- S



Source: M. Hebert

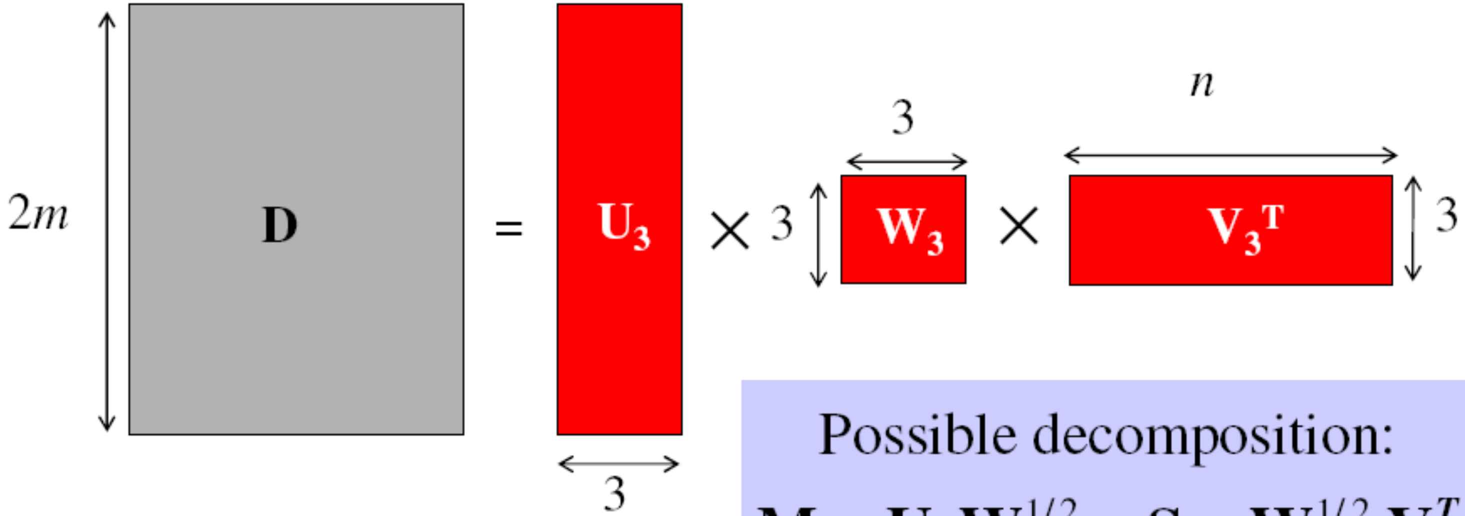
Factorizing the measurement matrix



Source: M. Hebert

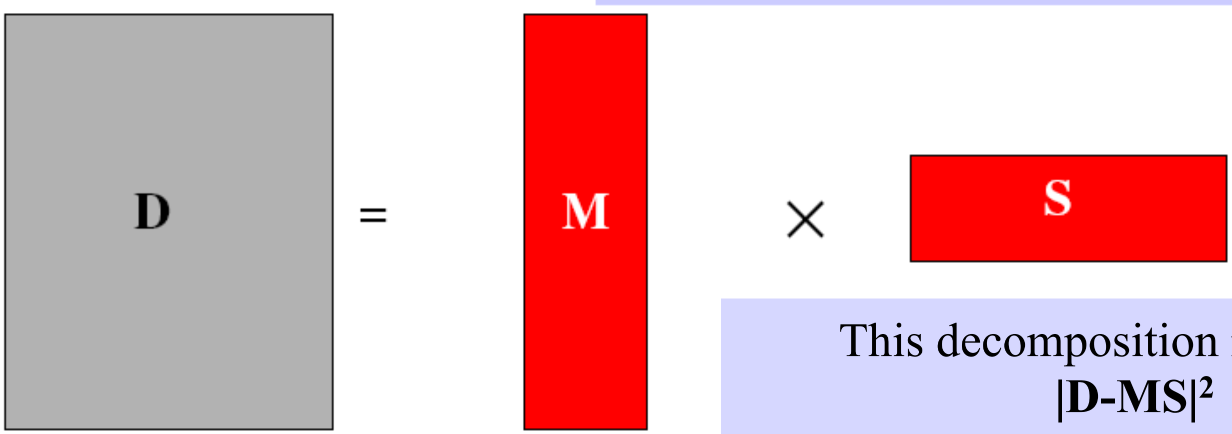
Factorizing the measurement matrix

- Obtaining a factorization from SVD:



Possible decomposition:

$$\mathbf{M} = \mathbf{U}_3 \mathbf{W}_3^{1/2} \quad \mathbf{S} = \mathbf{W}_3^{1/2}\, \mathbf{V}_3^T$$



