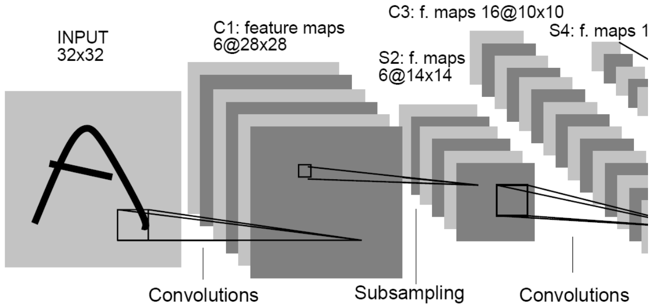
Convolutional Neural Networks

Computer Vision (UW EE/CSE 576)

Richard Szeliski

Facebook & UW

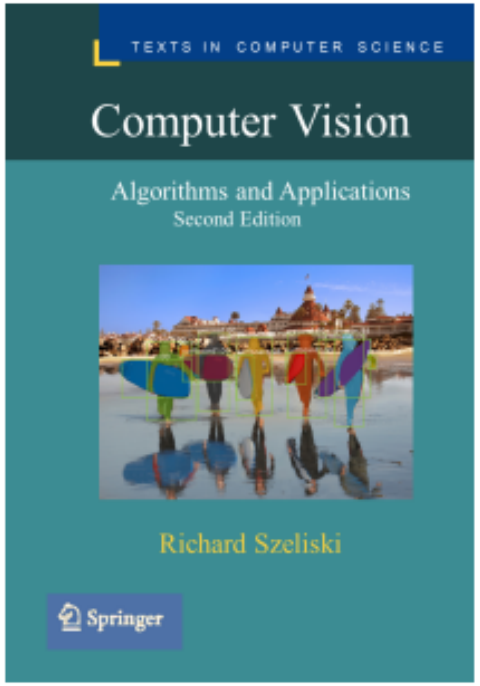
Lecture 8 – Apr 23, 2020

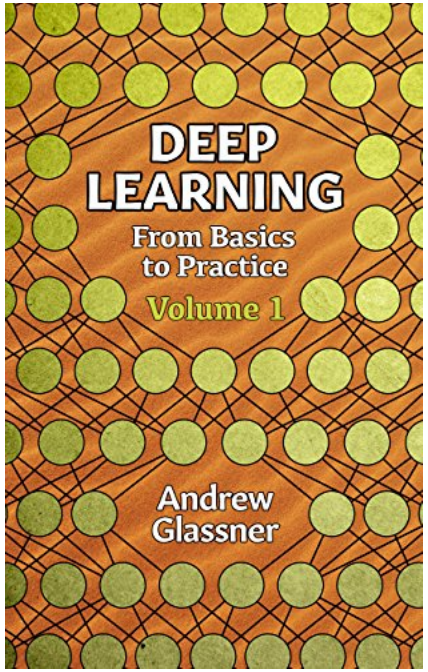


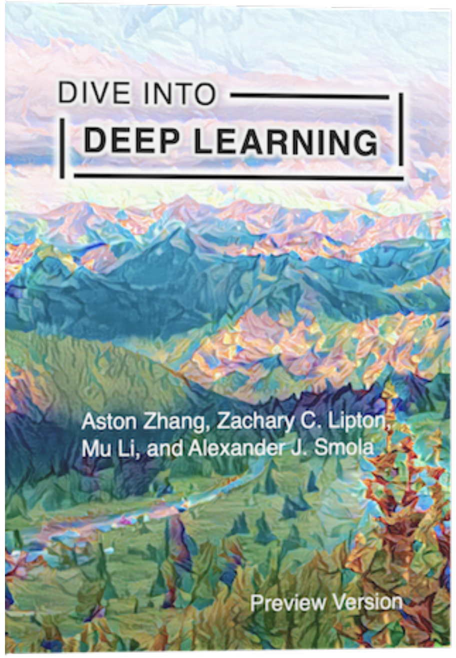
Class calendar

Date	Topic	Slides	Reading	Homework
April 9	Filters and convolutions	Google Slides	Szeliski, Chapter 3	HW1 due, HW2 assigned
April 14	Interpolation and Optimization	pdf, pptx	Szeliski, Chapter 4	
April 16	Machine Learning	pdf, pptx	Szeliski, Chapter 5.1-5.2	
April 21	Deep Neural Networks		Szeliski, Chapter 5.3	
April 23	Convolutional Neural Networks		Szeliski, Chapter 5.4	HW2 due, HW3 assigned
April 28	Network Architectures		Szeliski, Chapter 5.4-5.5	
April 30	Object Detection		Szeliski, Chapter 6.3	
May 5	Detection and Instance Segmentation		Szeliski, Chapter 6.4	
May 7	Edges, features, matching, RANSAC		Szeliski, Chapter 7.1-7.2, 8.1-8.2	HW3 due, HW4 assigned

References







https://d2l.ai/

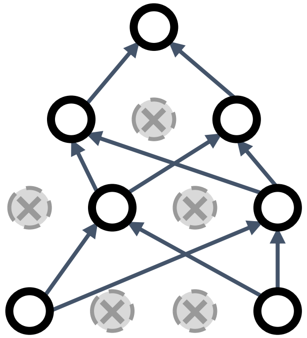
Readings

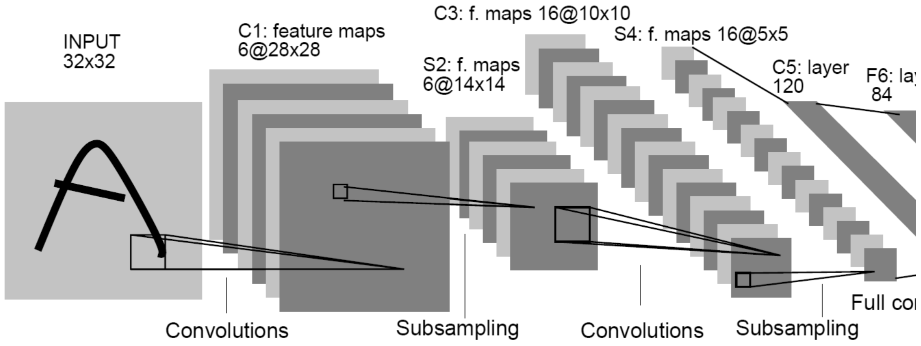
Chapter 5

Deep Learning

Convolutional neural networks++

- Training and optimization
- More regularization (dropout, …)
- Convolutional neural networks
- Pooling
- Batch normalization





As before, I’m borrowing slides from

Lecture 13	Thursday February 20	**Intro to Machine Learning** Image warping / blending Supervised vs Unsupervised learning Train / Test splits Linear Regression Regularization	[slides (pdf)] [slides (pptx)]
Lecture 14	Tuesday February 25	**Linear Models** Cross-Validation K-Nearest Neighbors SVM loss Cross-Entropy loss	[slides (pdf)] [slides (pptx)] [CS231n Linear Classification]
Lecture 15	Thursday February 27	**Optimization** Stochastic Gradient Descent SGD + Momentum	[slides (pdf)] [slides (pptx)] [CS231n Optimization]
Lecture 16	Tuesday March 10	**Neural Networks** Overfitting / Underfitting Bias / Variance tradeoff Fully-connected neural networks Biological neurons	[slides (pdf)] [slides (pptx)] [CS231n Neural Networks]

Lecture 17	Tuesday March 17	**Backpropagation** Computational Graphs Backpropagation Matrix multiplication example	[video (from EECS 498/598)] [slides (from EECS 498/598)] [231n Backpropagation] [Backprop for Matrix Multiply] [Olah on Backprop] [Nielsen on Backprop]
Lecture 18	Thursday March 19	**Convolutional Networks** Convolution Pooling Batch Normalization	[video (from EECS 498/598)] [slides (from EECS 498/598)] [231n ConvNets] [Goodfellow, Chapter 9]
Lecture 19	Tuesday March 24	**CNN Architectures** AlexNet, VGG, ResNet Size vs Accuracy Neural Architecture Search	[video (from EECS 498/598)] [slides (from EECS 498/598)] [AlexNet paper] [VGG paper] [GoogLeNet paper] [ResNet paper]
Lecture 20	Thursday March 26	**Training Neural Networks I** Activation Functions Data preprocessing Weight initialization Data Augmentation Regularization	[video (from EECS 498/598)] [slides (from EECS 498/598)] [231n Training I]
Lecture 21	Tuesday March 31	**Training Neural Networks II** Learning rate schedules Hyperparameter optimization Model ensembles Transfer learning Large-batch training	[video (EECS 498/598)] [slides (from EECS 498/598)] [231n Training II] [Karpathy "Recipe for Training"]

Lecture 4: Optimization

Loss Functions quantify preferences

- We have some dataset of (x, y)
- We have a **score function:**
- We have a **loss function**:

Q: How do we find the best W?

$$s = f(x; W) = Wx$$

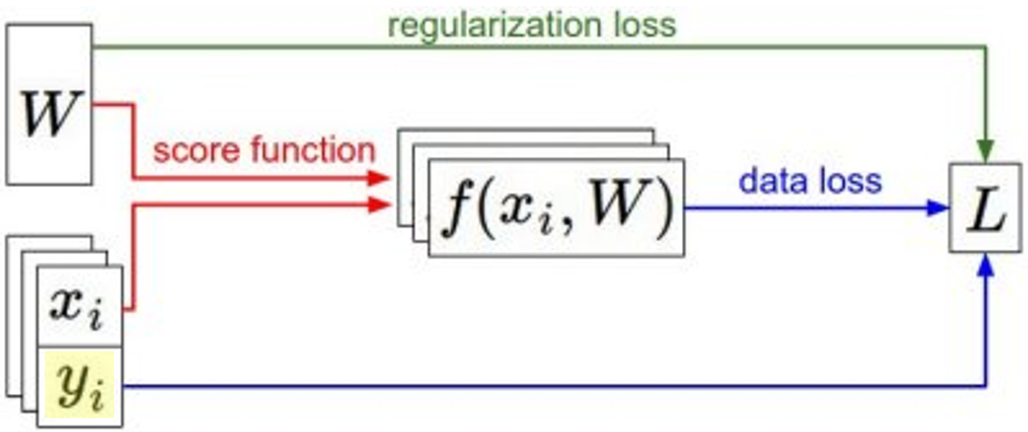
Linear classifier

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$ Softmax

SVM

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i + R(W)$$ Full loss



Follow the slope

In 1-dimension, the **derivative** of a function gives the slope:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

In multiple dimensions, the **gradient** is the vector of (partial derivatives) along each dimension

The slope in any direction is the **dot product** of the **direction** with the gradient
The direction of steepest descent is the **negative gradient**

Loss is a function of W: **Analytic Gradient**

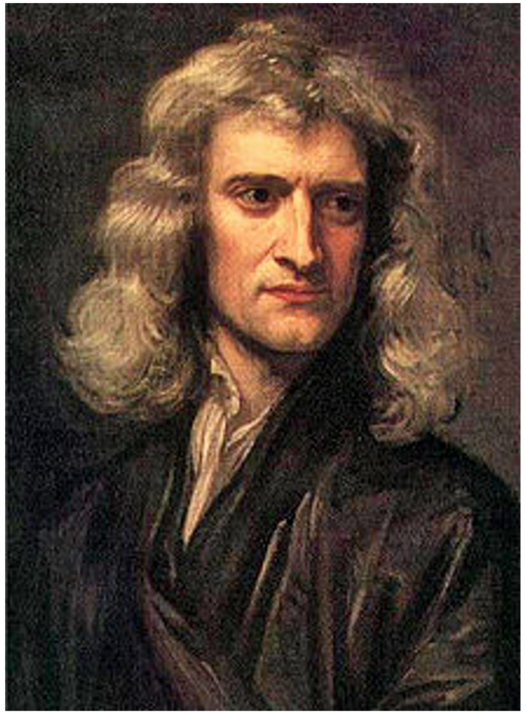
$$L = \frac{1}{N} \sum_{i=1}^{N} L_i + \sum_k W_k^2$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$$s = f(x; W) = Wx$$

want $\nabla_W L$

Use calculus to compute an **analytic gradient**



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Computing Gradients

- **Numeric gradient**: approximate, slow, easy to write
- **Analytic gradient**: exact, fast, error-prone

What's the difference?
Which one is better?

Computing Gradients

- **Numeric gradient**: approximate, slow, easy to write
- **Analytic gradient**: exact, fast, error-prone

<u>In practice:</u> Always use analytic gradient, but check implementation with numerical gradient. This is called a **gradient check.**

```
def grad_check_sparse(f, x, analytic_grad, num_checks=10, h=1e-7):
    """
    sample a few random elements and only return numerical
    in this dimensions.
    """
```

Gradient Descent

Iteratively step in the direction of
the negative gradient
(direction of local steepest descent)

```
# Vanilla gradient descent
w = initialize_weights()
for t in range(num_steps):
  dw = compute_gradient(loss_fn, data, w)
  w -= learning_rate * dw
```

$$\mathbf{w} \leftarrow \mathbf{w} - \alpha \mathbf{g} \qquad \text{or}$$

$$\mathbf{w}_{t+i} = \mathbf{w}_t - \alpha_t \mathbf{g}_t$$

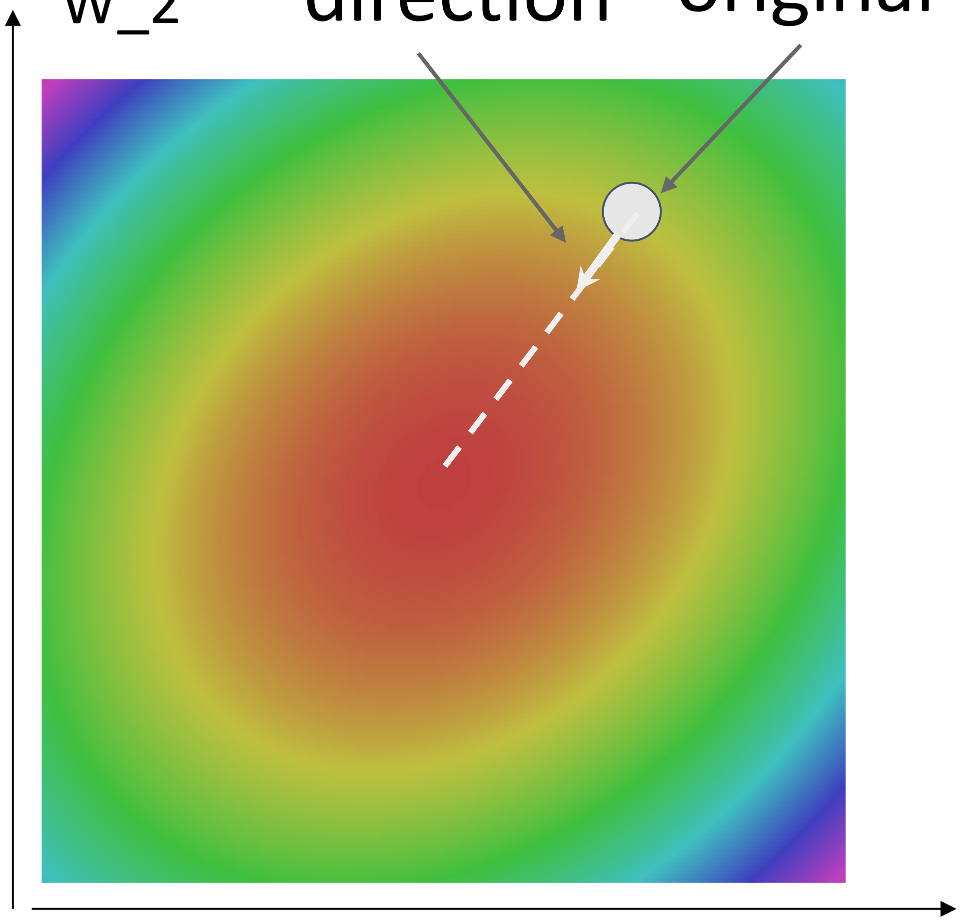
Hyperparameters:
- Weight initialization method
- Number of steps
- Learning rate

Gradient Descent

Iteratively step in the direction of
the negative gradient
(direction of local steepest descent)

```python
# Vanilla gradient descent
w = initialize_weights()
for t in range(num_steps):
  dw = compute_gradient(loss_fn, data, w)
  w -= learning_rate * dw
```

Hyperparameters:
- Weight initialization method
- Number of steps
- Learning rate



Batch Gradient Descent

$$L(W) = \frac{1}{N}\sum_{i=1}^{N} L_i(x_i, y_i, W) + \lambda R(W)$$

$$\nabla_W L(W) = \frac{1}{N}\sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W) + \lambda \nabla_W R(W)$$

Full sum expensive when N is large!

[Minibatch] Stochastic Gradient Descent (SGD)

$$L(W) = \frac{1}{N} \sum_{i=1}^{N} L_i(x_i, y_i, W) + \lambda R(W)$$

$$\nabla_W L(W) = \frac{1}{N} \sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W) + \lambda \nabla_W R(W)$$

Full sum expensive when N is large!

Approximate sum using a **minibatch** of examples 32 / 64 / 128 common

```
# Stochastic gradient descent
w = initialize_weights()
for t in range(num_steps):
  minibatch = sample_data(data, batch_size)
  dw = compute_gradient(loss_fn, minibatch, w)
  w -= learning_rate * dw
```

Hyperparameters:
- Weight initialization
- Number of steps
- Learning rate
- Batch size
- Data sampling

Stochastic Gradient Descent (SGD)

$$L(W) = \mathbb{E}_{(x,y)\sim p_{data}}[L(x,y,W)] + \lambda R(W)$$

$$\approx \frac{1}{N}\sum_{i=1}^{N} L(x_i, y_i, W) + \lambda R(W)$$

Think of loss as an expectation over the full **data distribution** p_{data}

Approximate expectation via sampling

Stochastic Gradient Descent (SGD)

$$L(W) = \mathbb{E}_{(x,y)\sim p_{data}}[L(x,y,W)] + \lambda R(W)$$

$$\approx \frac{1}{N}\sum_{i=1}^{N} L(x_i, y_i, W) + \lambda R(W)$$

Think of loss as an expectation over the full **data distribution** p_{data}

Approximate expectation via sampling

$$\nabla_W L(W) = \nabla_W \mathbb{E}_{(x,y)\sim p_{data}}[L(x,y,W)] + \lambda \nabla_W R(W))$$

$$\approx \sum_{i=1}^{N} \nabla_W L_W(x_i, y_i, W) + \nabla_W R(W)$$

Recall: Reverse-Mode Automatic Differentiation

$$x_0 \xrightarrow{f_1} x_1 \xrightarrow{f_2} x_2 \xrightarrow{f_3} x_3 \xrightarrow{f_4} L$$

D_0 D_1 D_2 D_3 scalar

Matrix multiplication is **associative**: we can compute products in any order
Computing products right-to-left avoids matrix-matrix products; only needs matrix-vector

Chain rule
$$\frac{\partial L}{\partial x_0} = \left(\frac{\partial x_1}{\partial x_0}\right)\left(\frac{\partial x_2}{\partial x_1}\right)\left(\frac{\partial x_3}{\partial x_2}\right)\left(\frac{\partial L}{\partial x_3}\right)$$

D_0 x D_1 D_1 x D_2 D_2 x D_3 D_3

Mini-batch evaluation with matrices (HW3)

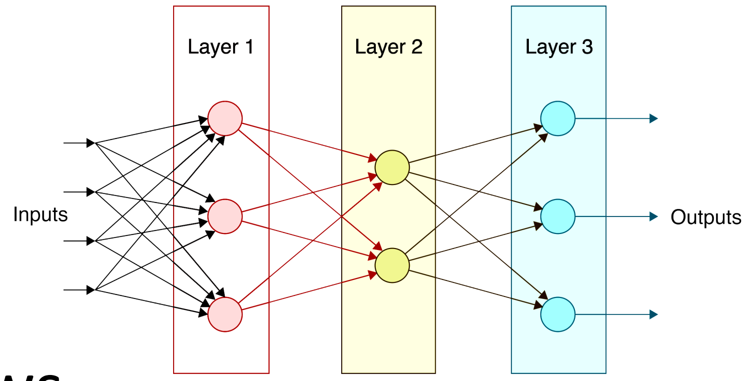
- DNNs are described as passing ***vectors*** between layers
- Why not pass all samples in a mini-batch as a ***matrix***?
- What used to be column vectors are now rows
- Need to adjust weight-vector multiplies

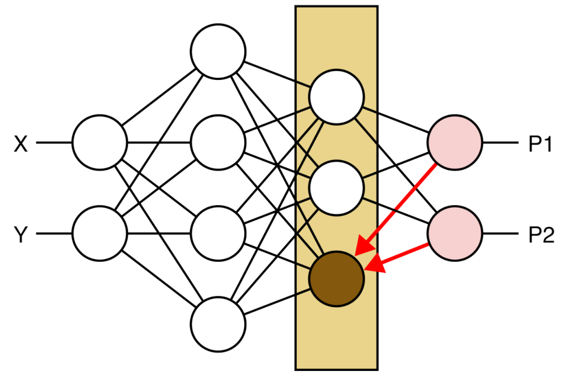
  $$\boldsymbol{s} = \boldsymbol{W}\,\boldsymbol{x}$$

  becomes

  $$\boldsymbol{S} = \boldsymbol{X}\,\boldsymbol{W}^{T}$$

- Need to adjust gradients (Jacobians) as well





Homework 3: Neural Networks in C++

- Quick overview by Keunhong Park

What you'll be implementing

We will be training a fully-connected neural network for this assignment.

