Geometric Transformations

CSE 576 Ali Farhadi

Many slides from Steve Seitz and Larry Zitnick

What are geometric transformations?









Translation



 $\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$

Preserves: Orientation

Translation and rotation



$$\begin{bmatrix} \cos(\theta) - \sin(\theta) \ t_x \\ \sin(\theta) \ \cos(\theta) \ t_y \\ 0 \ 0 \ 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Scale





$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Similarity transformations



Similarity transform (4 DoF) = translation + rotation + scale

Preserves: Angles

Aspect ratio



$$\begin{bmatrix} a & 0 & 0 \\ 0 & \frac{1}{a} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Shear



$$\begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Affine transformations



Affine transform (6 DoF) = translation + rotation + scale + aspect ratio + shear

Preserves: Parallelism

What is missing?





Canaletto

Are there any other planar transformations?

General affine

We already used these $\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$

How do we compute projective transformations?

Homogeneous coordinates

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

One extra step:

$$x' = u/w$$
$$y' = v/w$$

Projective transformations

a.k.a. Homographies

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \qquad x' = u/w \\ y' = v/w$$

"keystone" distortions







Preserves: Straight Lines

Finding the transformation

Translation 2 degrees of = freedom 4 degrees of Similarity freedom Affine 6 degrees of = freedom Homograph for responding paints for we need to solve?

Finding the transformation



- How can we find the transformation between these images?
- How many corresponding points do we need to solve?