This decomposition minimizes
$|\mathbf{D}\text{-}\mathbf{MS}|^2$

Source: M. Hebert

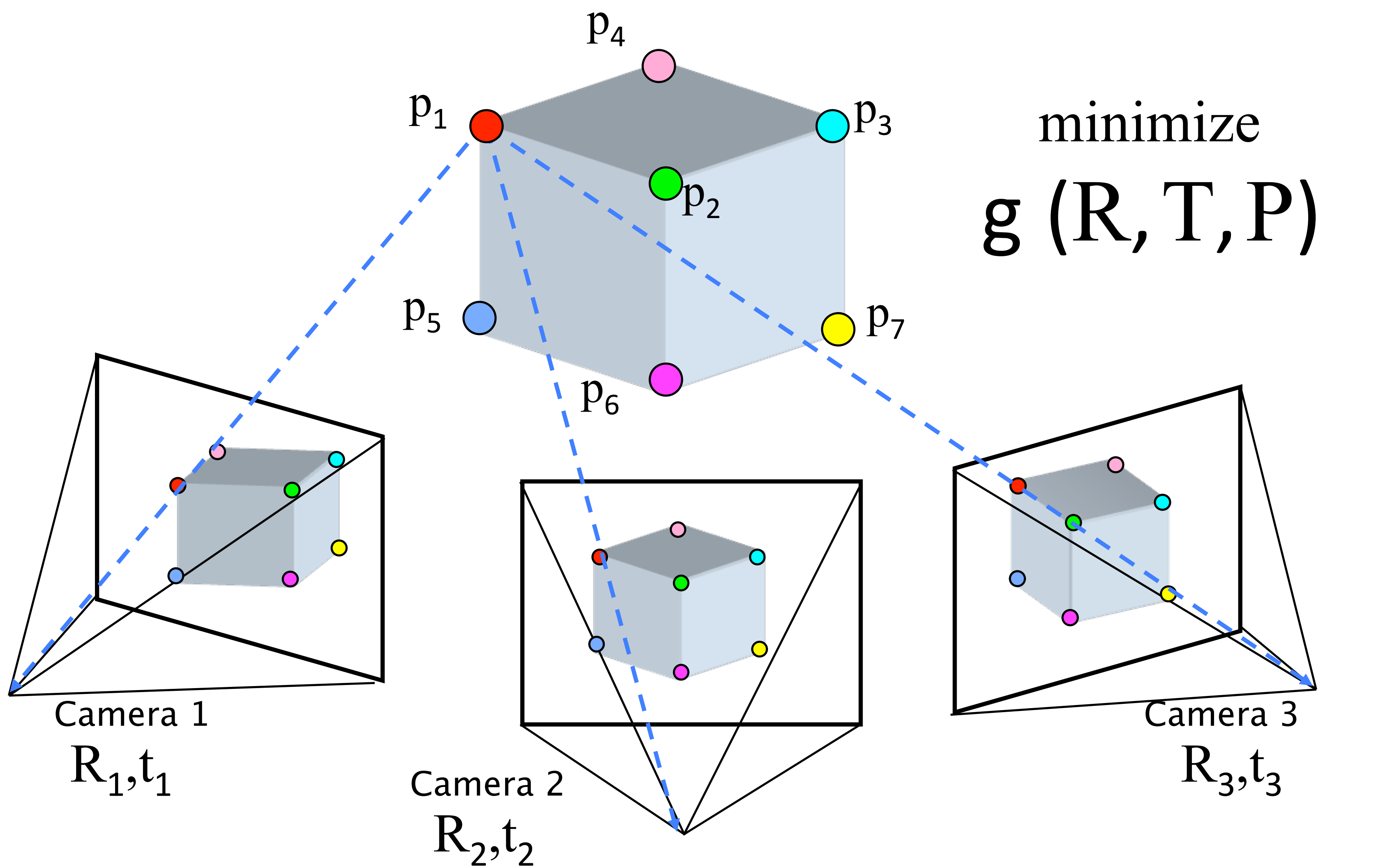
Algorithm summary

- Given: *m* images and *n* features $\mathbf{x}_{ij}$
- For each image *i, c*enter the feature coordinates
- Construct a 2*m* × *n* measurement matrix **D**:
  - Column *j* contains the projection of point *j* in all views
  - Row *i* contains one coordinate of the projections of all the *n* points in image *i*
- Factorize **D**:
  - Compute SVD: $\mathbf{D} = \mathbf{U}\ \mathbf{W}\ \mathbf{V}^T$
  - Create $\mathbf{U}_3$ by taking the first 3 columns of **U**
  - Create $\mathbf{V}_3$ by taking the first 3 columns of **V**
  - Create $\mathbf{W}_3$ by taking the upper left 3 × 3 block of **W**
- Create the motion and shape matrices:
  - $\mathbf{M} = \mathbf{U}_3\mathbf{W}_3^{½}$ and $\mathbf{S} = \mathbf{W}_3^{½}\ \mathbf{V}_3^T$ (**or** $\mathbf{M} = \mathbf{U}_3$ and $\mathbf{S} = \mathbf{W}_3\mathbf{V}_3^T$)

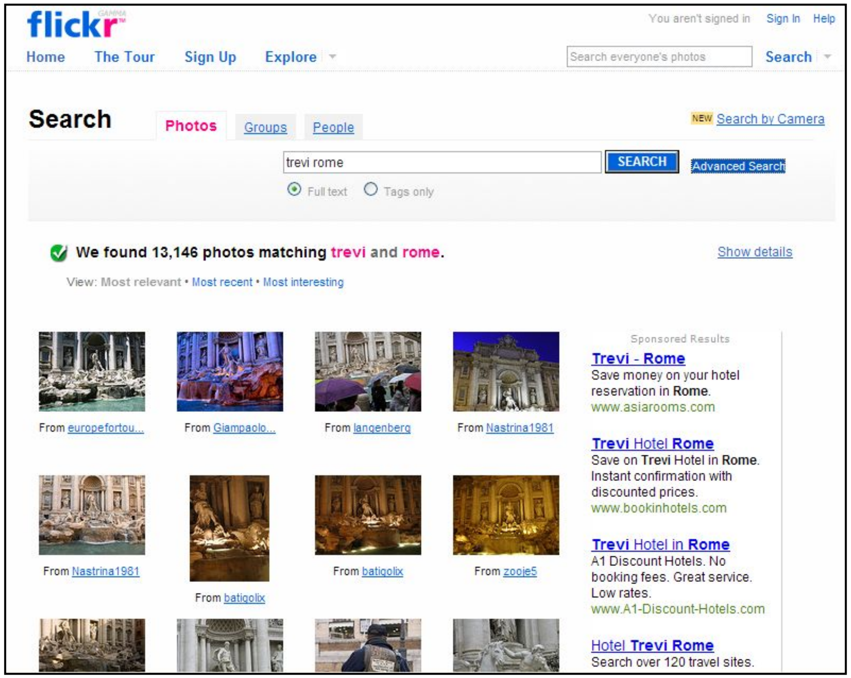
Source: M. Hebert

Structure from motion

- aka "bundle adjustment" (texts: Zisserman; Faugeras)