- `src/activation.cpp` : you will implement the forward and backward passes for several activation functions.
- `src/classifier.cpp` : you will implement gradient computation and parameter updates using algorithms we discussed in class.

You'll be training on two datasets, one is MNIST which is a digit-recognition dataset. The other is a simple visual recognition dataset called CIFAR.

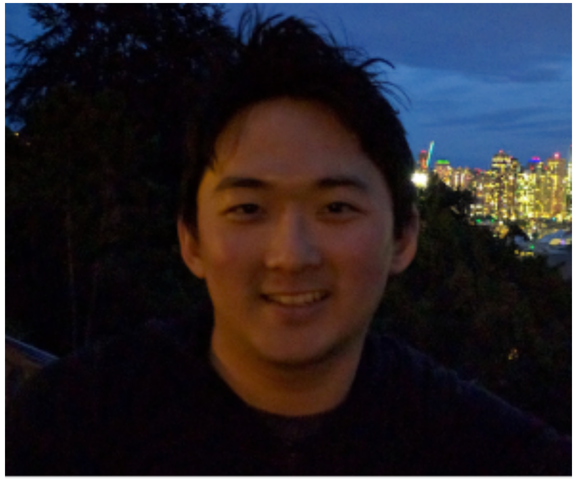
1. Implementing Neural Networks

1.1 Activation Functions

An important part of machine learning, be it linear classifiers or neural networks, is the activation function you use.

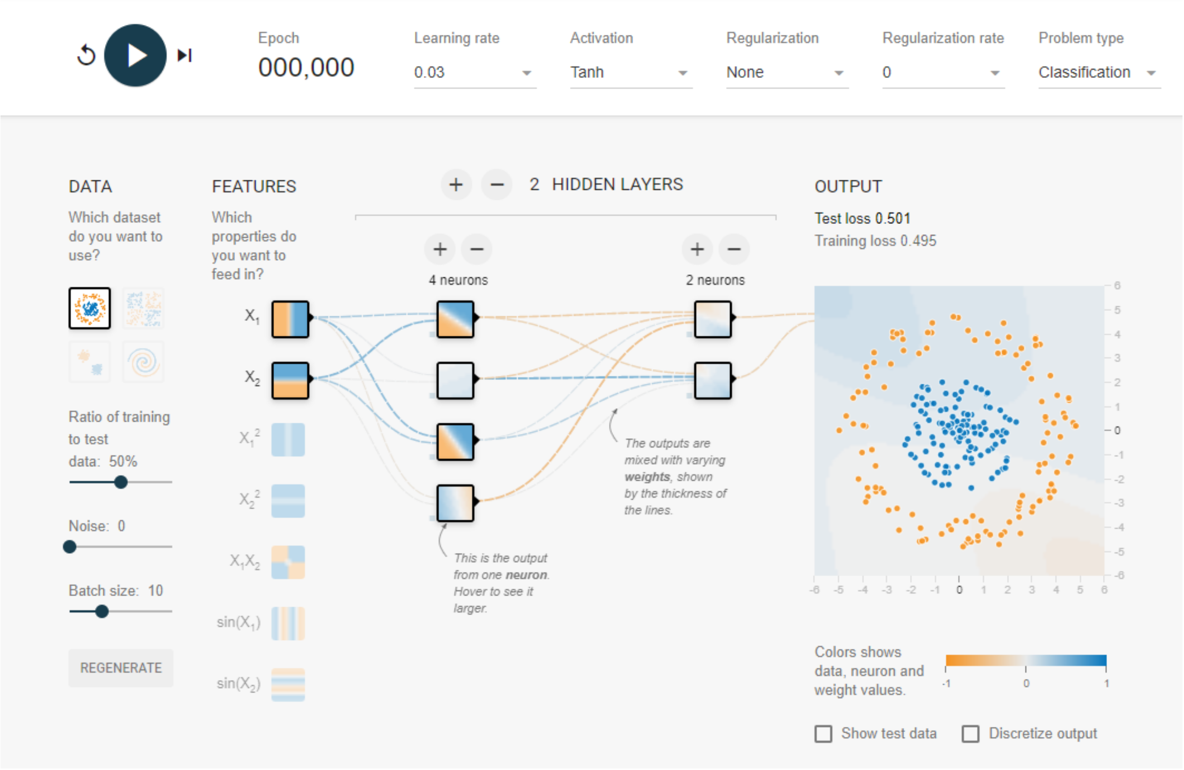
We will be implementing the following activation functions:

- Linear: `f(x) = x`
- Logistic: `f(x) = 1 / (1 + exp(-x))`
- tanh: `f(x) = tanh(x)`
- ReLU: `f(x) = max(0, x)`
- Leaky ReLU: `f(x) = 0.01*x if x < 0 else x`
- Softmax: https://en.wikipedia.org/wiki/Softmax_function



Keunhong Park (TA)
Ph.D Student
View full profile

Interactive Web demo

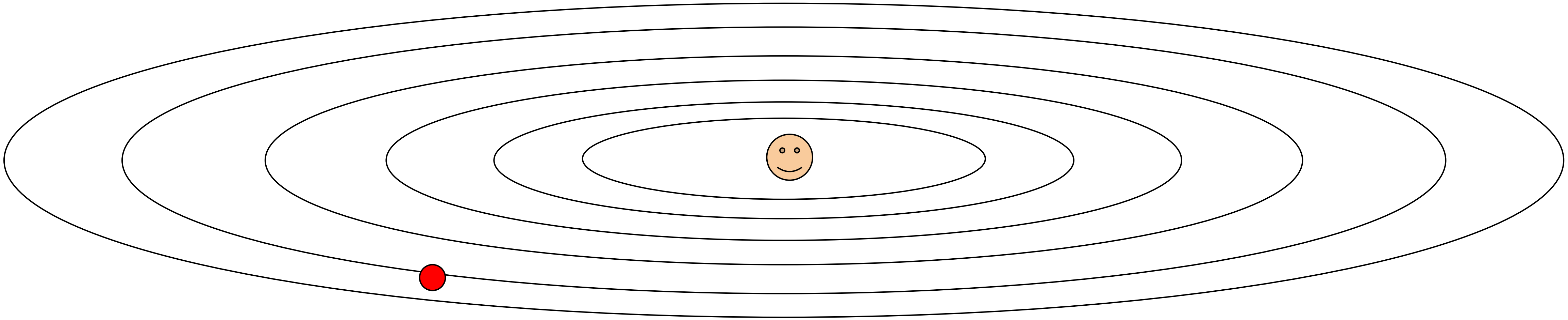


(Thanks, Doruk)

https://playground.tensorflow.org/

Problems with SGD

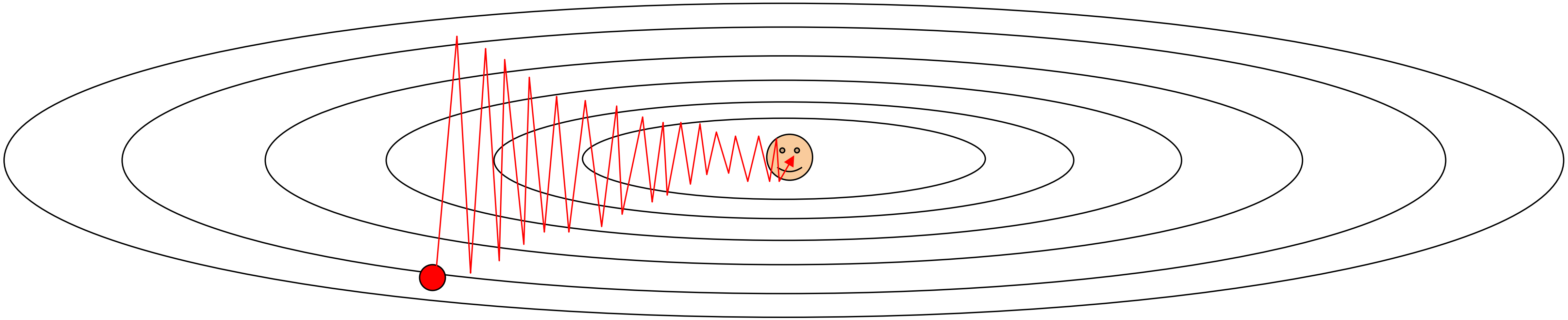
What if loss changes quickly in one direction and slowly in another?
What does gradient descent do?



Loss function has high **condition number**: ratio of largest to smallest singular value of the Hessian matrix is large

Problems with SGD

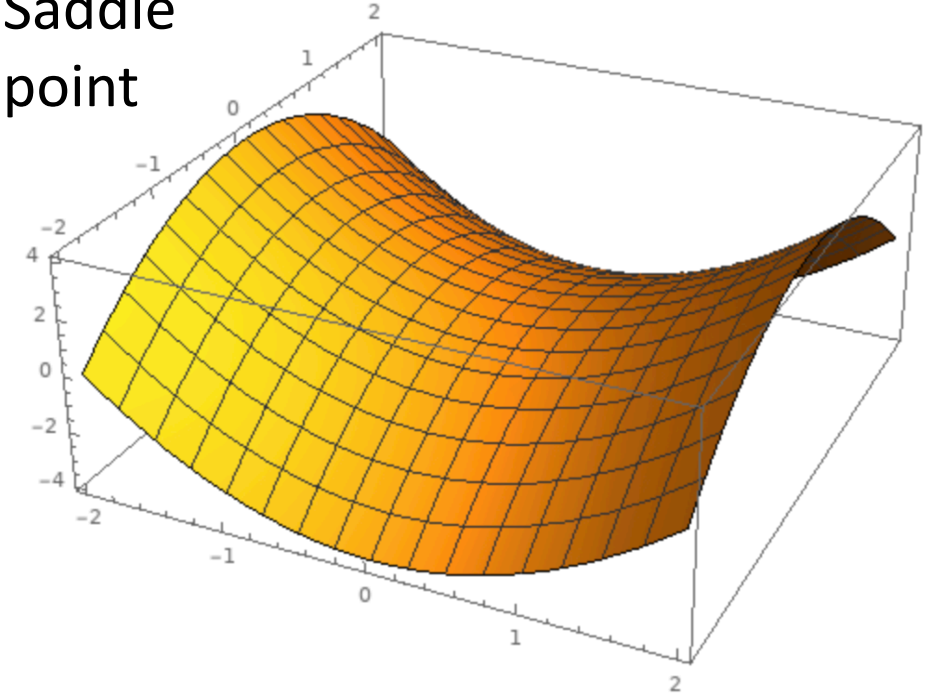
What if loss changes quickly in one direction and slowly in another?
What does gradient descent do?
Very slow progress along shallow dimension, jitter along steep direction



Loss function has high **condition number**: ratio of largest to smallest singular value of the Hessian matrix is large

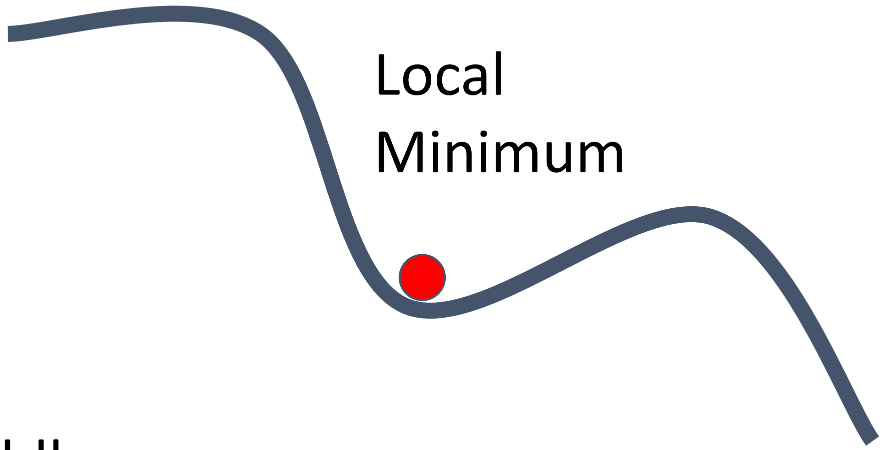
Problems with SGD

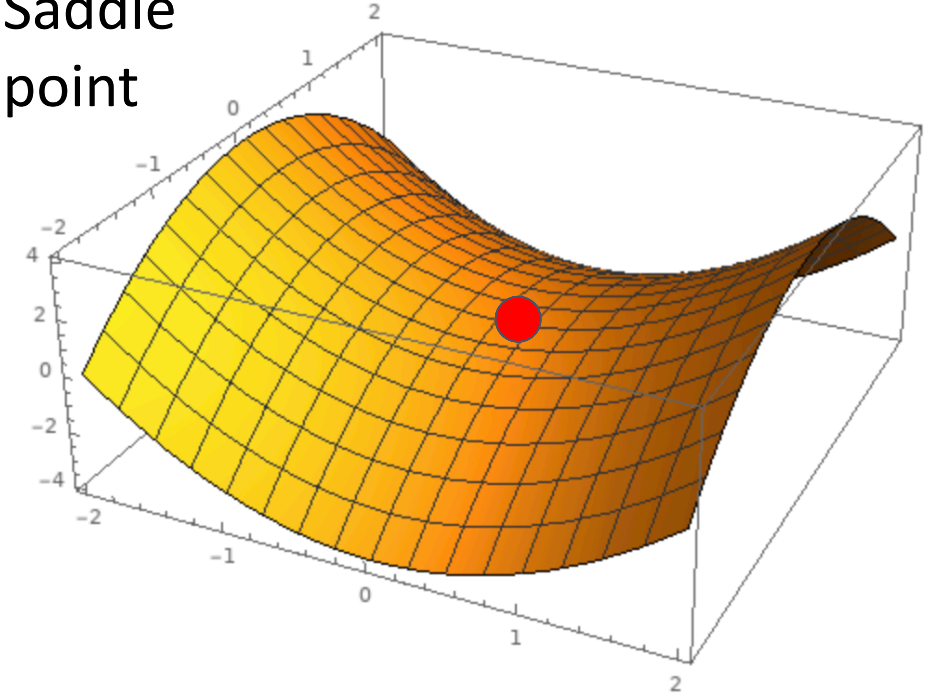
What if the loss function has a **local minimum** or **saddle point**?



Problems with SGD

What if the loss function has a **local minimum** or **saddle point**?





Zero gradient, gradient descent gets stuck

Problems with SGD

Our gradients come from minibatches so they can be noisy!

$$L(W) = \frac{1}{N} \sum_{i=1}^{N} L_i(x_i, y_i, W)$$

$$\nabla_W L(W) = \frac{1}{N} \sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W)$$

SGD

SGD

$$x_{t+1} = x_t - \alpha \nabla f(x_t)$$

```
for t in range(num_steps):
  dw = compute_gradient(w)
  w -= learning_rate * dw
```

SGD + Momentum

SGD

$$x_{t+1} = x_t - \alpha \nabla f(x_t)$$

```
for t in range(num_steps):
  dw = compute_gradient(w)
  w -= learning_rate * dw
```

SGD+Momentum

$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

```
v = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  v = rho * v + dw
  w -= learning_rate * v
```

- Build up “velocity” as a running mean of gradients
- Rho gives “friction”; typically rho=0.9 or 0.99

Sutskever et al, “On the importance of initialization and momentum in deep learning”, ICML 2013

SGD + Momentum

SGD+Momentum

$$v_{t+1} = \rho v_t - \alpha \nabla f(x_t)$$
$$x_{t+1} = x_t + v_{t+1}$$

```
v = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  v = rho * v - learning_rate * dw
  w += v
```

SGD+Momentum

$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

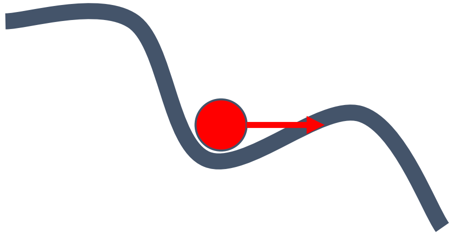
```
v = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  v = rho * v + dw
  w -= learning_rate * v
```

You may see SGD+Momentum formulated different ways, but they are equivalent - give same sequence of x

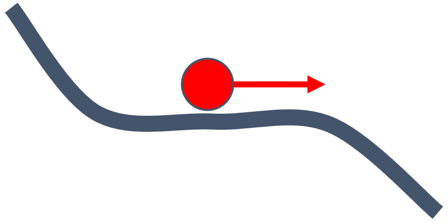
Sutskever et al, “On the importance of initialization and momentum in deep learning”, ICML 2013

SGD + Momentum

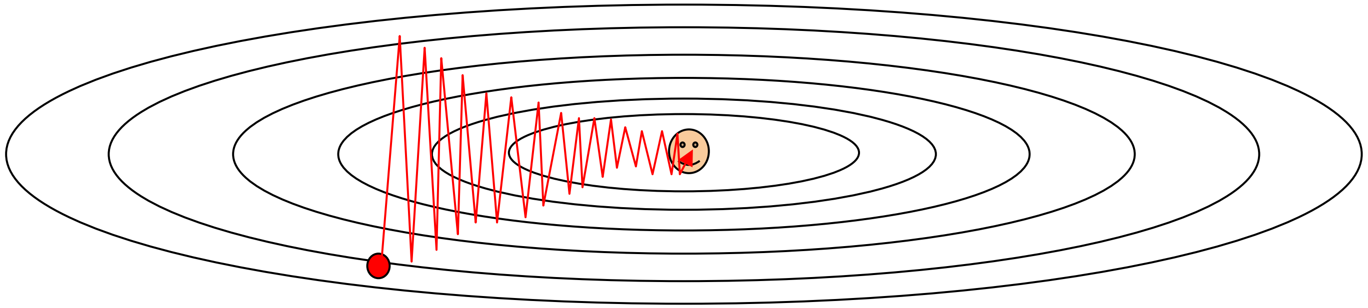
Local Minima



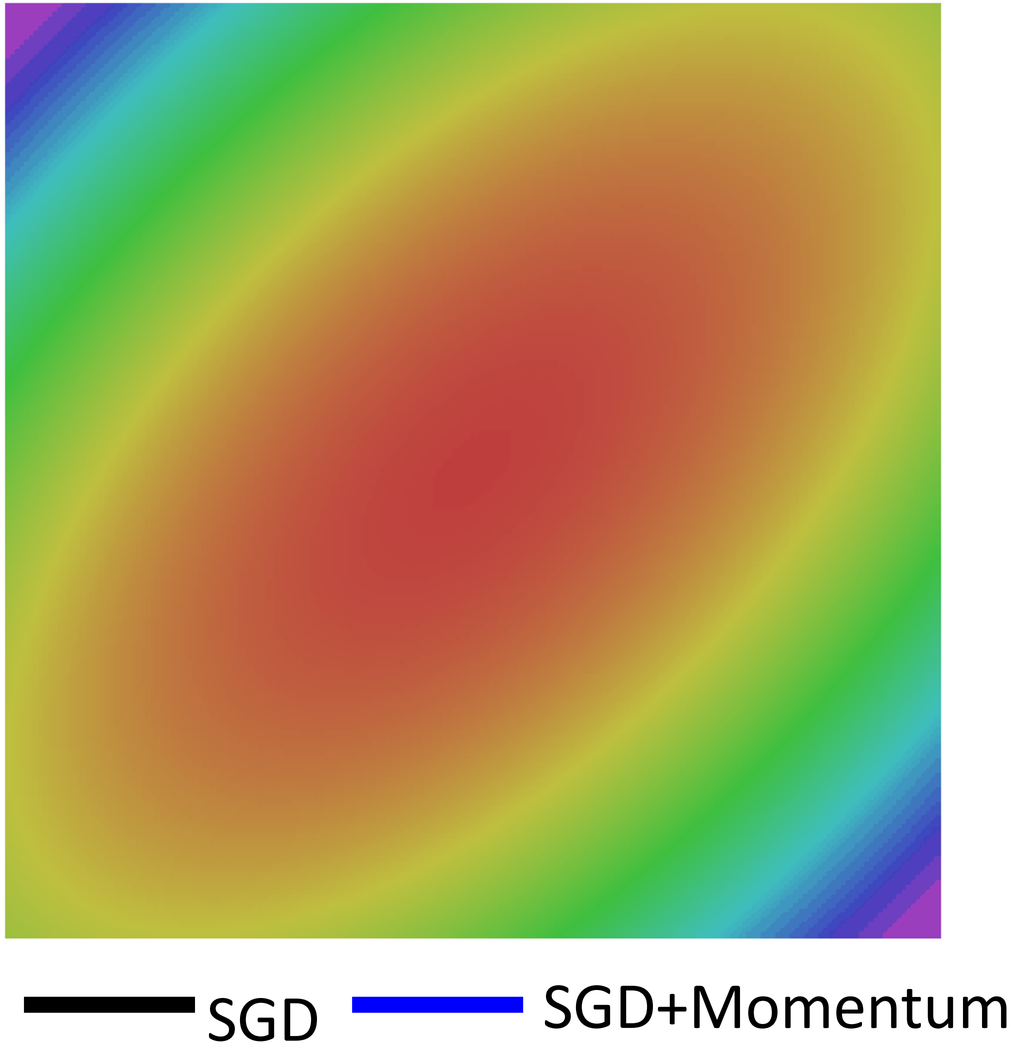
Saddle points



Poor Conditioning



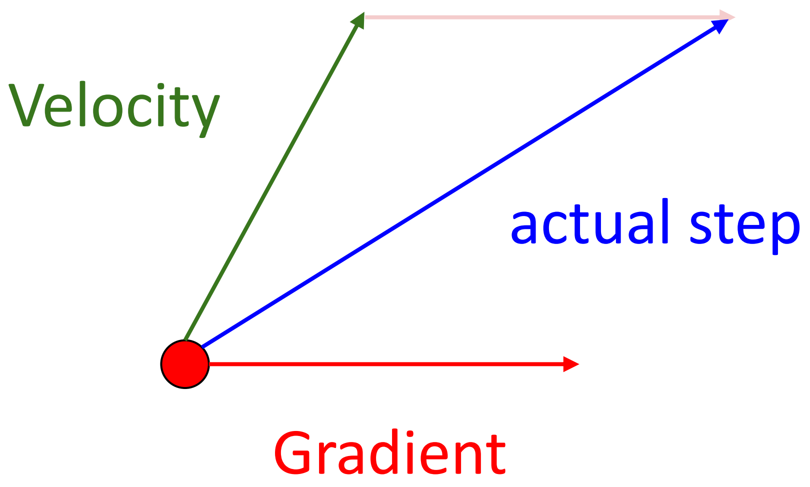
Gradient Noise



Sutskever et al, "On the importance of initialization and momentum in deep learning", ICML 2013

SGD + Momentum

Momentum update:

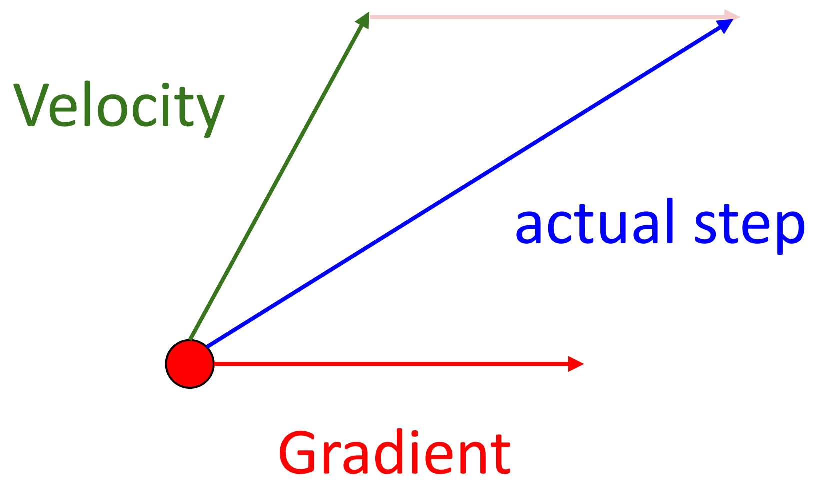


Combine gradient at current point with velocity to get step used to update weights

Nesterov, “A method of solving a convex programming problem with convergence rate O(1/k^2)”, 1983
Nesterov, “Introductory lectures on convex optimization: a basic course”, 2004
Sutskever et al, “On the importance of initialization and momentum in deep learning”, ICML 2013

Nesterov Momentum

Momentum update:



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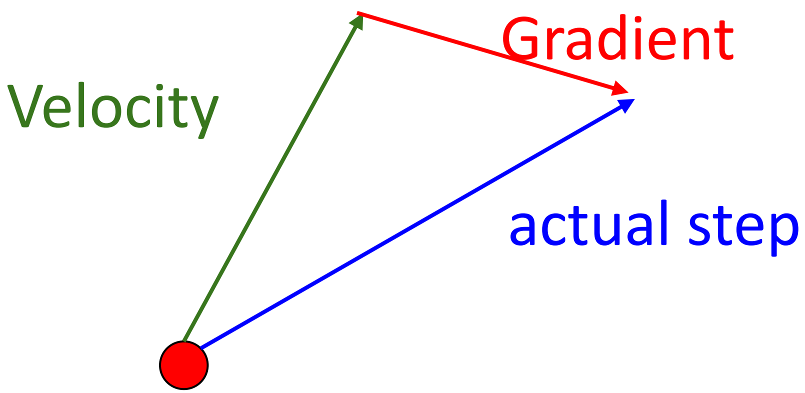

"Look ahead" to the point where updating using velocity would take us; compute gradient there and mix it with velocity to get actual update direction

Nesterov, "A method of solving a convex programming problem with convergence rate O(1/k^2)", 1983
Nesterov, "Introductory lectures on convex optimization: a basic course", 2004
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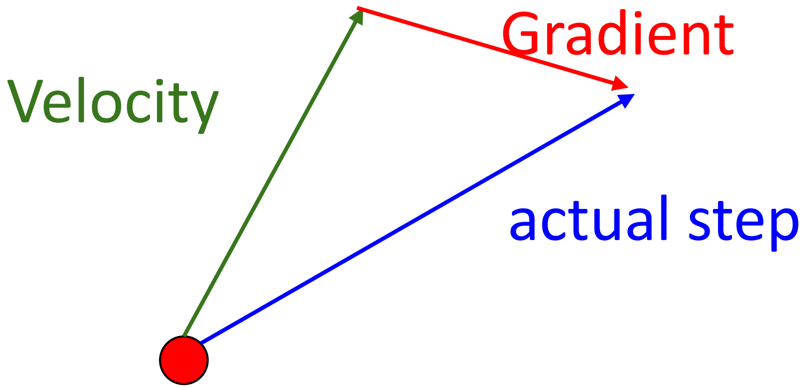
Nesterov Momentum

$$v_{t+1} = \rho v_t - \alpha \nabla f(x_t + \rho v_t)$$
$$x_{t+1} = x_t + v_{t+1}$$



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Nesterov Momentum

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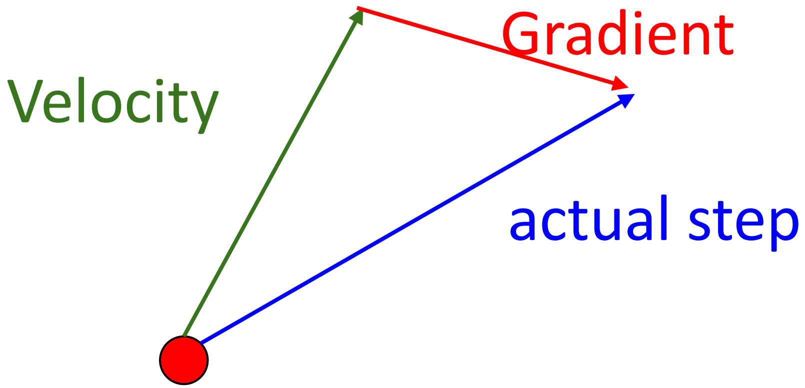
$$x_{t+1} = x_t + v_{t+1}$$

Annoying, usually we want update in terms of $x_t, \nabla f(x_t)$



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Nesterov Momentum

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$$x_{t+1} = x_t + v_{t+1}$$

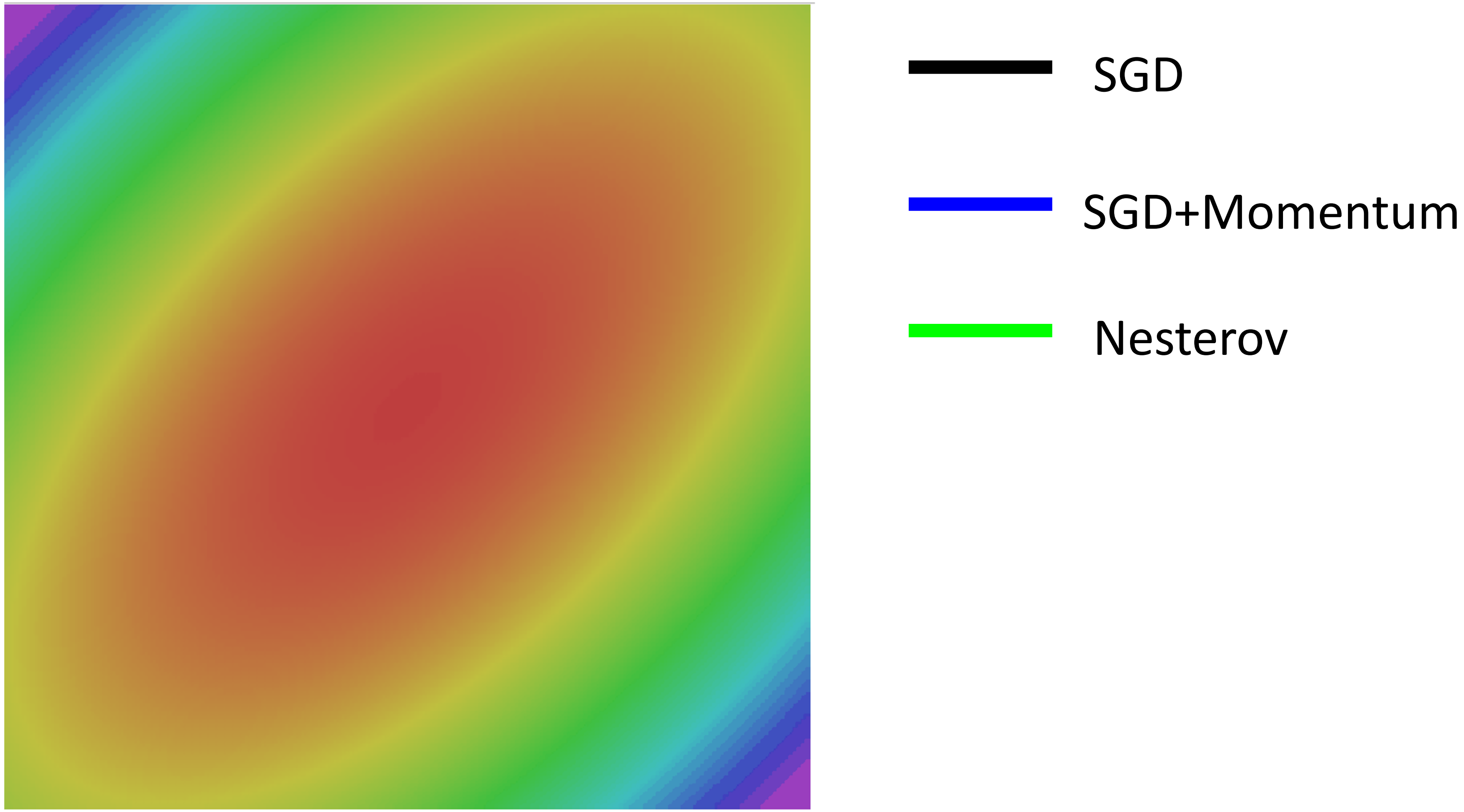
Annoying, usually we want update in terms of $x_t, \nabla f(x_t)$

Change of variables $\tilde{x}_t = x_t + \rho v_t$ and rearrange:

$$v_{t+1} = \rho v_t - \alpha \nabla f(\tilde{x}_t)$$
$$\begin{aligned} \tilde{x}_{t+1} &= \tilde{x}_t - \rho v_t + (1+\rho) v_{t+1} \\ &= \tilde{x}_t + v_{t+1} + \rho(v_{t+1} - v_t) \end{aligned}$$

```
v = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  old_v = v
  v = rho * v - learning_rate * dw
  w -= rho * old_v - (1 + rho) * v
```

Nesterov Momentum



AdaGrad

```
grad_squared = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  grad_squared += dw * dw
  w -= learning_rate * dw / (grad_squared.sqrt() + 1e-7)
```

Added element-wise scaling of the gradient based on the historical sum of squares in each dimension

"Per-parameter learning rates"
or "adaptive learning rates"

Duchi et al, "Adaptive subgradient methods for online learning and stochastic optimization", JMLR 2011

AdaGrad

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grad_squared = 0
for t in range(num_steps):
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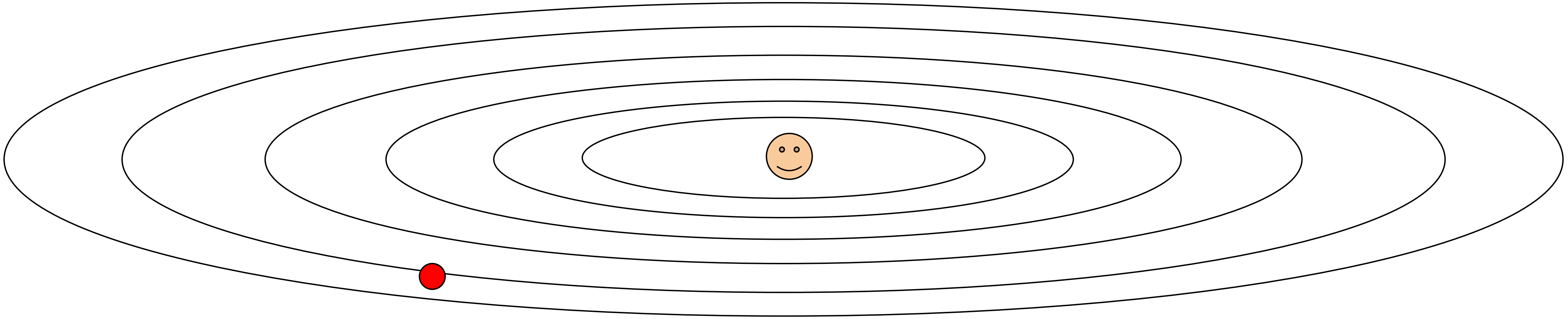
```



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for t in range(num_steps):
  dw = compute_gradient(w)
  grad_squared += dw * dw
  w -= learning_rate * dw / (grad_squared.sqrt() + 1e-7)
```
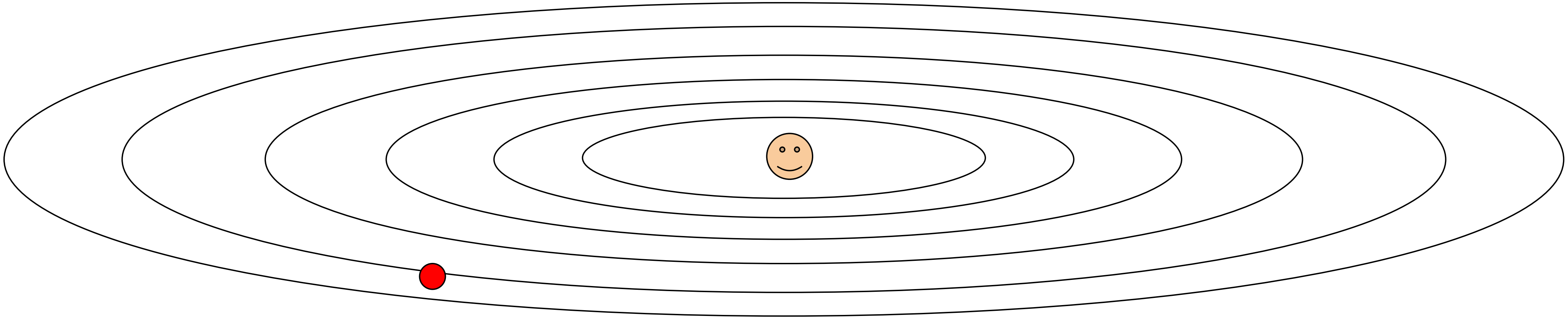
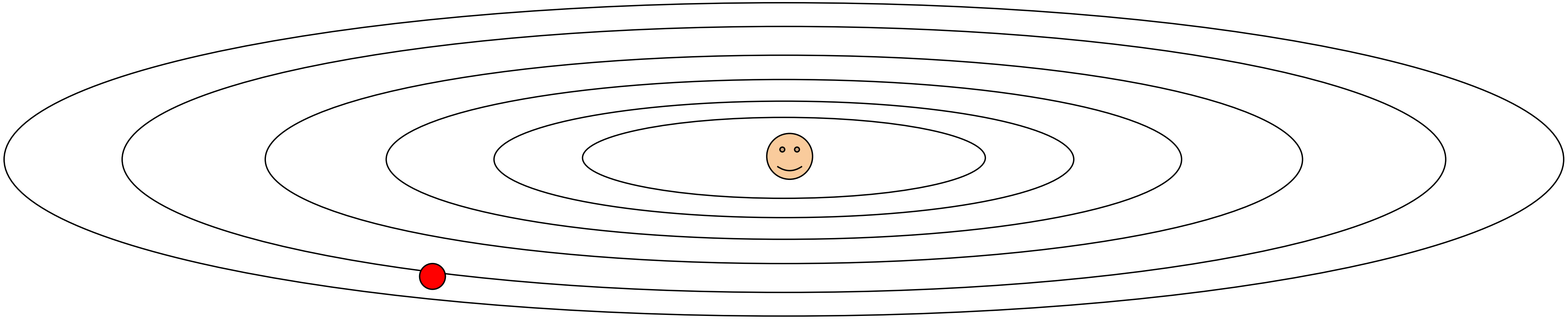



Q: What happens with AdaGrad?

AdaGrad

```
grad_squared = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  grad_squared += dw * dw
  w -= learning_rate * dw / (grad_squared.sqrt() + 1e-7)
```



Q: What happens with AdaGrad?

Progress along “steep” directions is damped; progress along “flat” directions is accelerated

RMSProp: “Leaky Adagrad”

```
grad_squared = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  grad_squared += dw * dw
  w -= learning_rate * dw / (grad_squared.sqrt() + 1e-7)
```

AdaGrad

```
grad_squared = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  grad_squared = decay_rate * grad_squared + (1 - decay_rate) * dw * dw
  w -= learning_rate * dw / (grad_squared.sqrt() + 1e-7)
```

RMSProp

Tieleman and Hinton, 2012

RMSProp

SGD

SGD+Momentum

RMSProp

Adam (almost): RMSProp + Momentum

```
moment1 = 0
moment2 = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  moment1 = beta1 * moment1 + (1 - beta1) * dw
  moment2 = beta2 * moment2 + (1 - beta2) * dw * dw
  w -= learning_rate * moment1 / (moment2.sqrt() + 1e-7)
```

Kingma and Ba, "Adam: A method for stochastic optimization", ICLR 2015

Adam (almost): RMSProp + Momentum

```
moment1 = 0
moment2 = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  moment1 = beta1 * moment1 + (1 - beta1) * dw
  moment2 = beta2 * moment2 + (1 - beta2) * dw * dw
  w -= learning_rate * moment1 / (moment2.sqrt() + 1e-7)
```

Adam

Momentum

```
v = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  v = rho * v + dw
  w -= learning_rate * v
```

SGD+Momentum

Kingma and Ba, "Adam: A method for stochastic optimization", ICLR 2015

Adam (almost): RMSProp + Momentum

```
moment1 = 0
moment2 = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  moment1 = beta1 * moment1 + (1 - beta1) * dw
  moment2 = beta2 * moment2 + (1 - beta2) * dw * dw
  w -= learning_rate * moment1 / (moment2.sqrt() + 1e-7)
```

Adam

Momentum

AdaGrad / RMSProp

```
grad_squared = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  grad_squared = decay_rate * grad_squared + (1 - decay_rate) * dw * dw
  w -= learning_rate * dw / (grad_squared.sqrt() + 1e-7)
```

RMSProp

Kingma and Ba, “Adam: A method for stochastic optimization”, ICLR 2015

Adam: Very Common in Practice!

for input to the CNN; each colored pixel in the image yields a 7D one-hot vector. Following common practice, the network is trained end-to-end using stochastic gradient descent with the Adam optimizer [22]. We anneal the learning rate to 0 using a half cosine schedule without restarts [28].

Bakhtin, van der Maaten, Johnson, Gustafson, and Girshick, NeurIPS 2019

We train all models using Adam [23] with learning rate 10^{-4} and batch size 32 for 1 million iterations; training takes about 3 days on a single Tesla P100. For each minibatch we first update f, then update D_{img} and D_{obj}.

Johnson, Gupta, and Fei-Fei, CVPR 2018

ganized into three residual blocks. We train for 25 epochs using Adam [27] with learning rate 10^{-4} and 32 images per batch on 8 Tesla V100 GPUs. We set the cubify thresh-

Gkioxari, Malik, and Johnson, ICCV 2019

sampled with each bit drawn uniformly at random. For gradient descent, we use Adam [29] with a learning rate of 10^{-3} and default hyperparameters. All models are trained with batch size 12. Models are trained for 200 epochs, or 400 epochs if being trained on multiple noise layers.