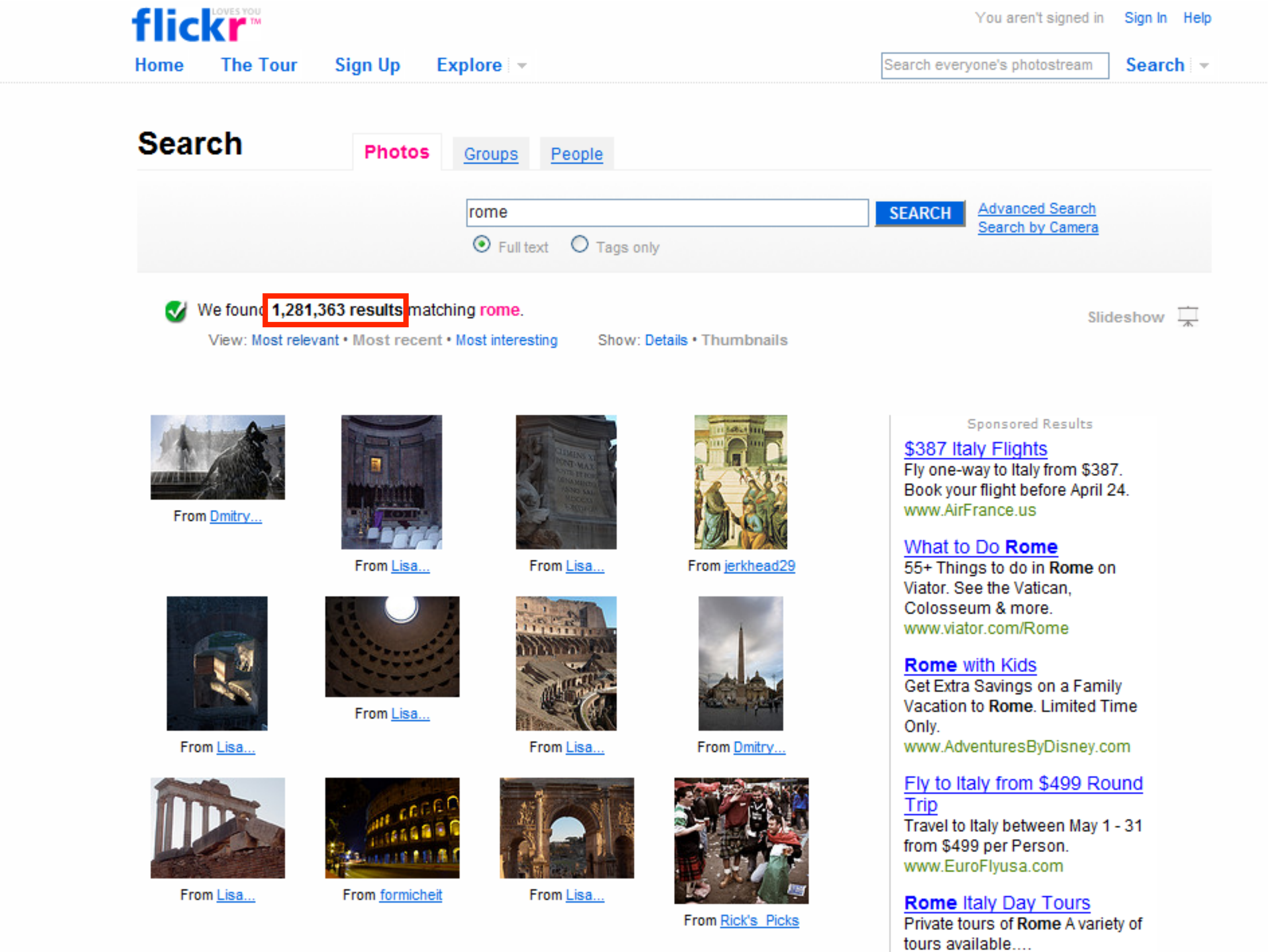
Structure from motion



Images on the Internet

Computed 3D structure

- Photo tourism video: http://www.youtube.com/watch?v=5Ji84zb2r8s
- Photosynth: http://photosynth.net/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View: Most relevant • Most recent • Most interesting
Show: Details • Thumbnails
Slideshow
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From Lisa...
From Lisa...
From jerkhead29
From Lisa...
From Lisa...
From Lisa...
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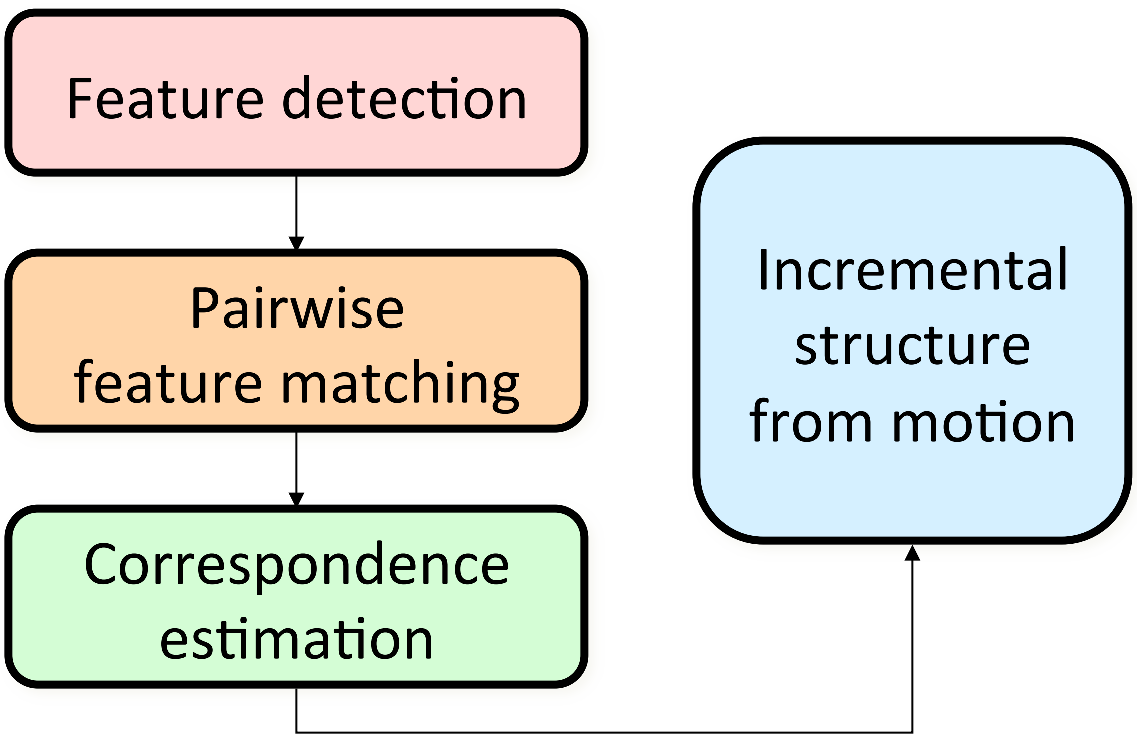
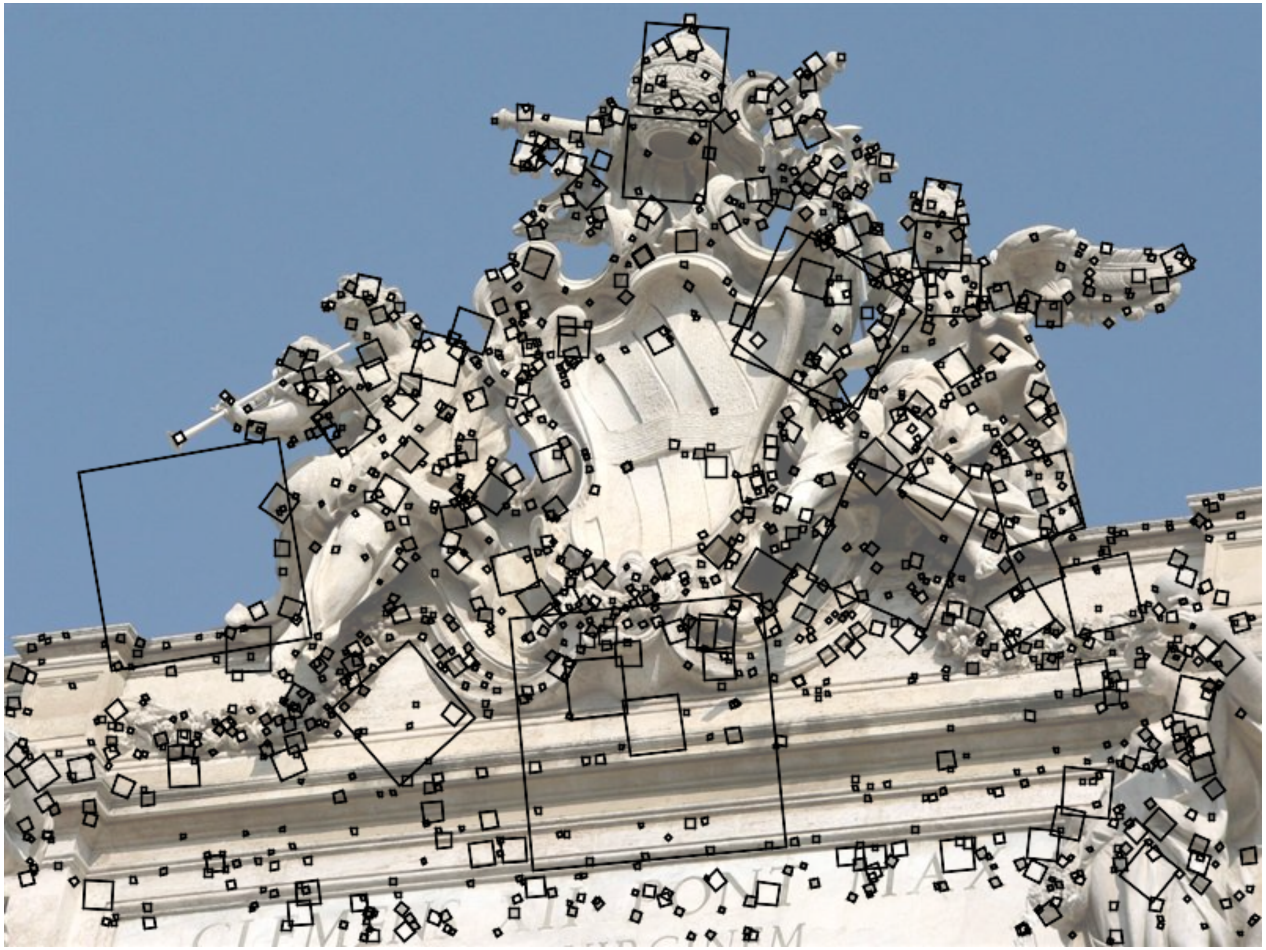
Scene reconstruction