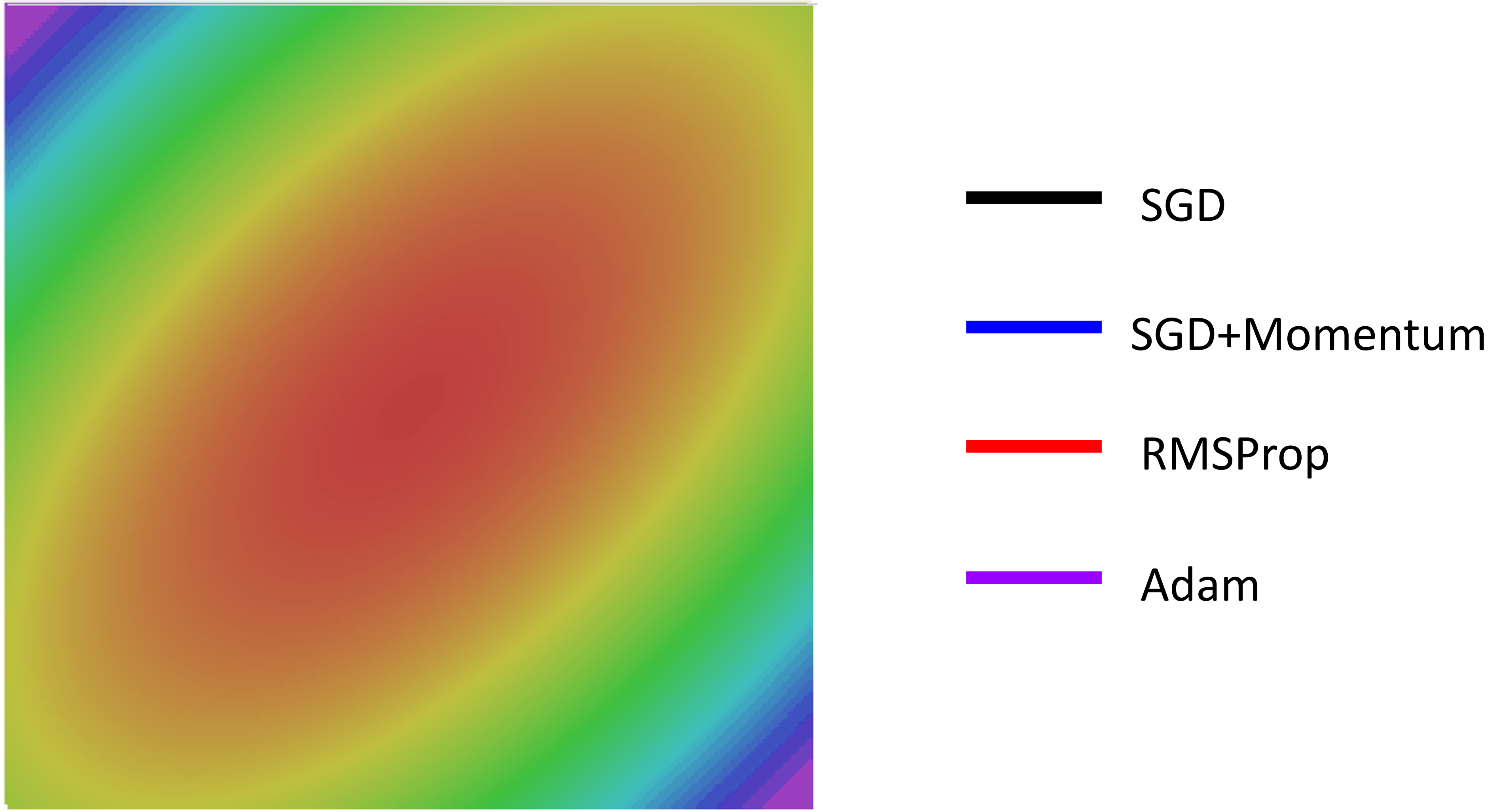
Zhu, Kaplan, Johnson, and Fei-Fei, ECCV 2018

16 dimensional vectors. We iteratively train the Generator and Discriminator with a batch size of 64 for 200 epochs using Adam [22] with an initial learning rate of 0.001.

Gupta, Johnson, et al, CVPR 2018

Adam with beta1 = 0.9, beta2 = 0.999, and learning_rate = 1e-3, 5e-4, 1e-4 is a great starting point for many models!

Adam



Optimization Algorithm Comparison

Algorithm	Tracks first moments (Momentum)	Tracks second moments (Adaptive learning rates)	Leaky second moments	Bias correction for moment estimates
SGD	✗	✗	✗	✗
SGD+Momentum	✓	✗	✗	✗
Nesterov	✓	✗	✗	✗
AdaGrad	✗	✓	✗	✗
RMSProp	✗	✓	✓	✗
Adam	✓	✓	✓	✓

In practice:

- **Adam** is a good default choice in many cases
  SGD+Momentum can outperform Adam but may require more tuning

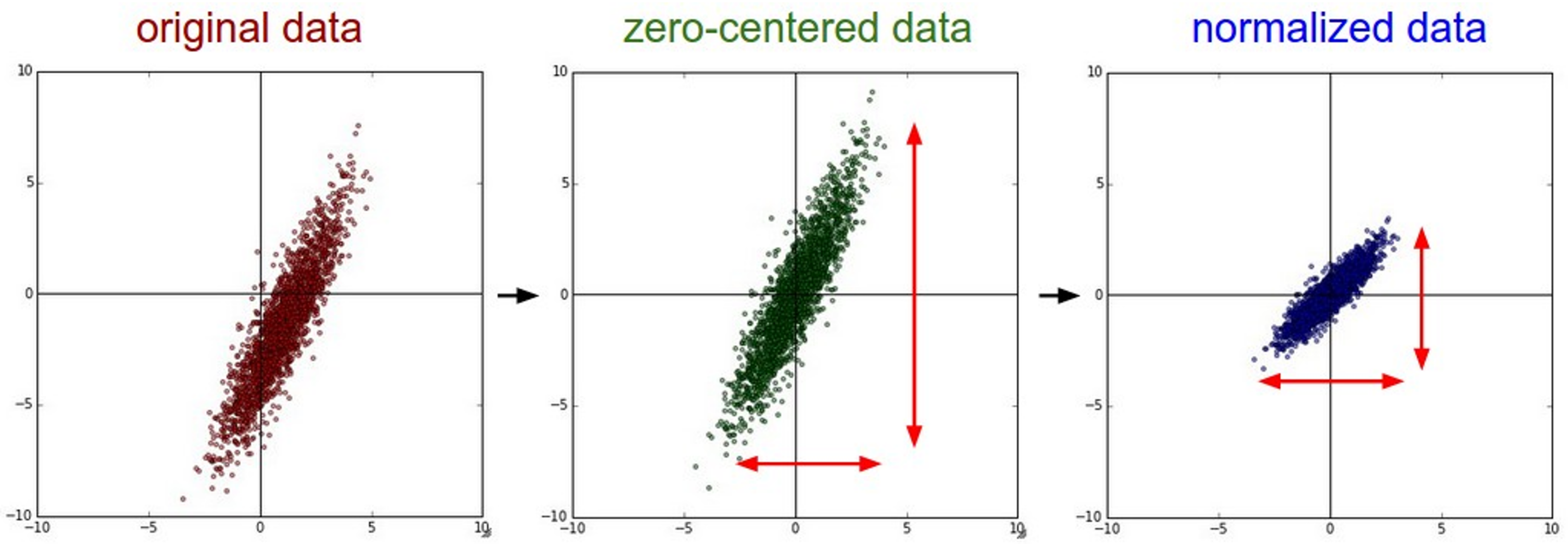
- If you can afford to do full batch updates then try out **L-BFGS** (and don't forget to disable all sources of noise)

Lecture 10: Training Neural Networks (Part 1)

Overview

1. **One time setup**
   Activation functions, data preprocessing, weight initialization, regularization
2. **Training dynamics**
   Learning rate schedules; large-batch training; hyperparameter optimization
3. **After training**
   Model ensembles, transfer learning

Snapshot: Data Preprocessing

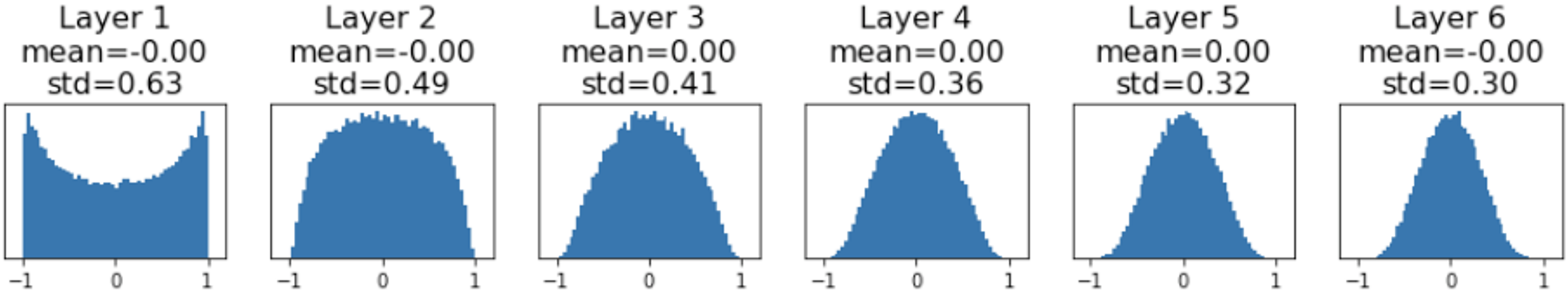


Snapshot: Weight Initialization

```python
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

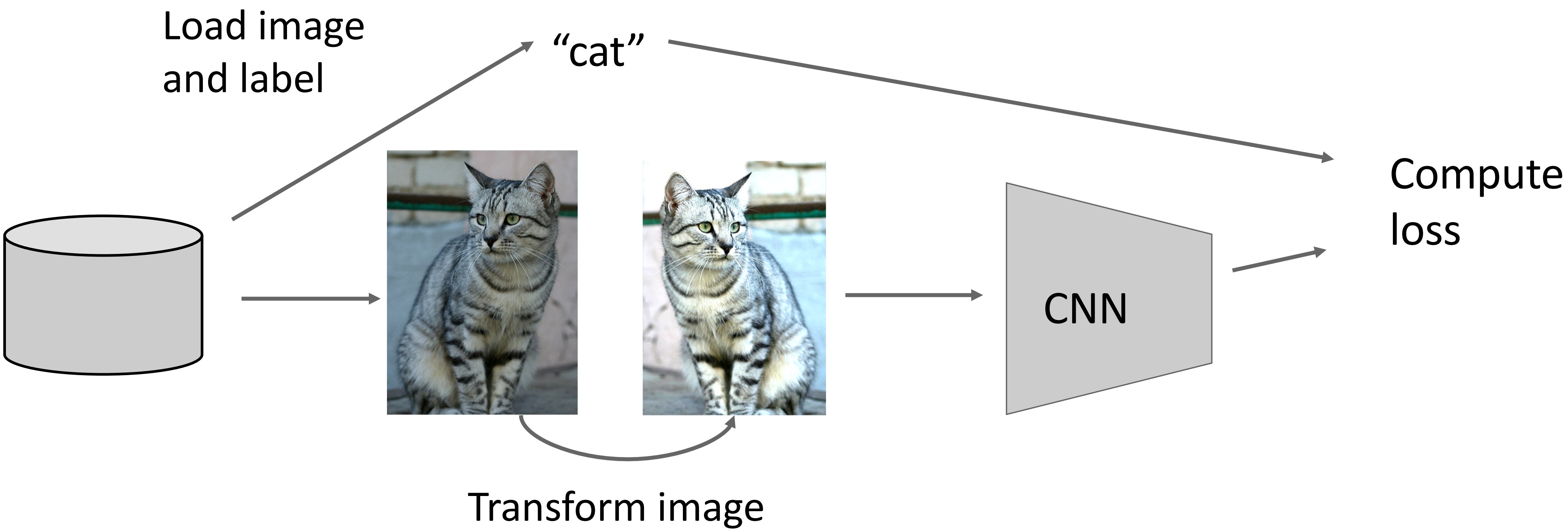
"Xavier" initialization:
std = 1/sqrt(Din)

"Just right": Activations are nicely scaled for all layers!



Glorot and Bengio, "Understanding the difficulty of training deep feedforward neural networks", AISTAT 2010

Snapshot: Data Augmentation

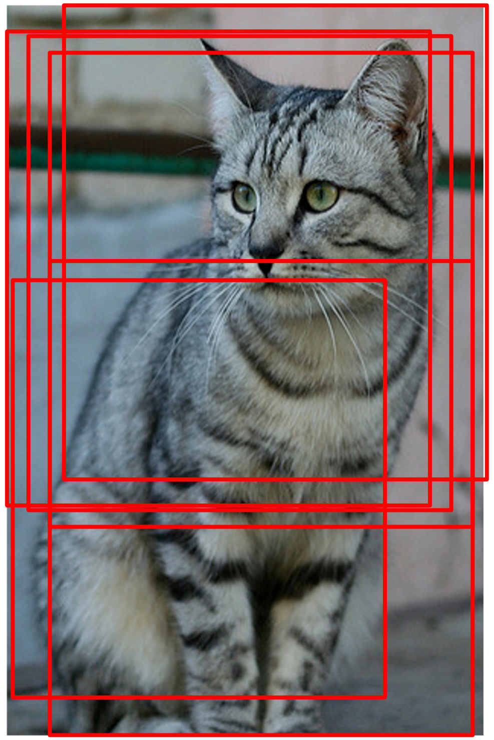


Data Augmentation: Random Crops and Scales

Training: sample random crops / scales

ResNet:

1. Pick random L in range [256, 480]
2. Resize training image, short side = L
3. Sample random 224 x 224 patch



Regularization

Training: Add randomness

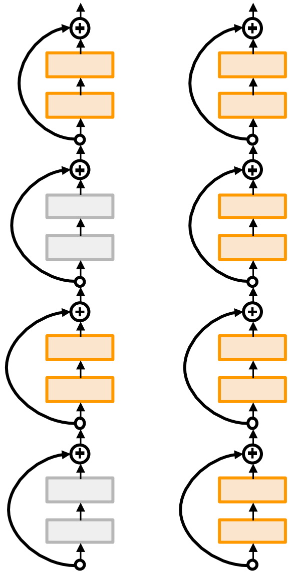
Testing: Marginalize out randomness

Examples:

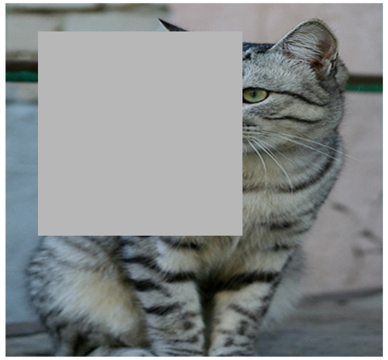
Batch Normalization

Data Augmentation

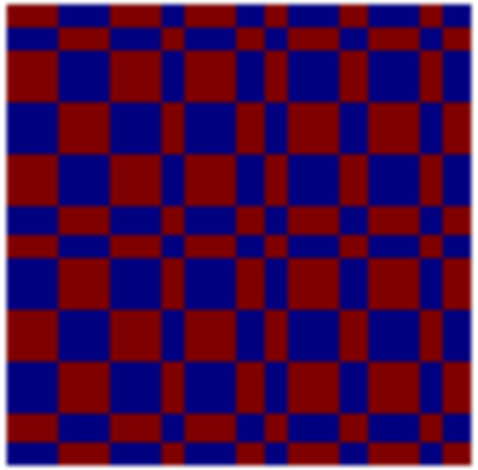
Stochastic Depth



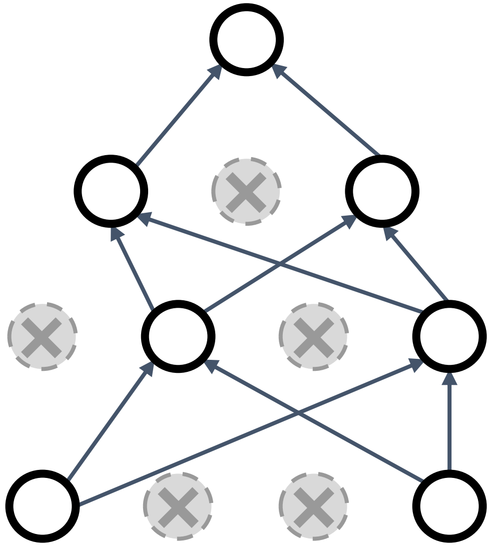
Cutout



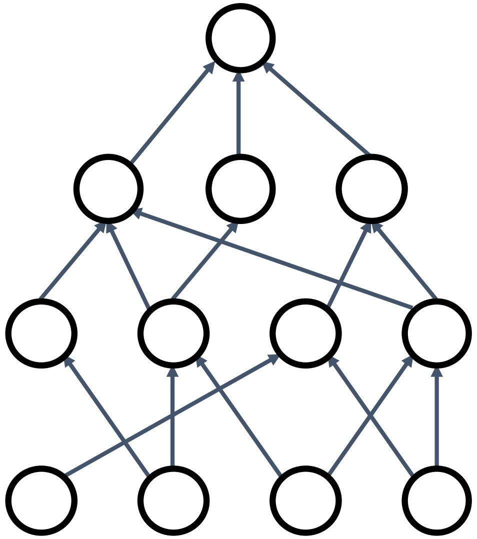
Fractional pooling



Dropout



DropConnect



Mixup

(Old style) regularization: Add term to the loss

$$L = \frac{1}{N} \sum_{i=1}^{N} \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1) + \boxed{\lambda R(W)}$$

In common use:

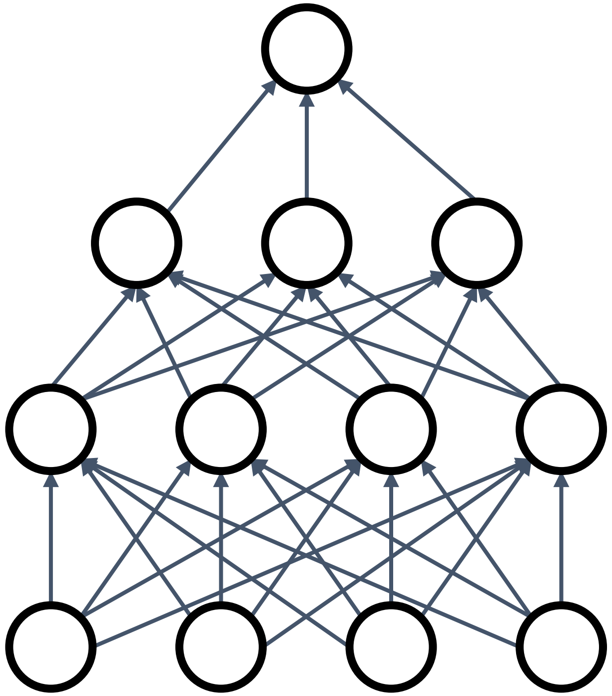
L2 regularization $R(W) = \sum_k \sum_l W_{k,l}^2$ (Weight decay)

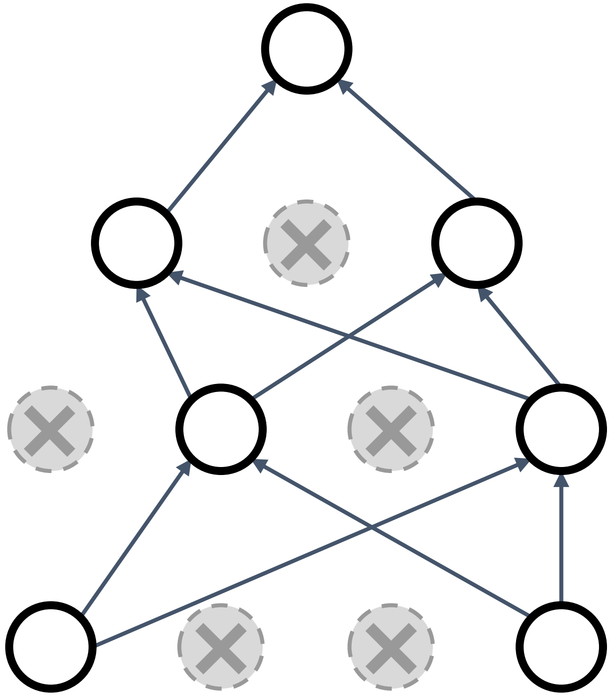
L1 regularization $R(W) = \sum_k \sum_l |W_{k,l}|$

Elastic net (L1 + L2) $R(W) = \sum_k \sum_l \beta W_{k,l}^2 + |W_{k,l}|$

Regularization: Dropout

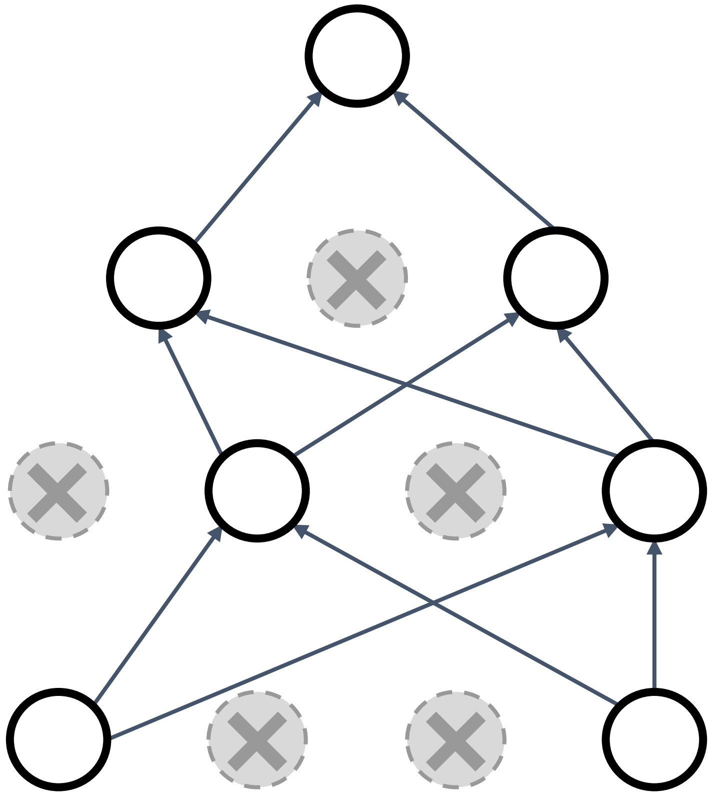
In each forward pass, randomly set some neurons to zero
Probability of dropping is a hyperparameter; 0.5 is common





Srivastava et al, "Dropout: A simple way to prevent neural networks from overfitting", JMLR 2014

Regularization: Dropout



Another interpretation:

Dropout is training a large **ensemble** of models (that share parameters).

Each binary mask is one model

An FC layer with 4096 units has $2^{4096} \sim 10^{1233}$ possible masks!
Only $\sim 10^{82}$ atoms in the universe...

Dropout: Test Time

```
def predict(X):
  # ensembled forward pass
  H1 = np.maximum(0, np.dot(W1, X) + b1) * p # NOTE: scale the activations
  H2 = np.maximum(0, np.dot(W2, H1) + b2) * p # NOTE: scale the activations
  out = np.dot(W3, H2) + b3
```

At test time all neurons are active always

=> We must scale the activations so that for each neuron:

<u>output at test time</u> = <u>expected output at training time</u>

More common: “Inverted dropout”

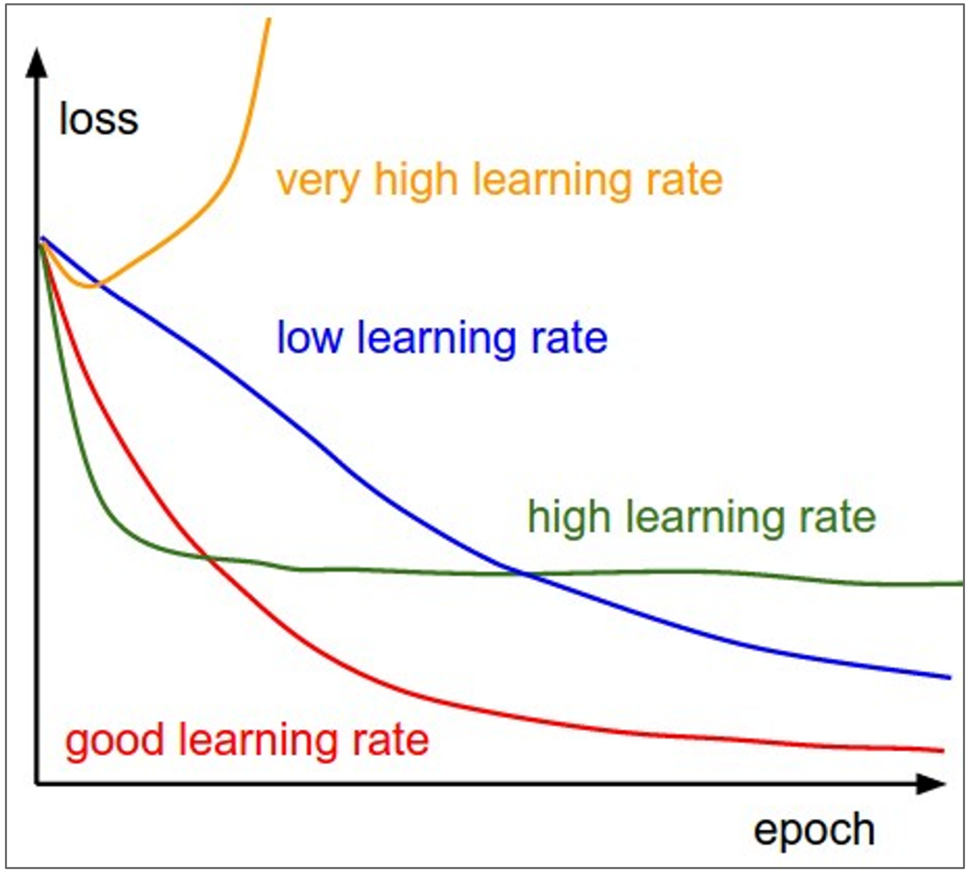
```python
p = 0.5 # probability of keeping a unit active. higher = less dropout

def train_step(X):
  # forward pass for example 3-layer neural network
  H1 = np.maximum(0, np.dot(W1, X) + b1)
  U1 = (np.random.rand(*H1.shape) < p) / p # first dropout mask. Notice /p!
  H1 *= U1 # drop!
  H2 = np.maximum(0, np.dot(W2, H1) + b2)
  U2 = (np.random.rand(*H2.shape) < p) / p # second dropout mask. Notice /p!
  H2 *= U2 # drop!
  out = np.dot(W3, H2) + b3