- Automatically estimate
  - position, orientation, and focal length of cameras
  - 3D positions of feature points



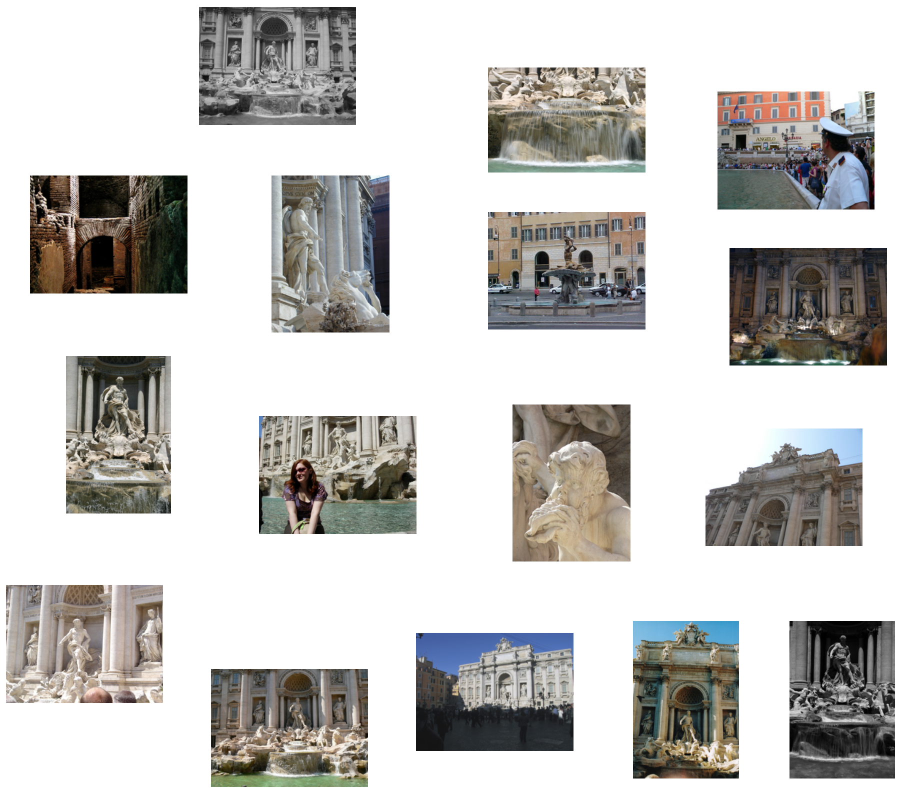
Feature detection

Detect features using SIFT [Lowe, IJCV 2004]



Feature detection

Detect features using SIFT [Lowe, IJCV 2004]



Feature detection

Detect features using SIFT [Lowe, IJCV 2004]

Correspondence estimation

- Link up pairwise matches to form connected components of matches across several images

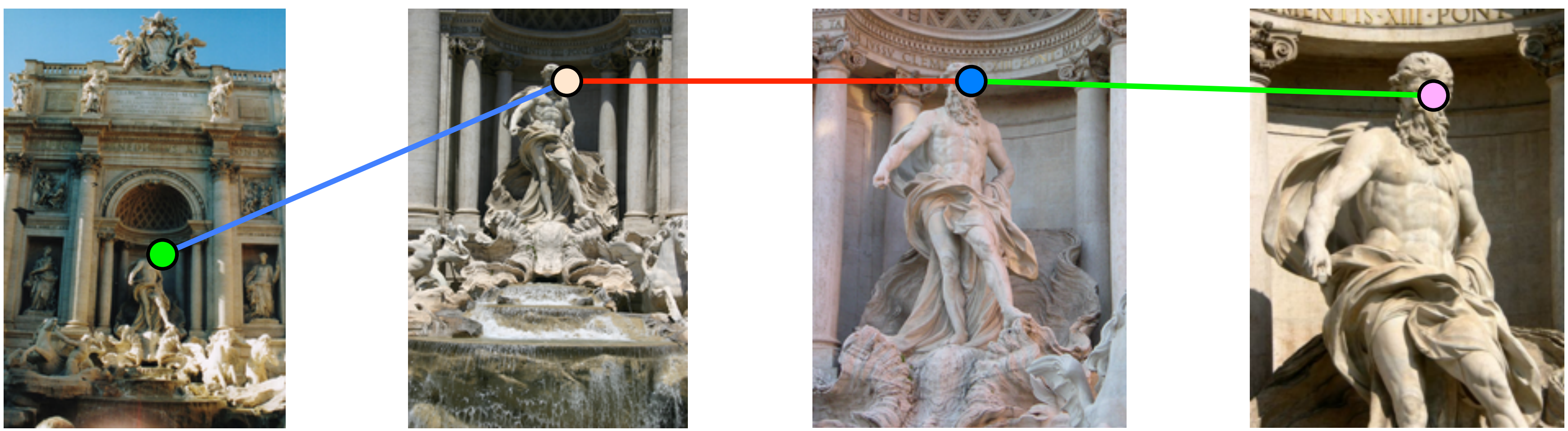
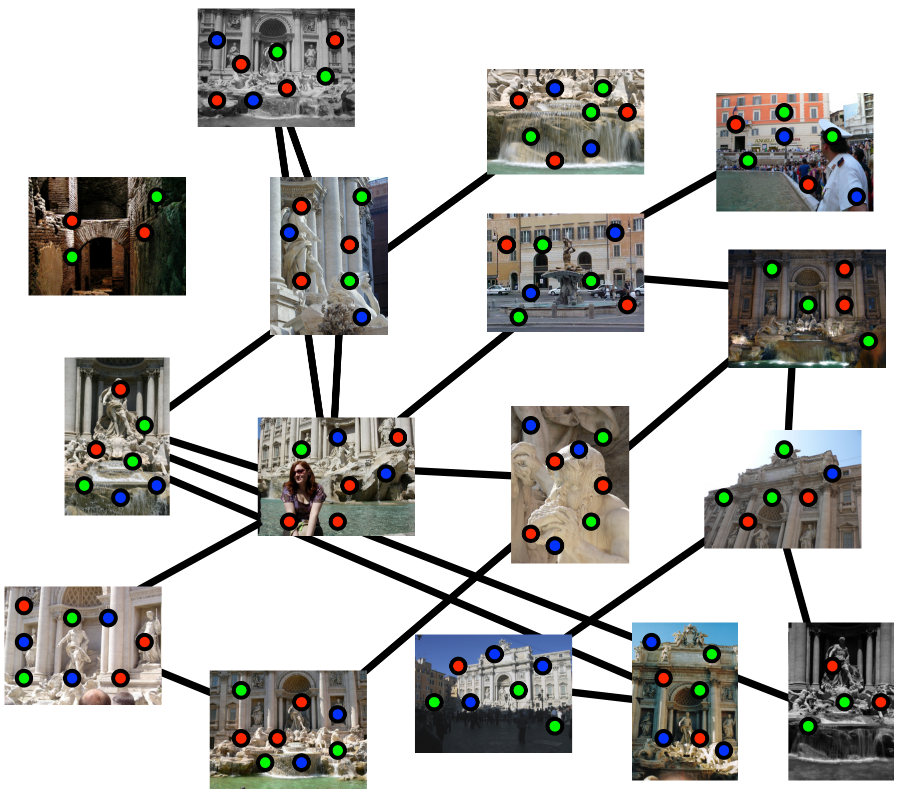