  # backward pass: compute gradients... (not shown)
  # perform parameter update... (not shown)

def predict(X):
  # ensembled forward pass
  H1 = np.maximum(0, np.dot(W1, X) + b1) # no scaling necessary
  H2 = np.maximum(0, np.dot(W2, H1) + b2)
  out = np.dot(W3, H2) + b3
```

Drop and scale during training

test time is unchanged!

Learning Rate Schedules

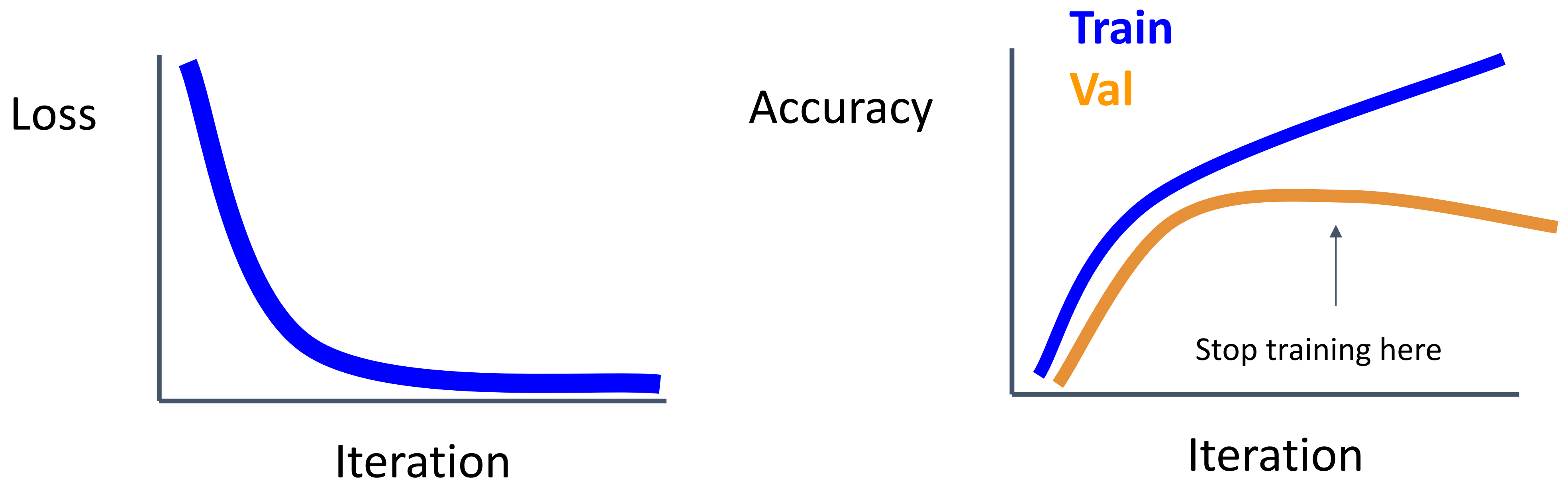
SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have **learning rate** as a hyperparameter.



Q: Which one of these learning rates is best to use?

A: All of them! Start with large learning rate and decay over time

How long to train? **Early Stopping**



Stop training the model when accuracy on the validation set decreases
Or train for a long time, but always keep track of the model snapshot that worked best on val. **Always a good idea to do this!**

Model Ensembles

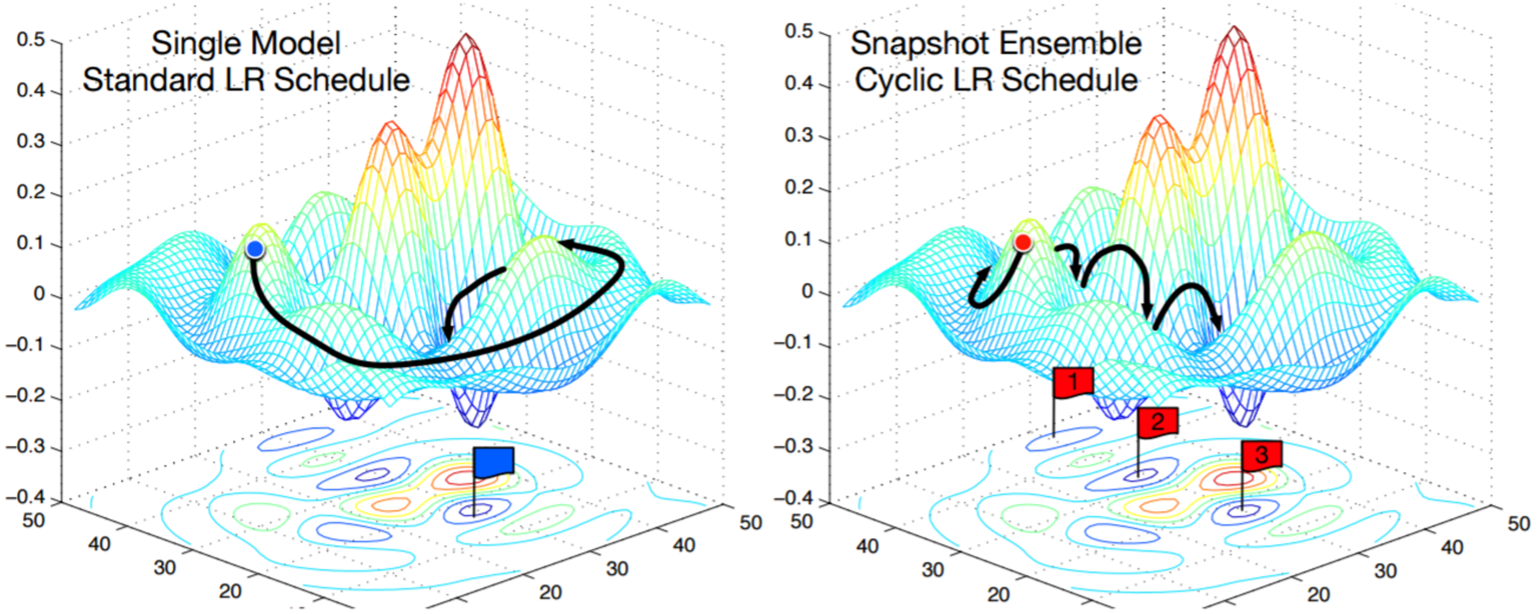
1. Train multiple independent models
2. At test time average their results

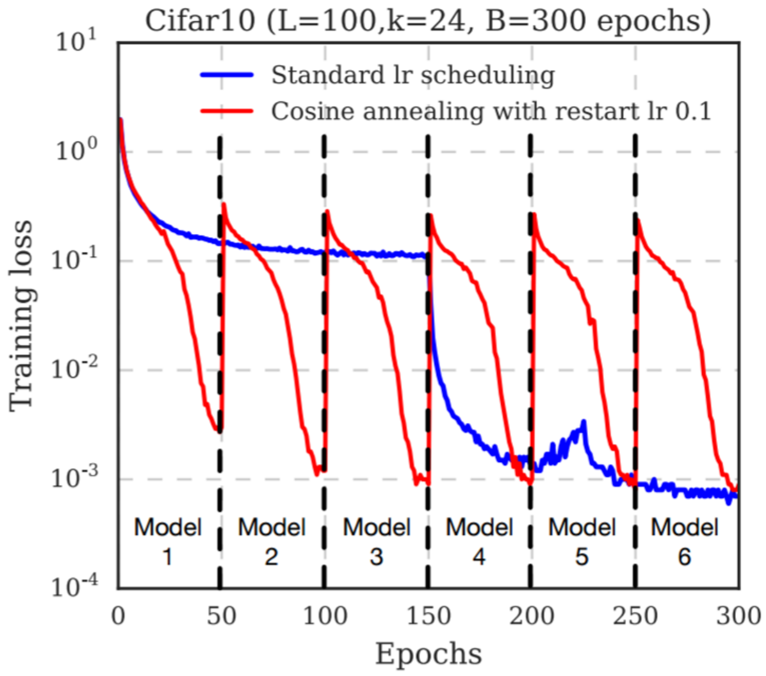
   (Take average of predicted probability distributions, then choose argmax)

Enjoy 2% extra performance

Model Ensembles: Tips and Tricks

Instead of training independent models, use multiple snapshots of a single model during training!



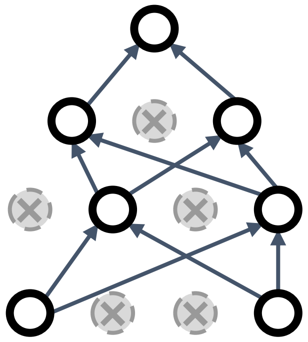


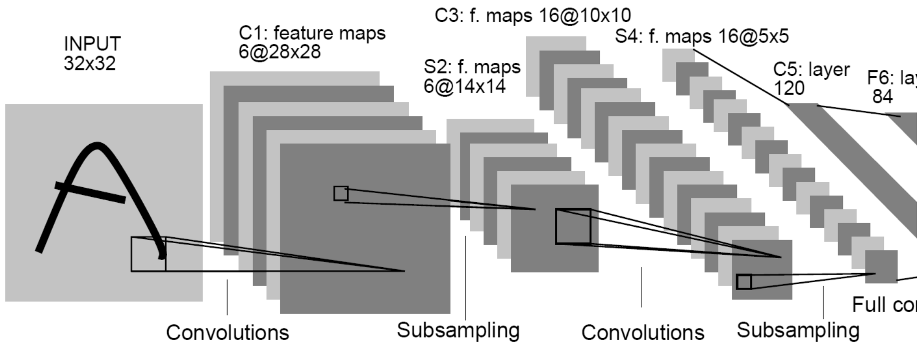
Cyclic learning rate schedules can make this work even better!

Loshchilov and Hutter, "SGDR: Stochastic gradient descent with restarts", arXiv 2016
Huang et al, "Snapshot ensembles: train 1, get M for free", ICLR 2017

Convolutional neural networks++

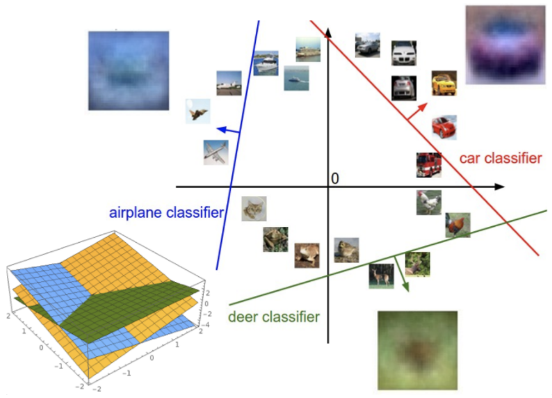
- Training and optimization
- More regularization (dropout, …)
- Convolutional neural networks
- Pooling
- Batch normalization



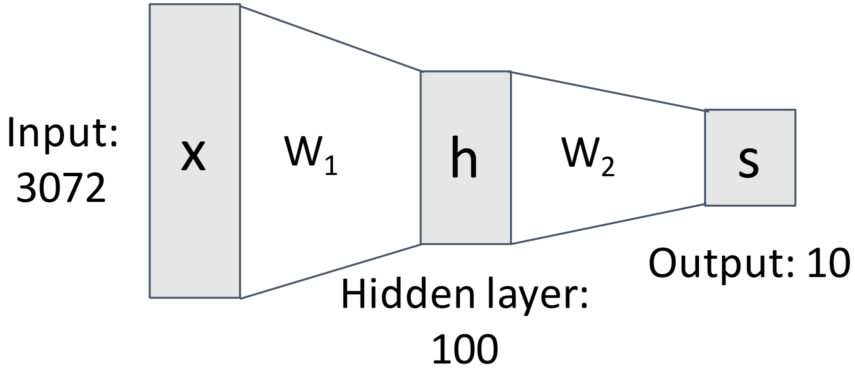


Lecture 7: Convolutional Networks

f(x,W) = Wx



$$f = W_2 \max(0, W_1 x)$$



Stretch pixels into column

56	231
24	2

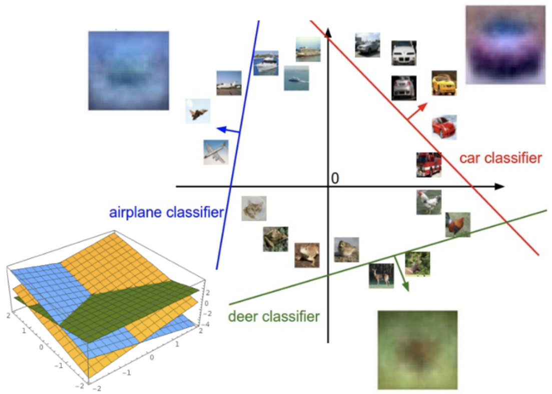
Input image (2, 2)

Problem: So far our classifiers don't respect the spatial structure of images!