Image 1 Image 2 Image 3 Image 4

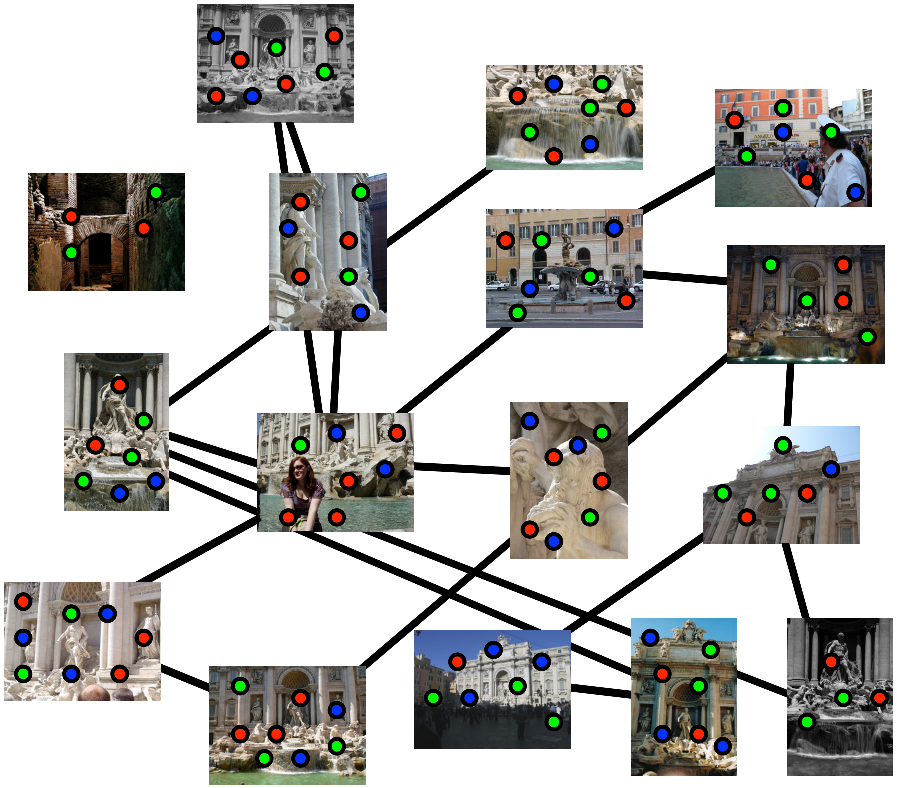
Feature matching

Match features between each pair of images

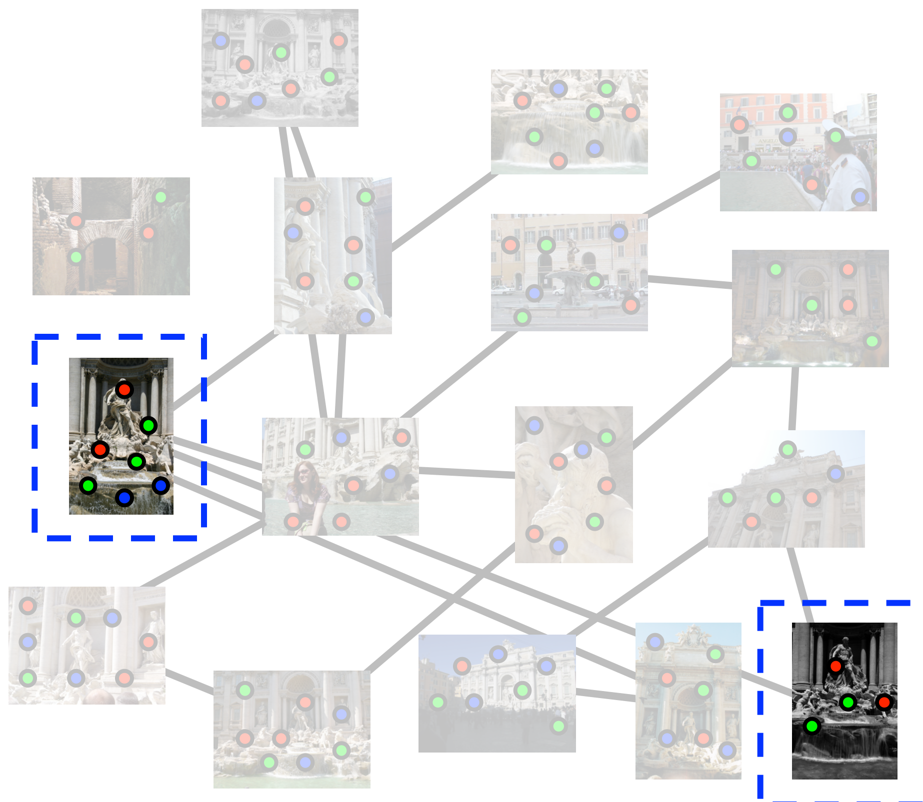


Feature matching

Refine matching using RANSAC [Fischler & Bolles 1987]

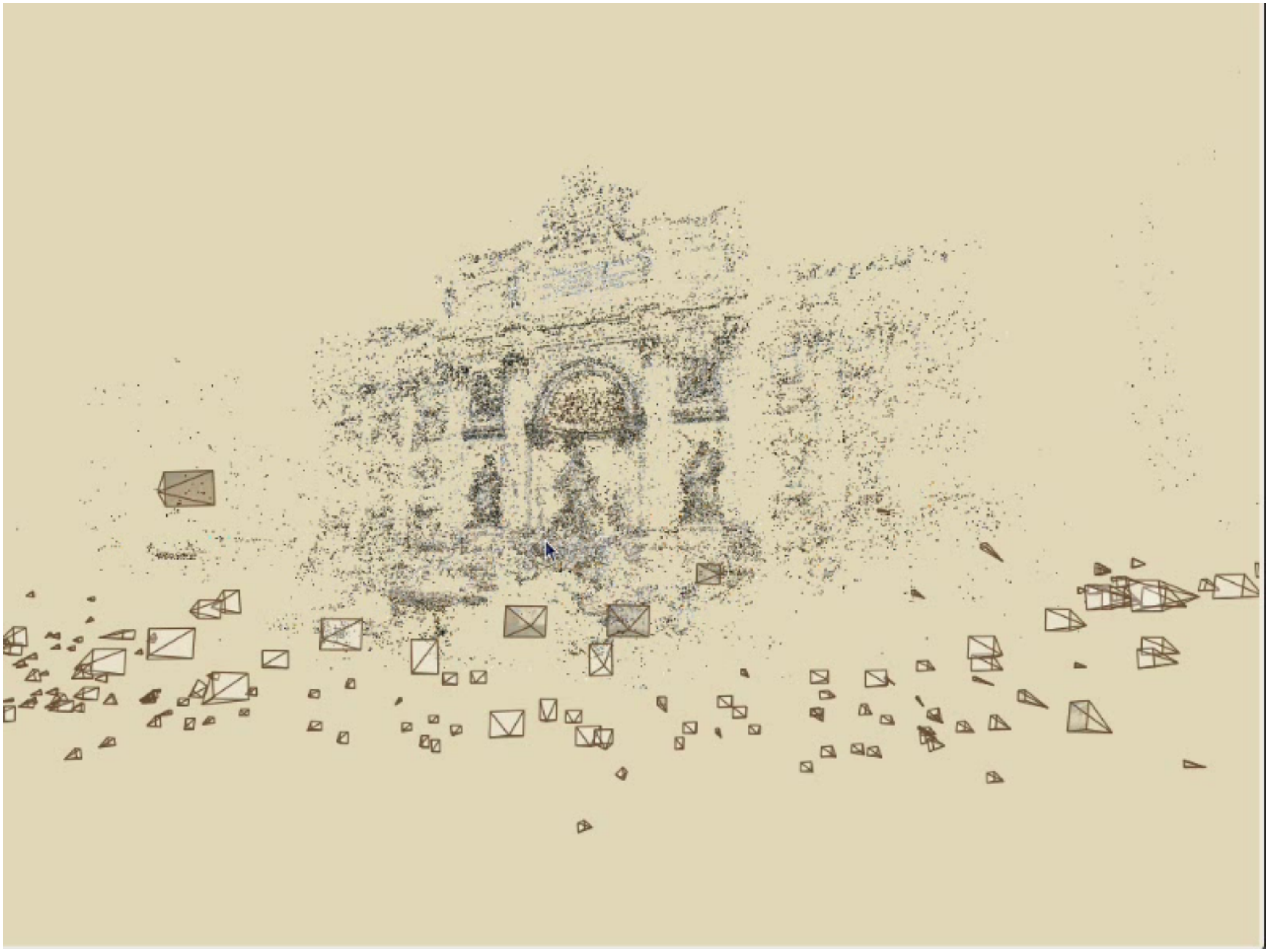


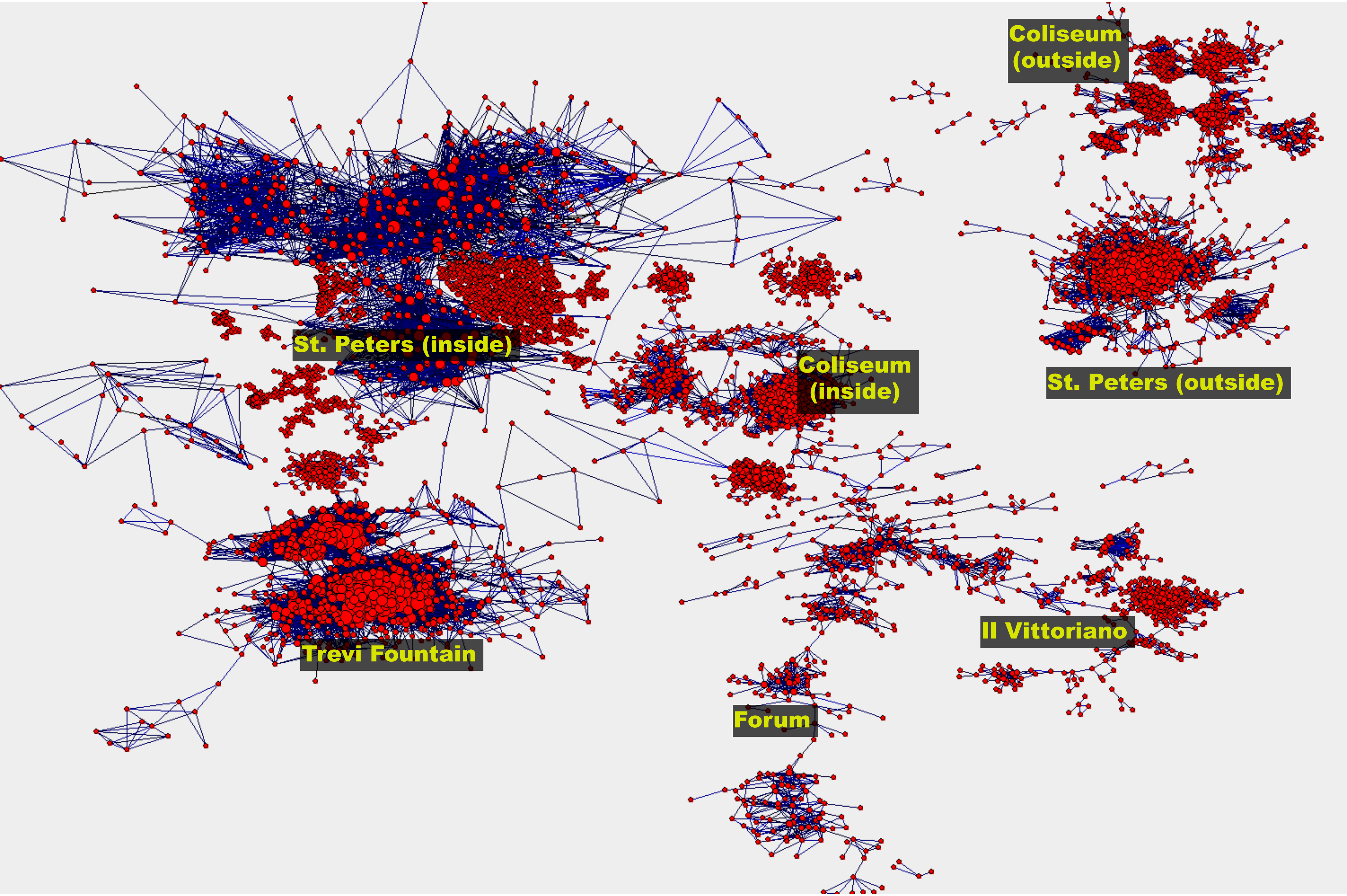
Incremental structure from motion



Incremental structure from motion

Photo Explorer



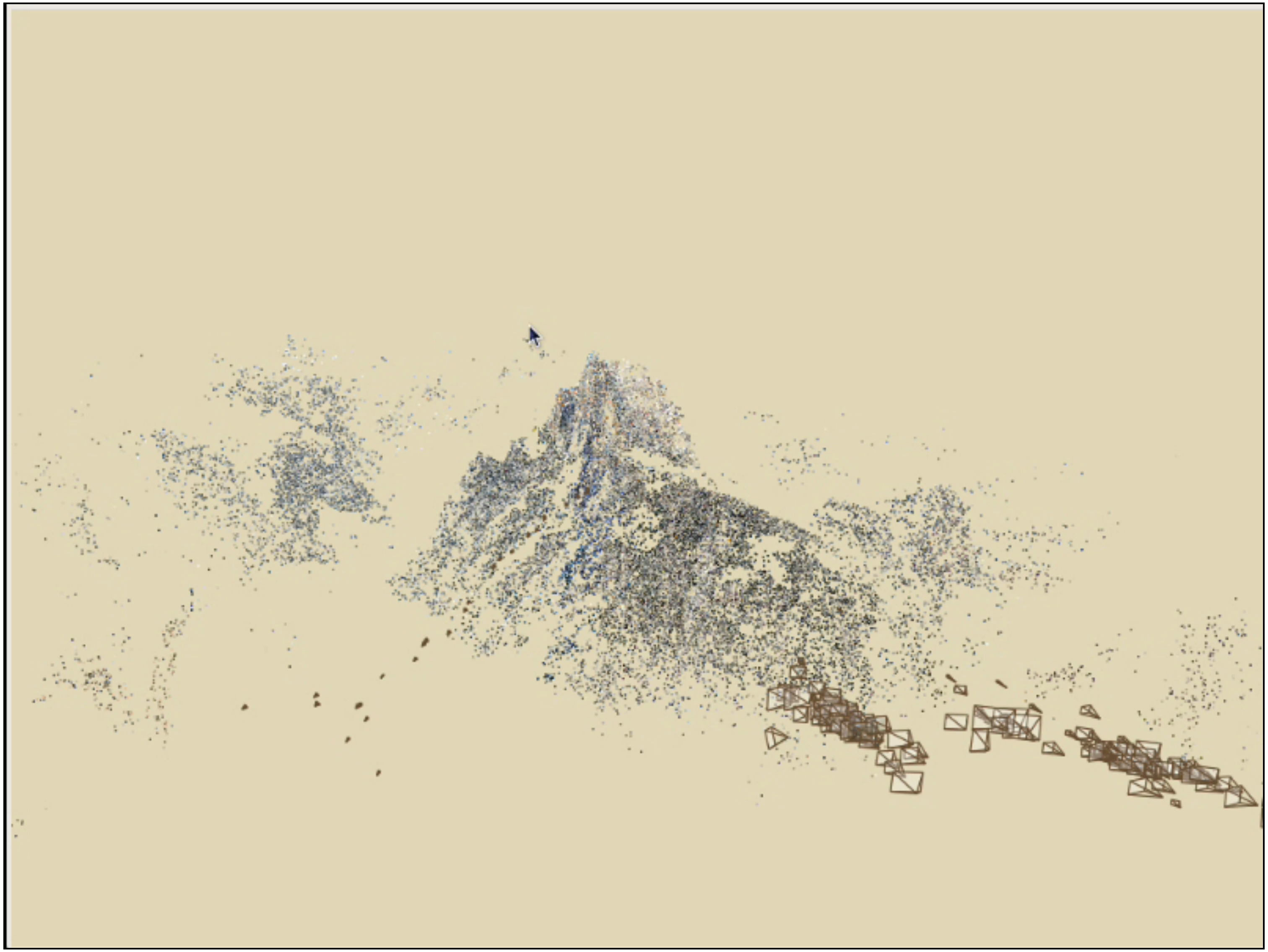


Coliseum (outside)
St. Peters (inside)
Coliseum (inside)
St. Peters (outside)
Il Vittoriano
Trevi Fountain
Forum

Navigation: Prague Old Town Square

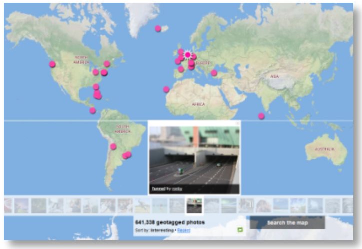
Hierarchical annotations

Locking the camera (stabilization)