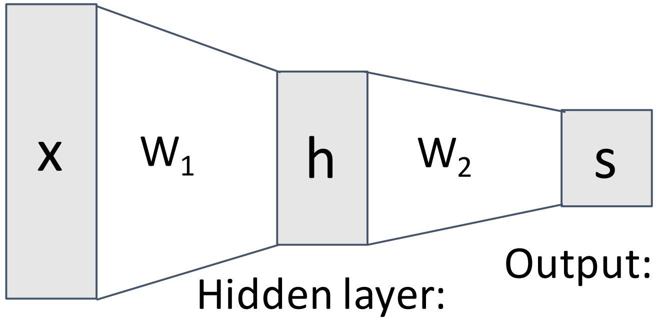
56
231
24
2

(4,)

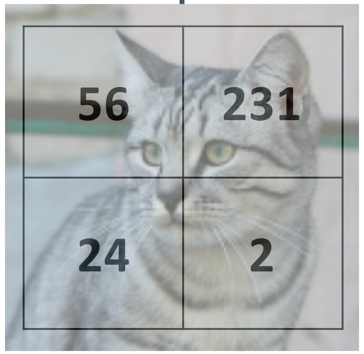
f(x,W) = Wx



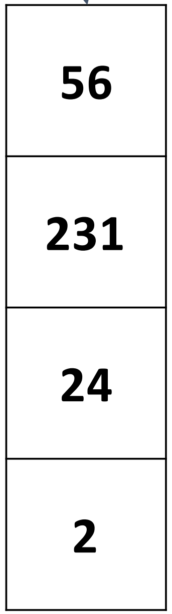
$$f = W_2 \max(0, W_1 x)$$



Stretch pixels into column



Input image
(2, 2)



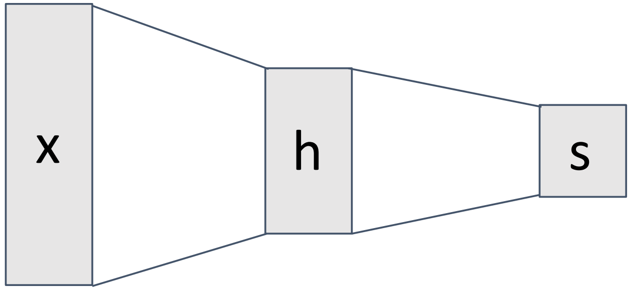
(4,)

Problem: So far our classifiers don't respect the spatial structure of images!

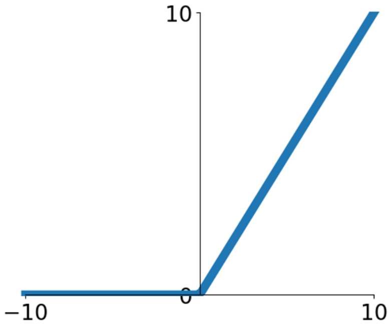
Solution: Define new computational nodes that operate on images!

Components of a Fully-Connected Network

Fully-Connected Layers

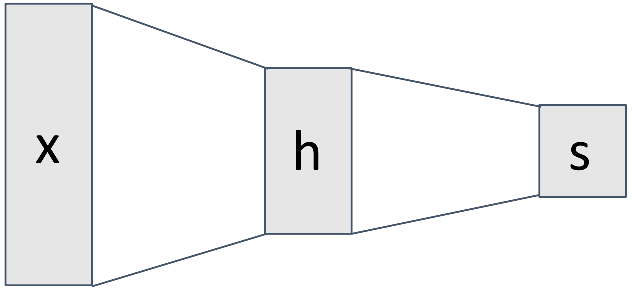


Activation Function

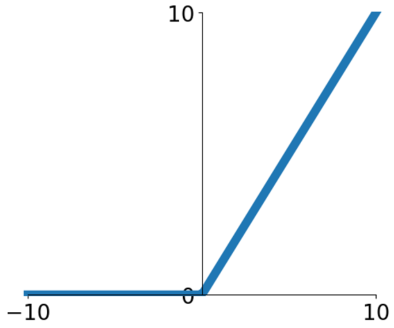


Components of a Convolutional Network

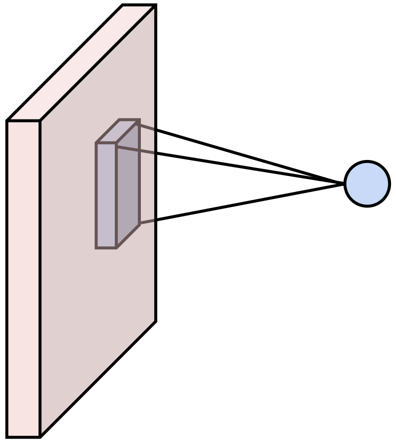
Fully-Connected Layers



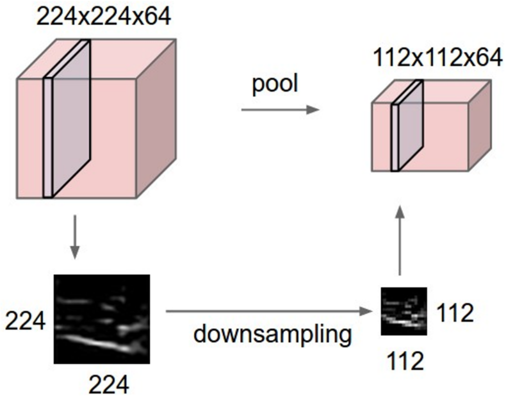
Activation Function



Convolution Layers



Pooling Layers

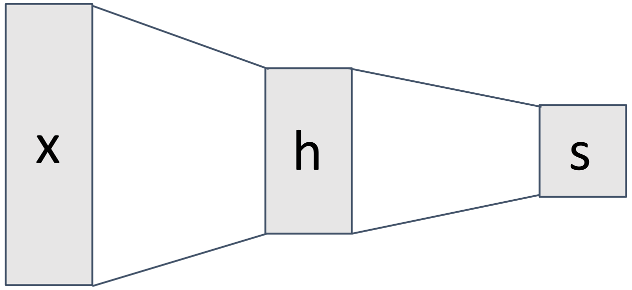


Normalization

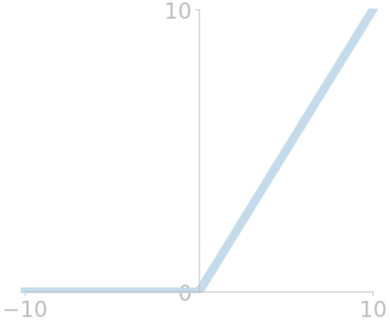
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Components of a Convolutional Network

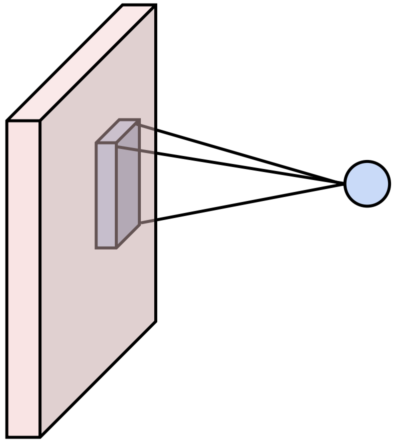
Fully-Connected Layers



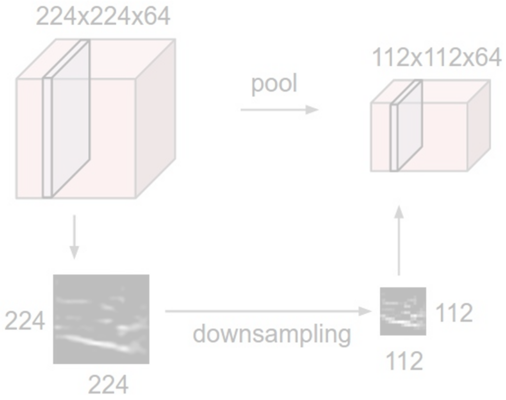
Activation Function



Convolution Layers



Pooling Layers

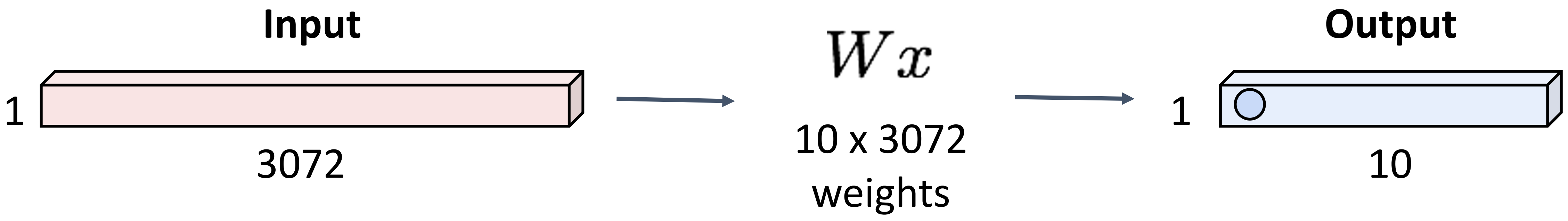


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

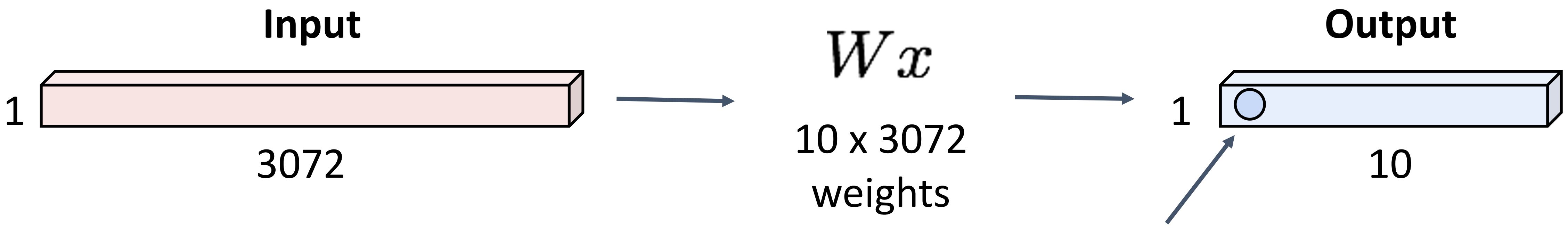
Fully-Connected Layer

32x32x3 image -> stretch to 3072 x 1



Fully-Connected Layer

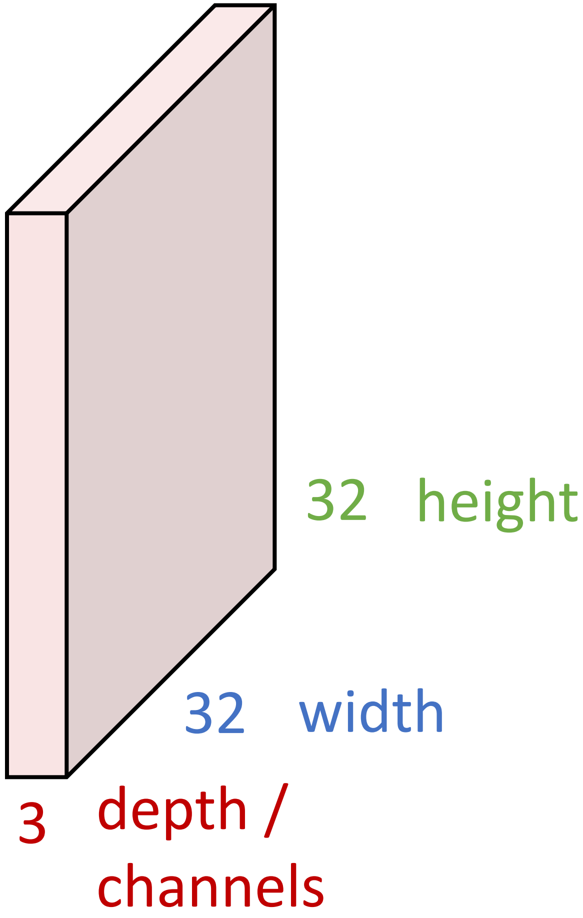
32x32x3 image -> stretch to 3072 x 1



1 number:
the result of taking a dot product between a row of W and the input (a 3072-dimensional dot product)

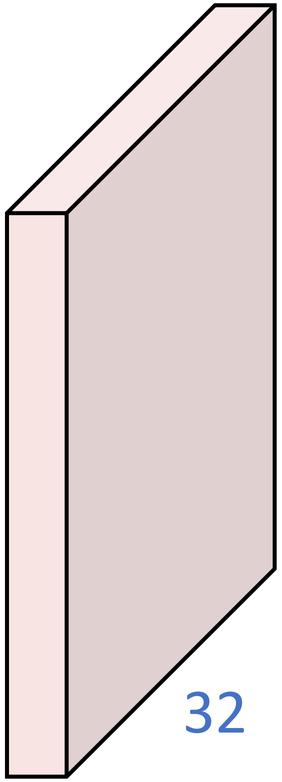
Convolution Layer

3x32x32 image: preserve spatial structure



Convolution Layer

3x32x32 image

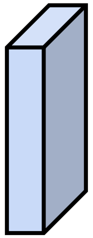


32 height

32 width

3 depth / channels

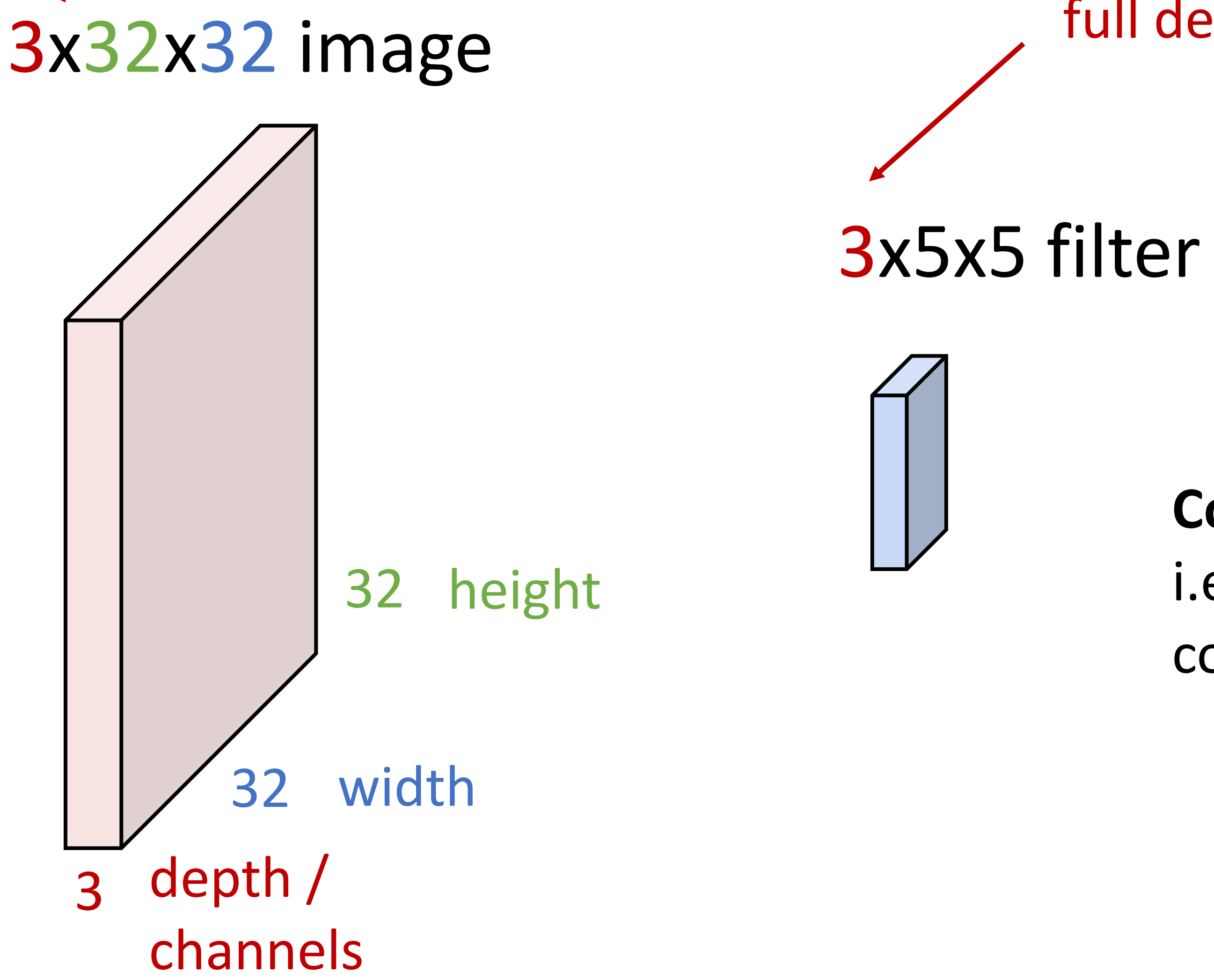
3x5x5 filter



Convolve the filter with the image i.e. “slide over the image spatially, computing dot products”

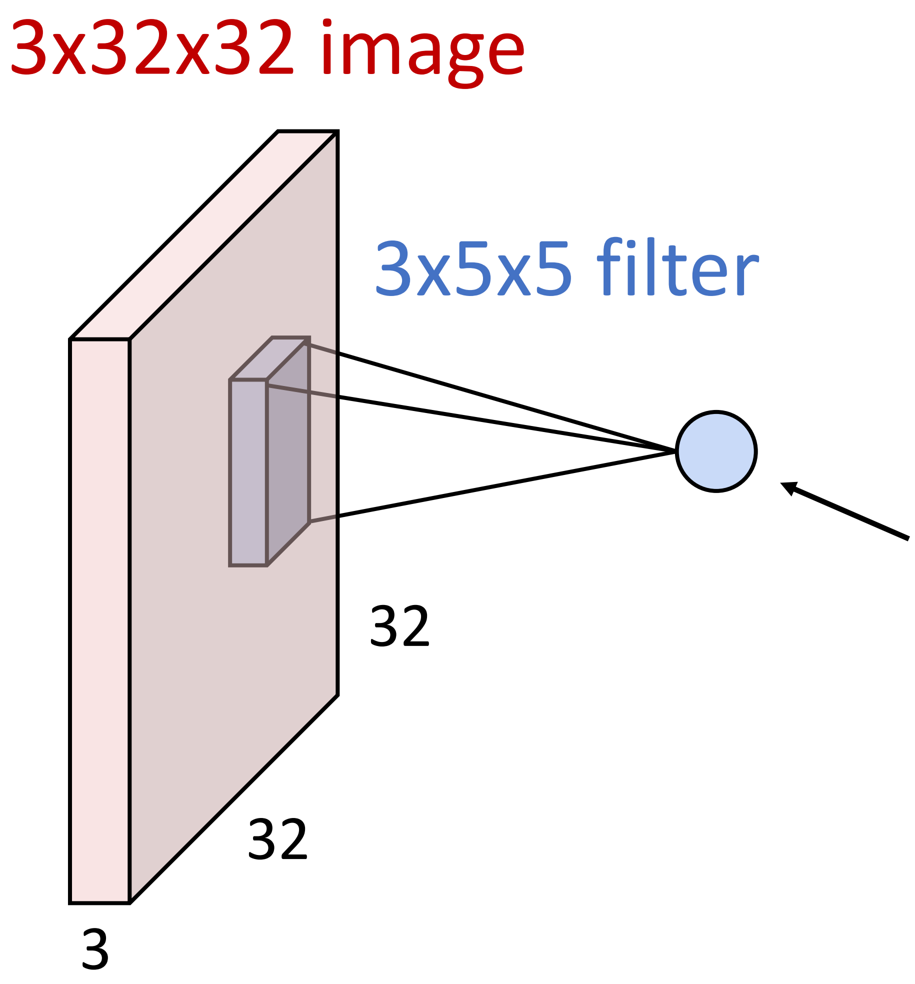
Convolution Layer