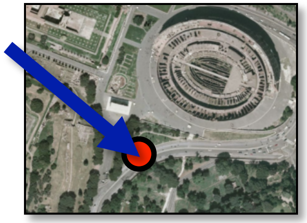
Applications

Community photo collections



- "Wikipedia for photos" – visual record of world through community of photographers
  - *Geograph British Isles* http://www.geograph.org.uk/

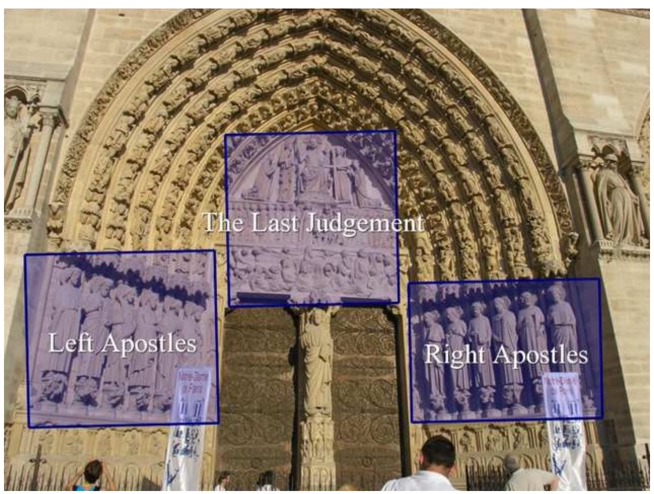
- Users can tag and comment on photos, link to other content
  - *World-wide telescope*

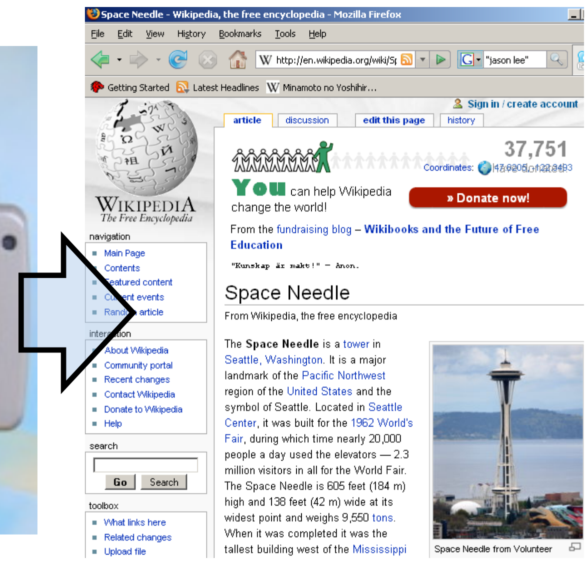


- "Where should I take a photo?" http://photocitygame.com/

Community photo collections

- Leveraging large databases of photos, large number of users
  - Annotations / augmented reality



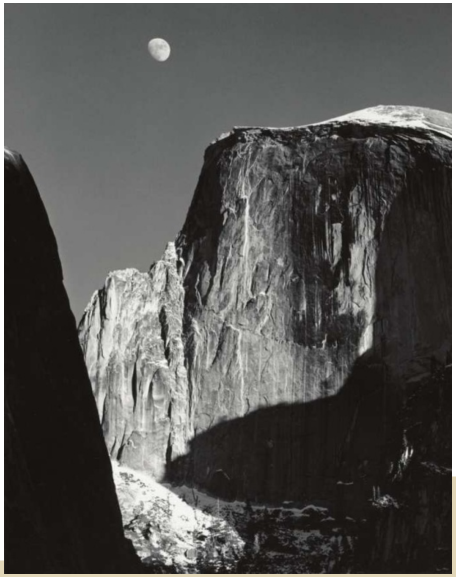


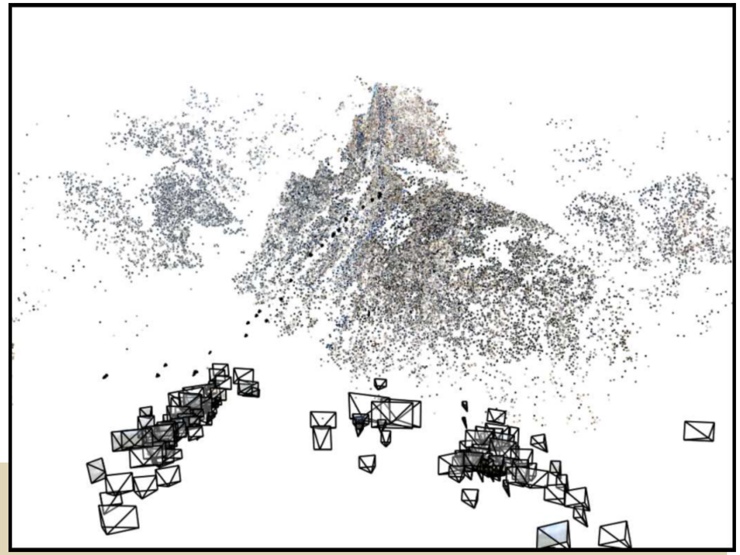
Virtual tour guide scenario

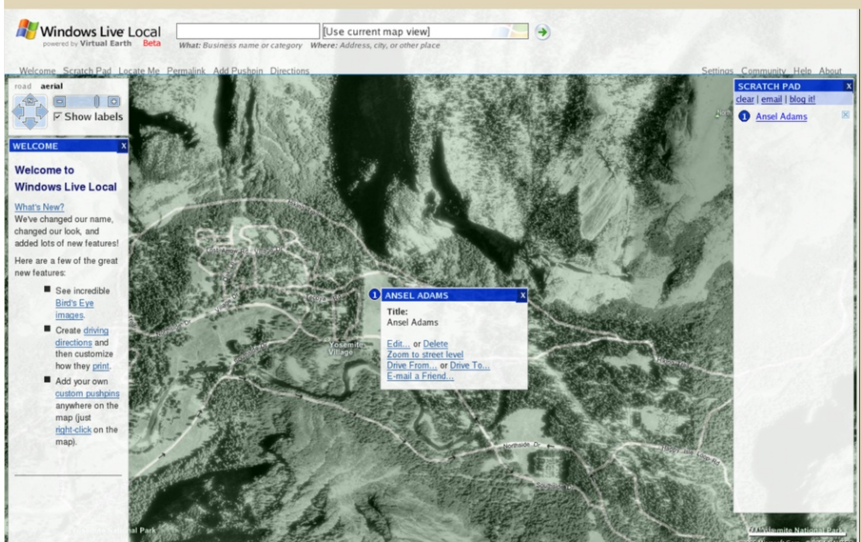
St. Peter's Basilica

- Built: 1506-1626
- http://en.wikipedia.org/wiki/St._Peter's_Basilica

Rephotography







Topographic data courtesy USGS