Filters (almost) always extend the full depth of the input volume



Convolve the filter with the image
i.e. “slide over the image spatially, computing dot products”

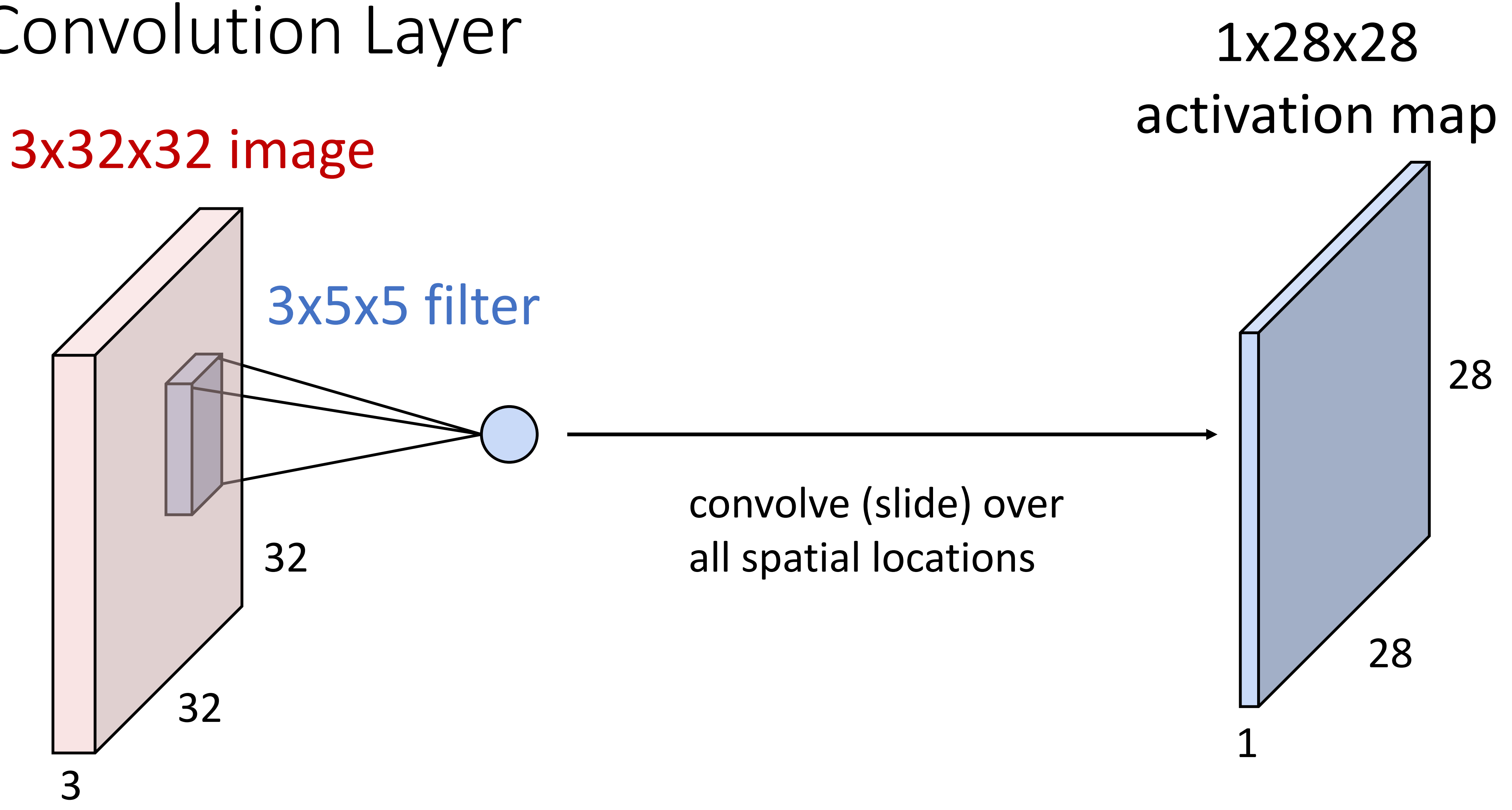
Convolution Layer



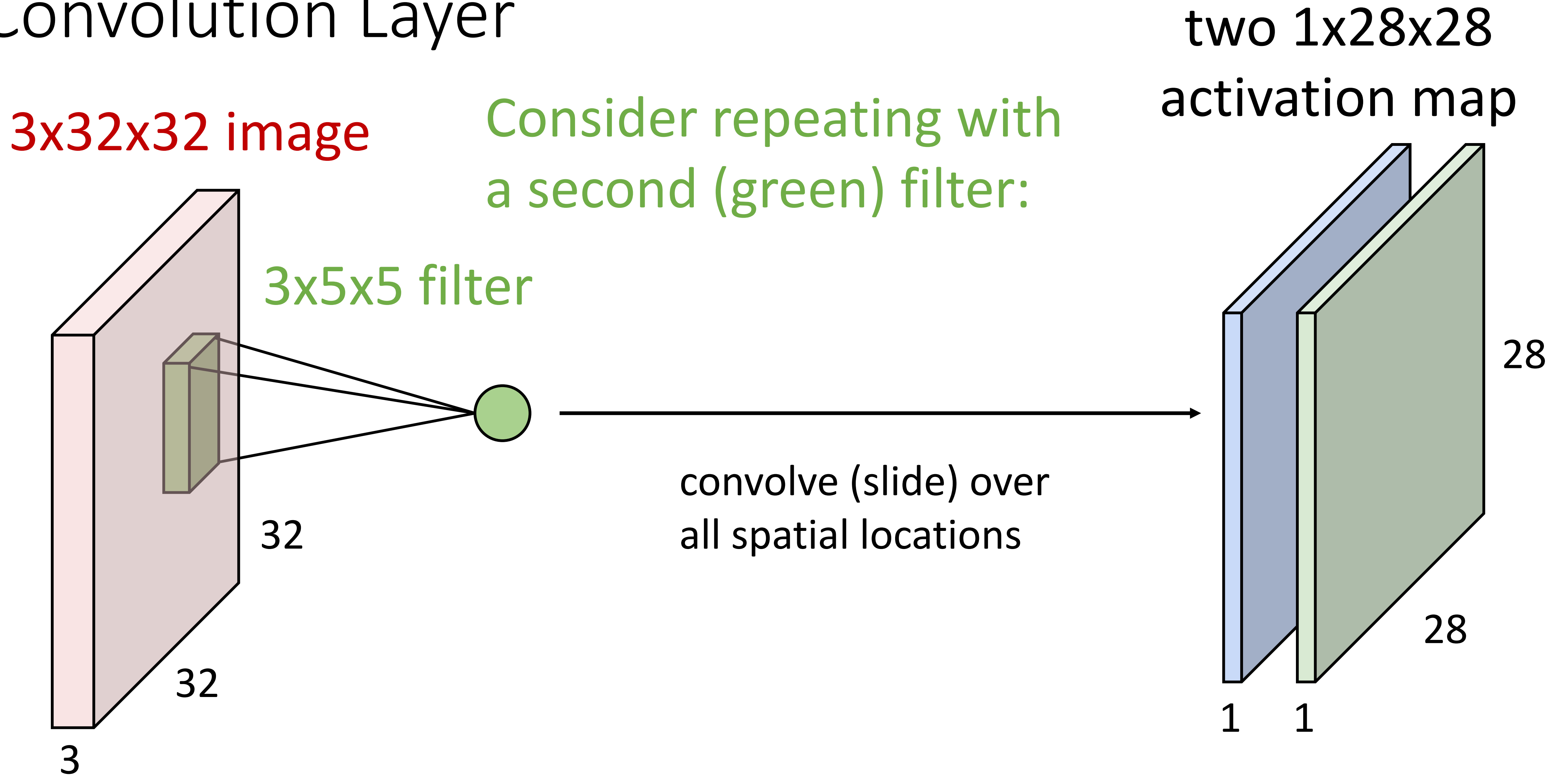
1 number:
the result of taking a dot product between the filter and a small 3x5x5 chunk of the image
(i.e. 3*5*5 = 75-dimensional dot product + bias)

$$w^T x + b$$

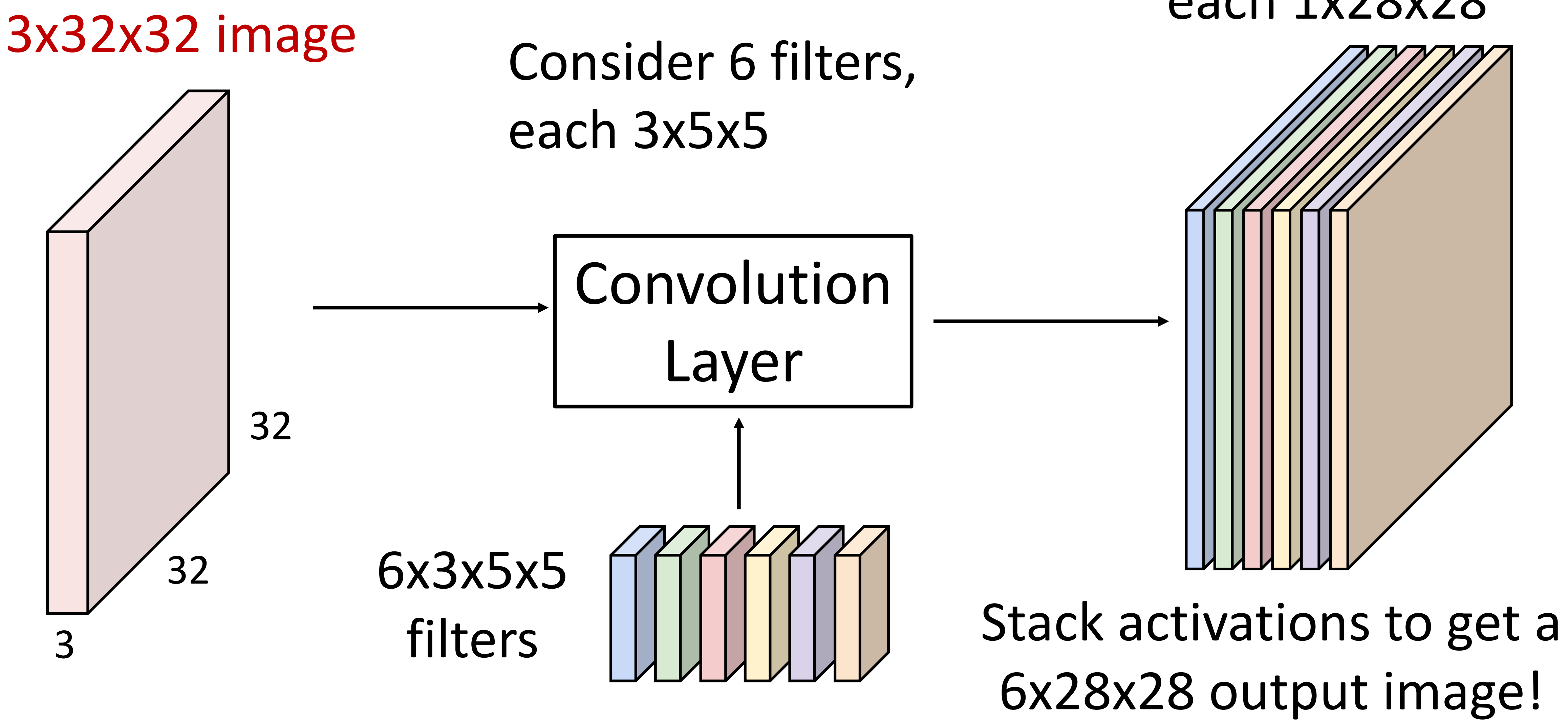
Convolution Layer



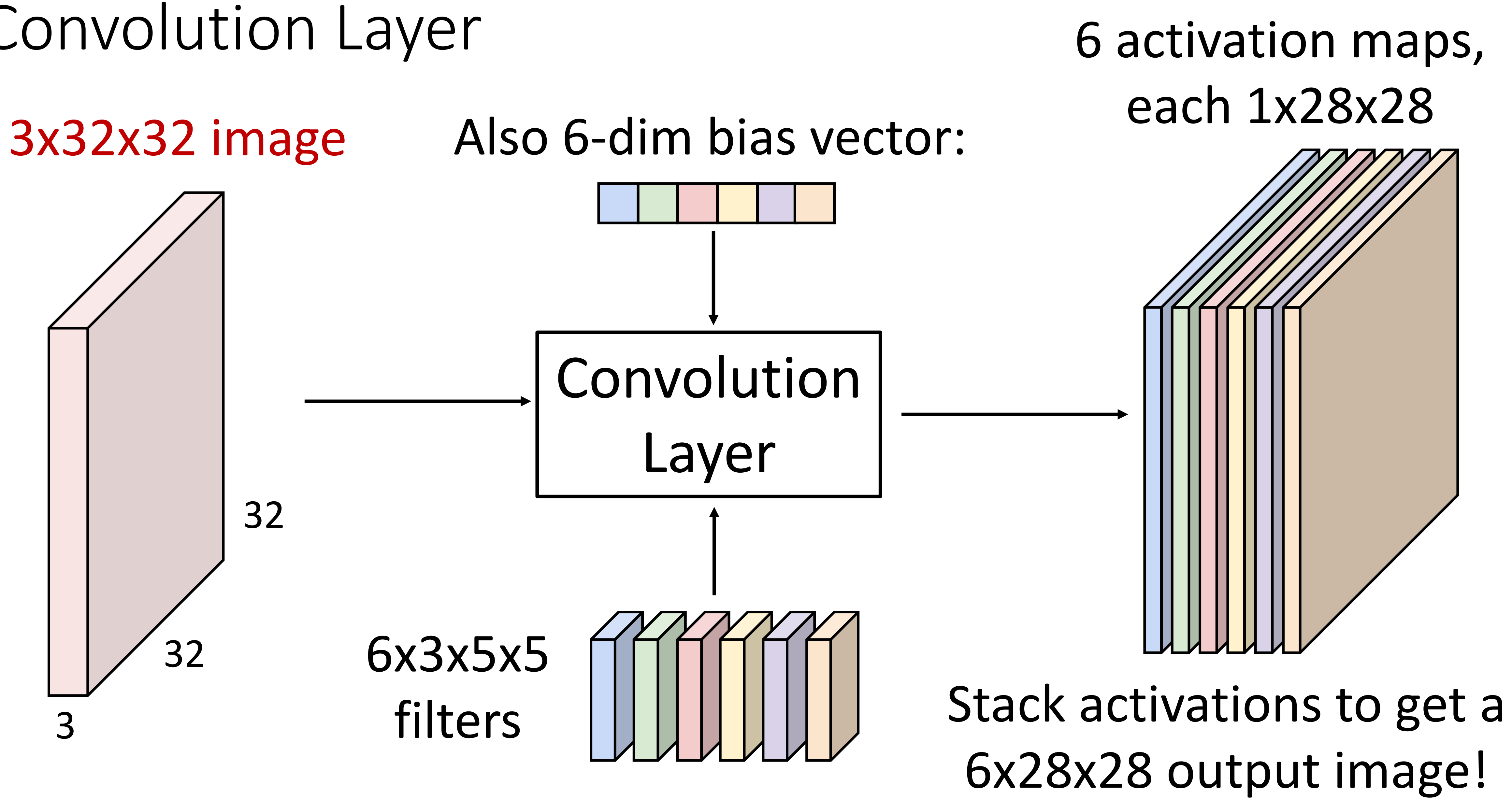
Convolution Layer



Convolution Layer



Convolution Layer

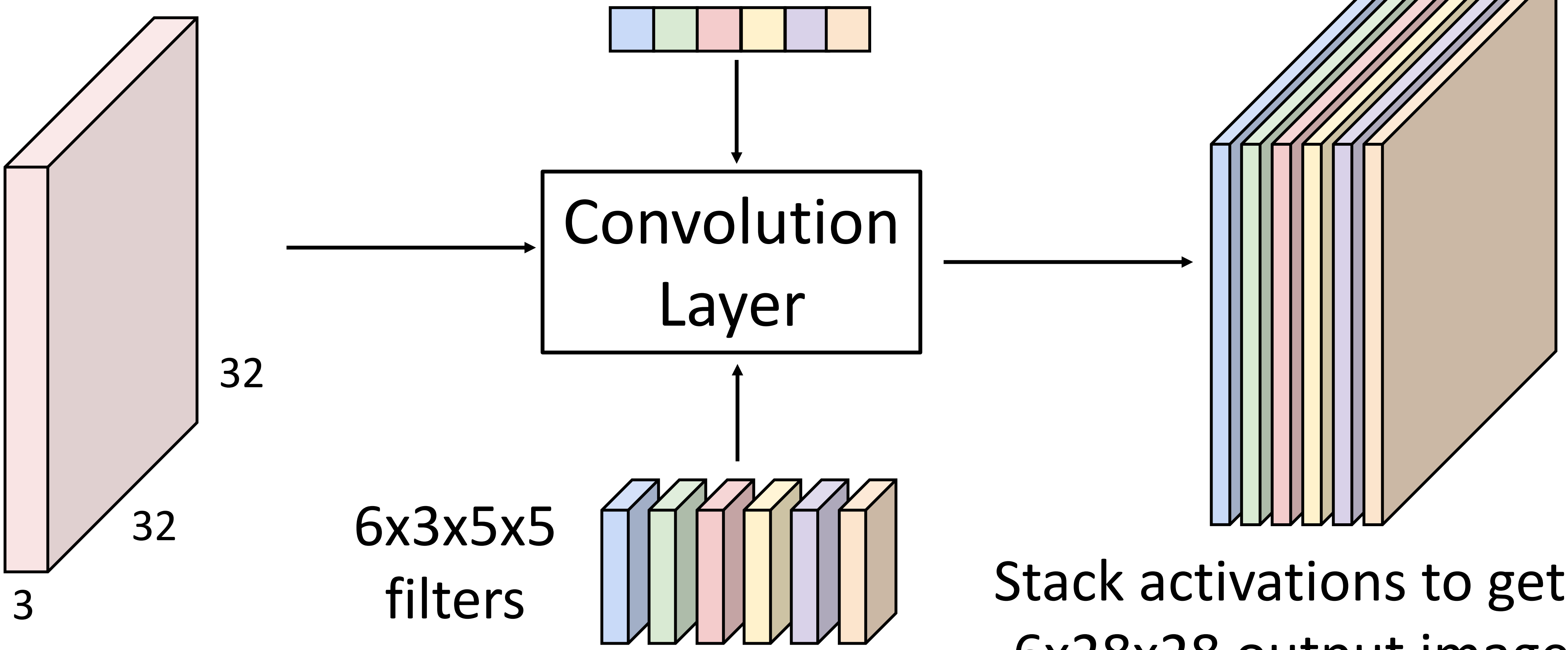


Convolution Layer

28x28 grid, at each point a 6-dim vector

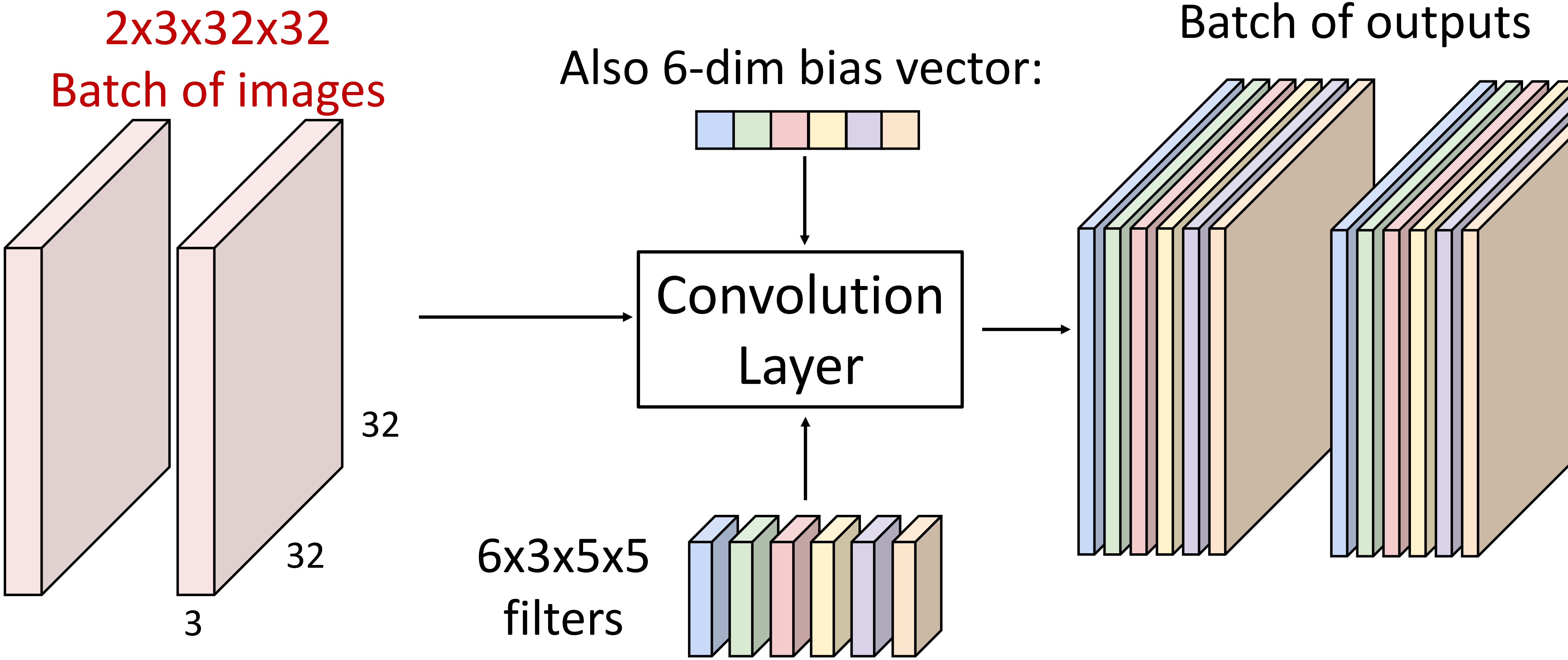
3x32x32 image

Also 6-dim bias vector:

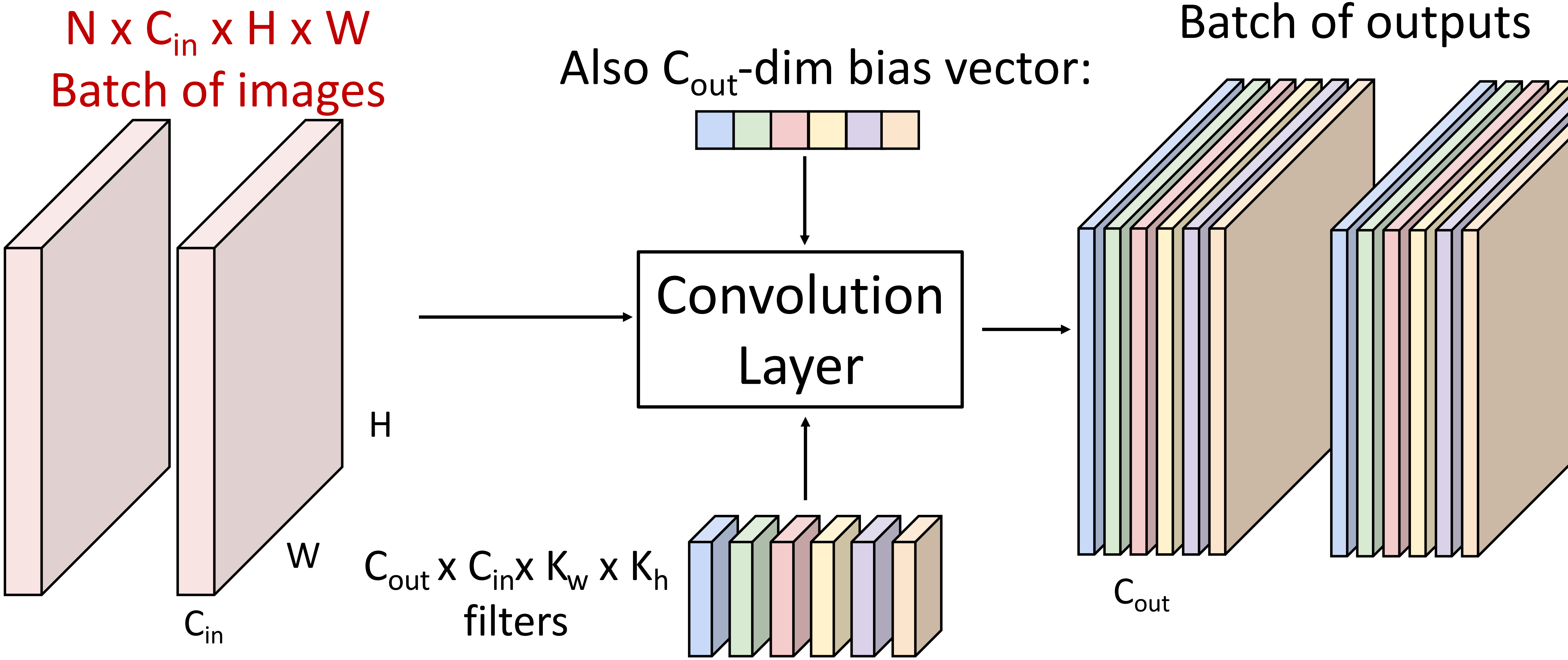


Stack activations to get a 6x28x28 output image!

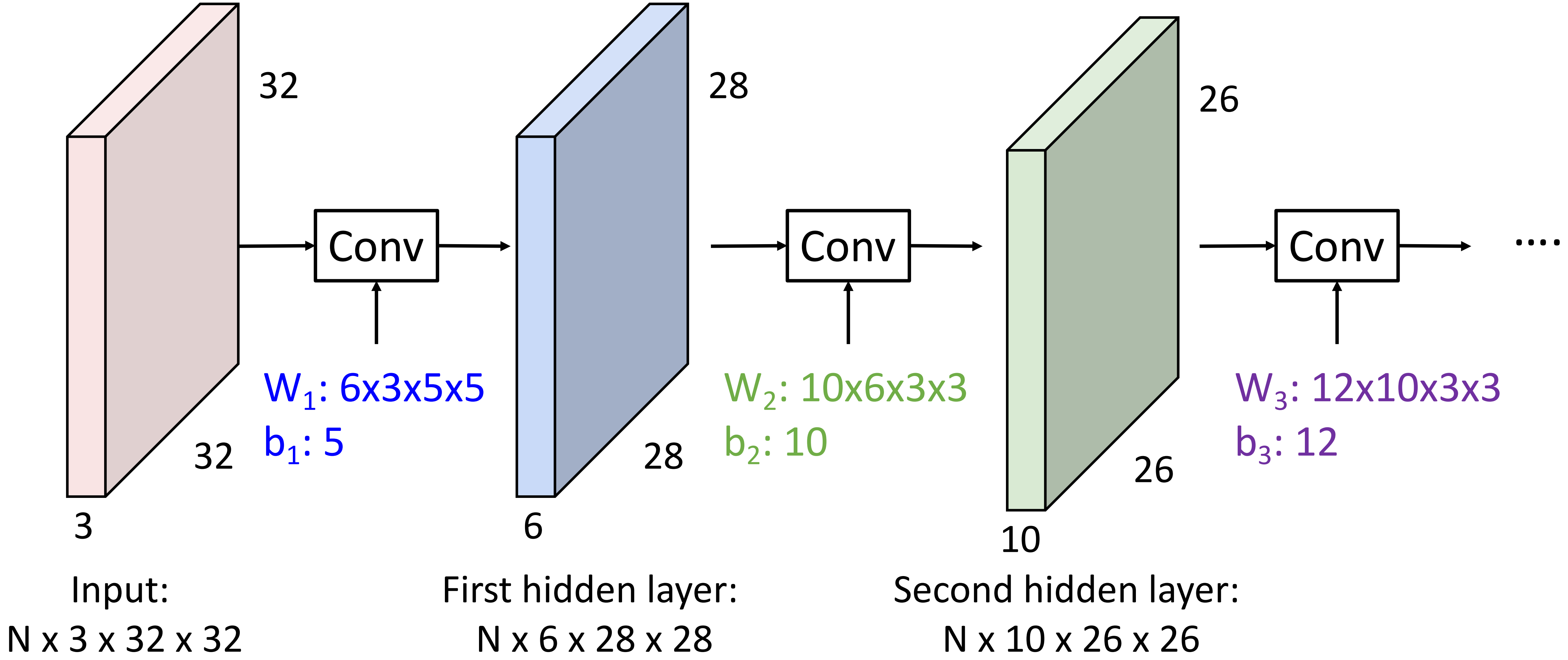
Convolution Layer



Convolution Layer



Stacking Convolutions



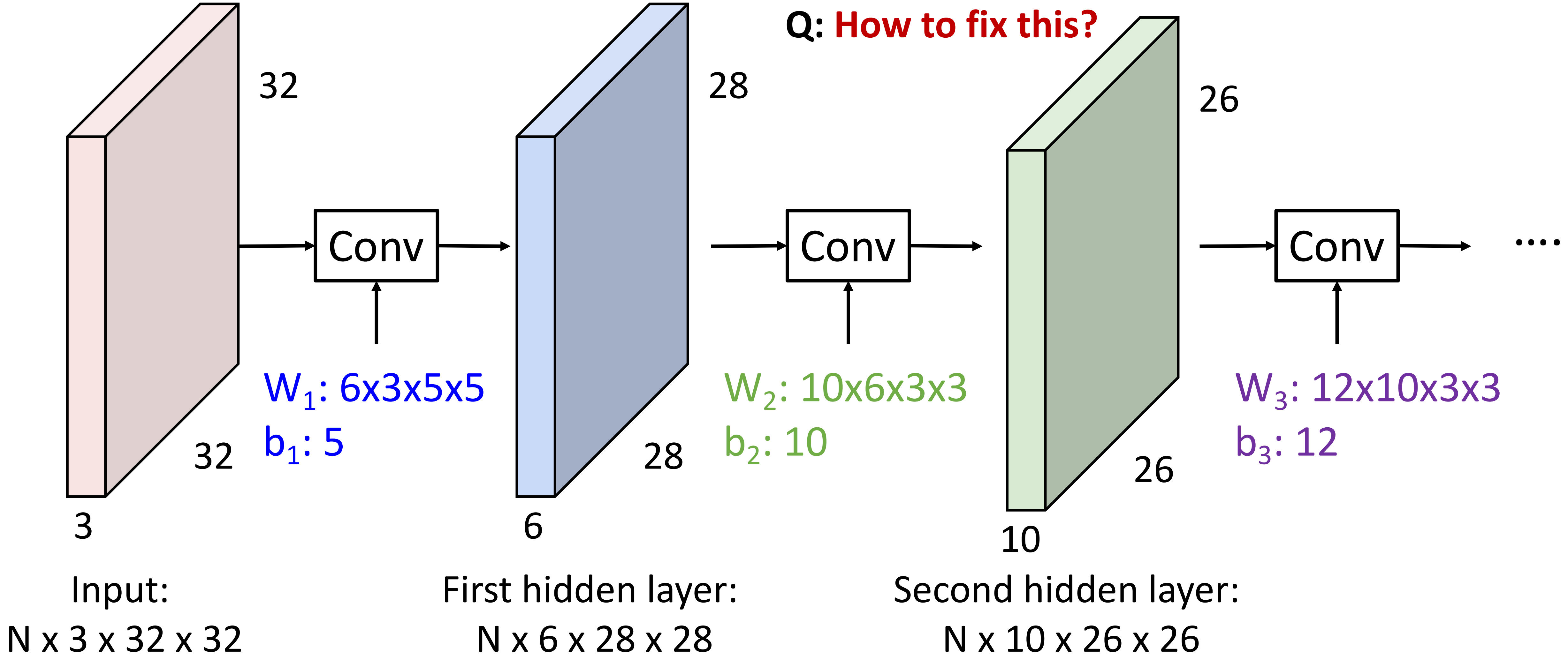
Stacking Convolutions

Q: What happens if we stack two convolution layers?

(Recall $y=W_2W_1x$ is a linear classifier)

A: We get another convolution!

Q: **How to fix this?**



Stacking Convolutions

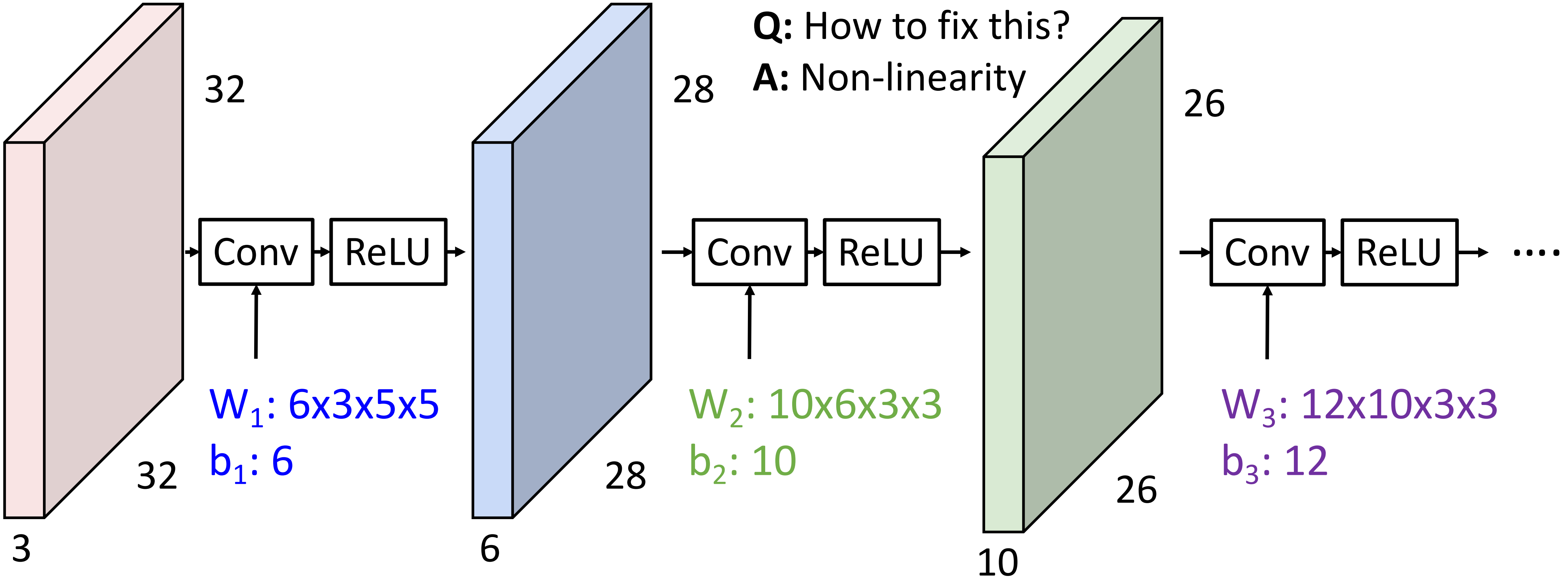
Q: What happens if we stack two convolution layers?

(Recall $y=W_2W_1x$ is a linear classifier)

A: We get another convolution!

Q: How to fix this?

A: Non-linearity

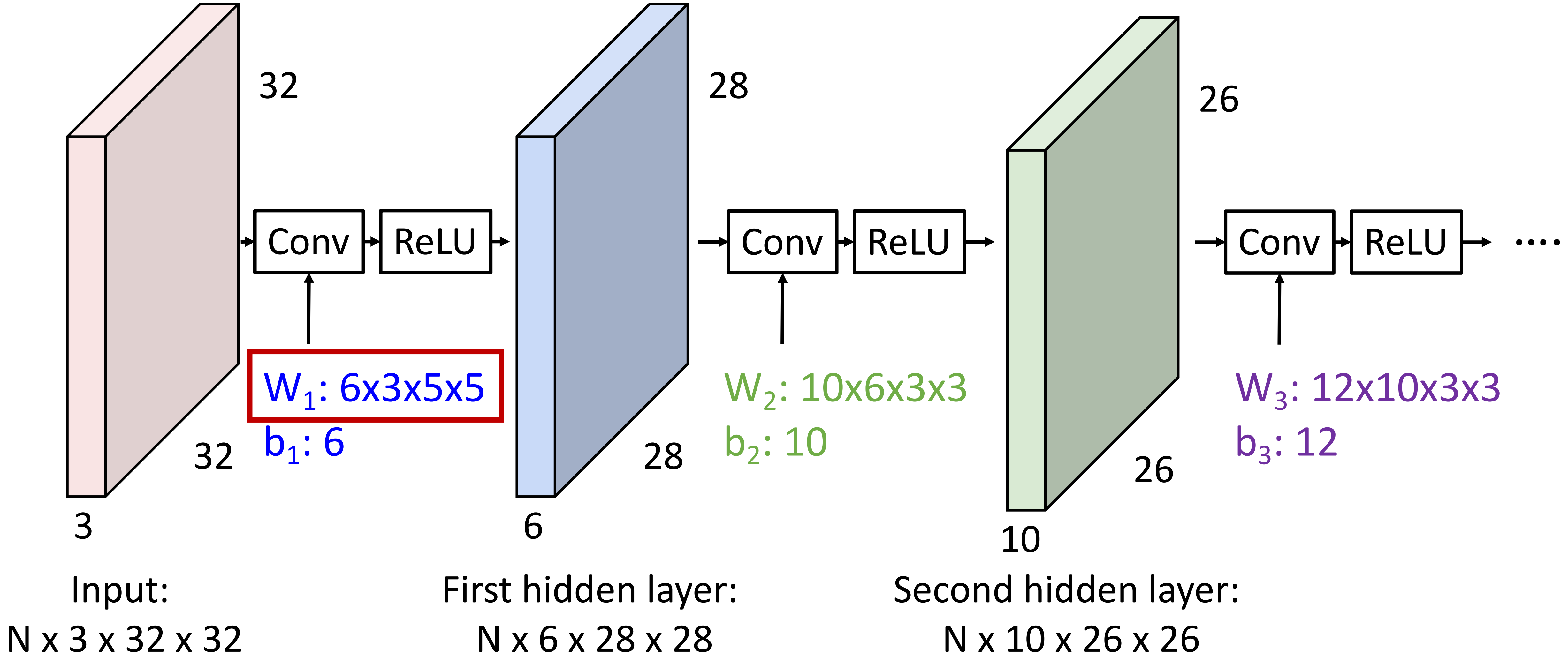


Input:
N x 3 x 32 x 32

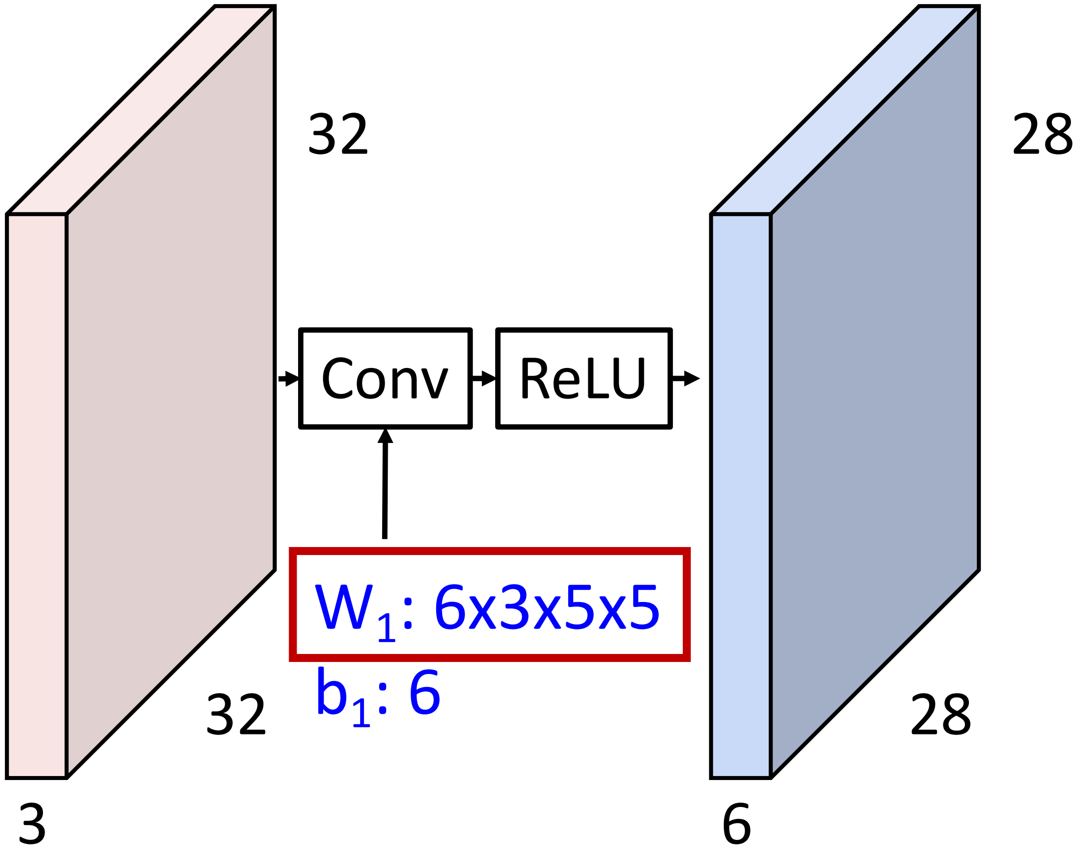
First hidden layer:
N x 6 x 28 x 28

Second hidden layer:
N x 10 x 26 x 26

What do convolutional filters learn?



What do convolutional filters learn?

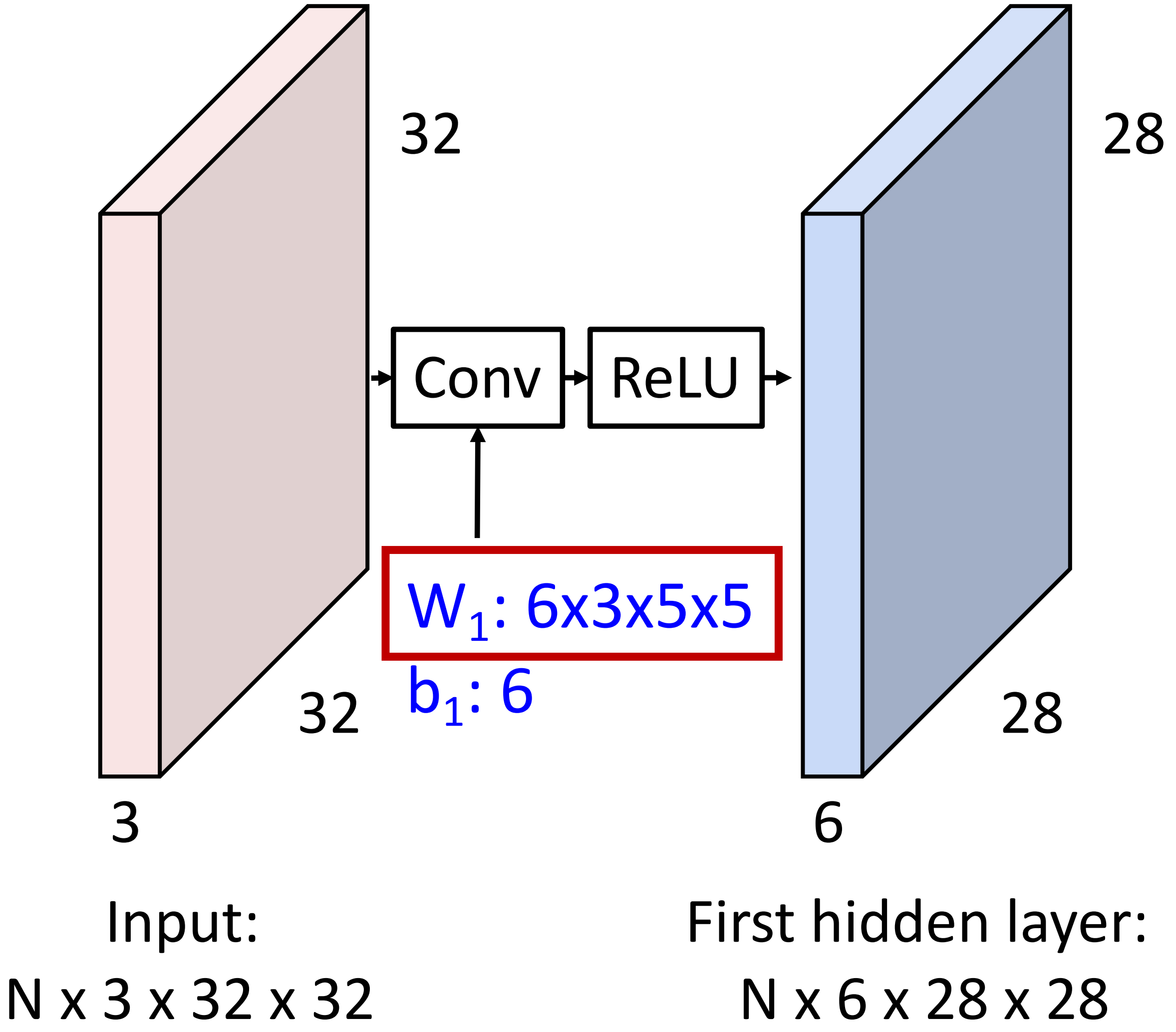


Input:
N x 3 x 32 x 32

First hidden layer:
N x 6 x 28 x 28

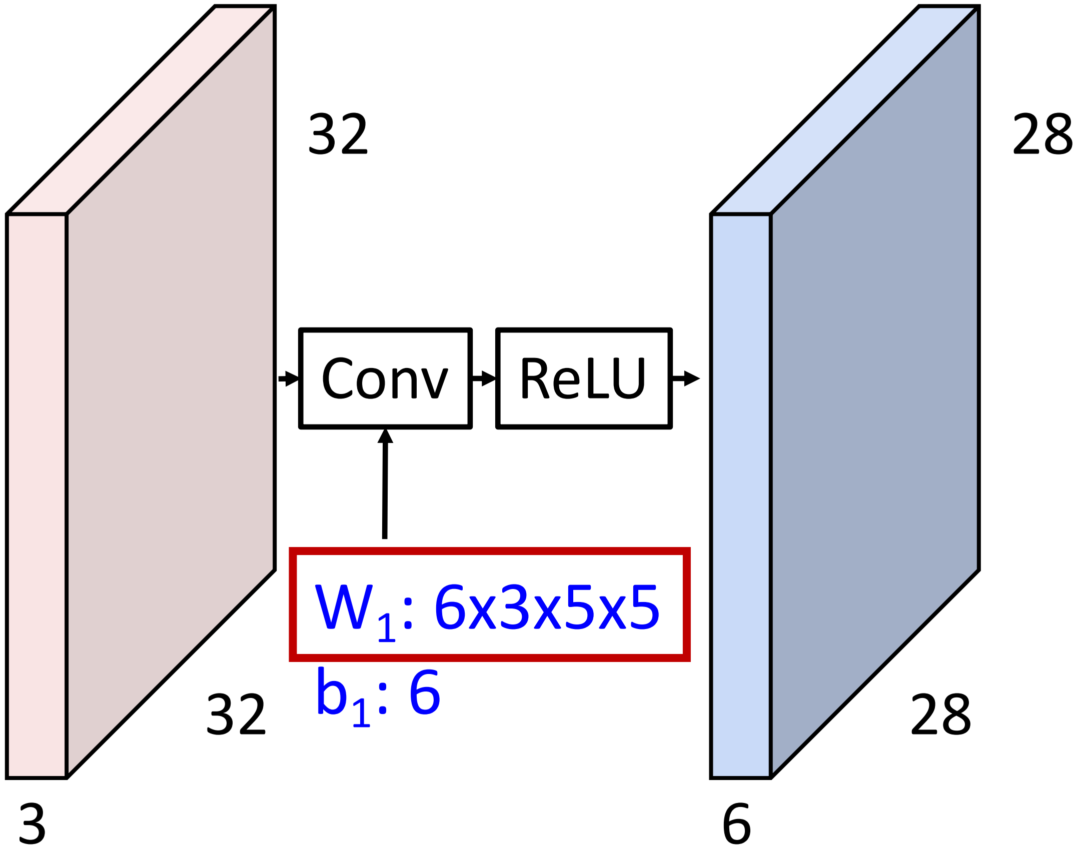
Linear classifier: One template per class

What do convolutional filters learn?



MLP: Bank of whole-image templates

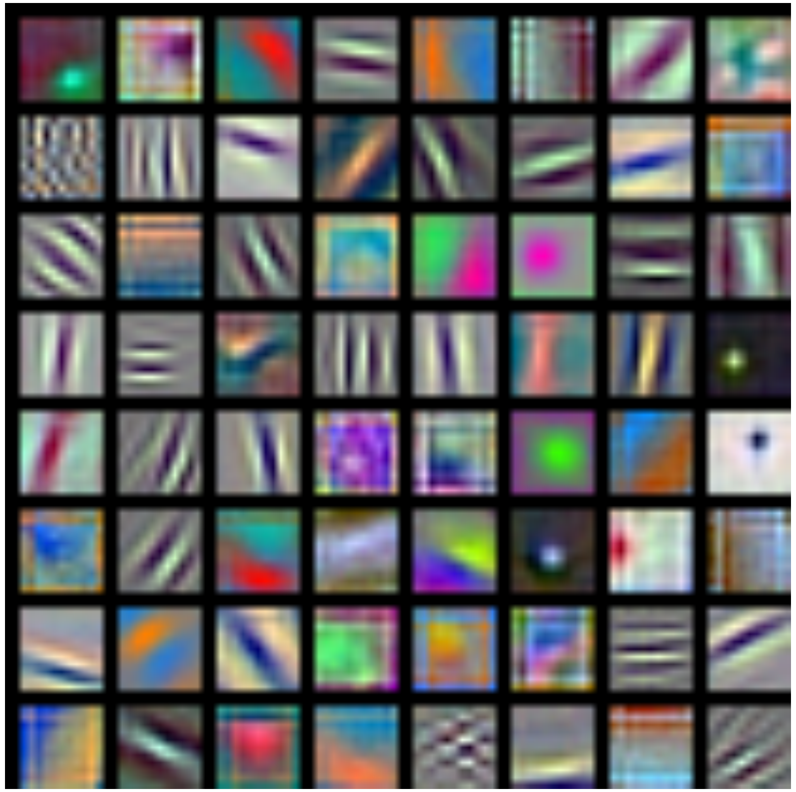
What do convolutional filters learn?



Input:
N x 3 x 32 x 32

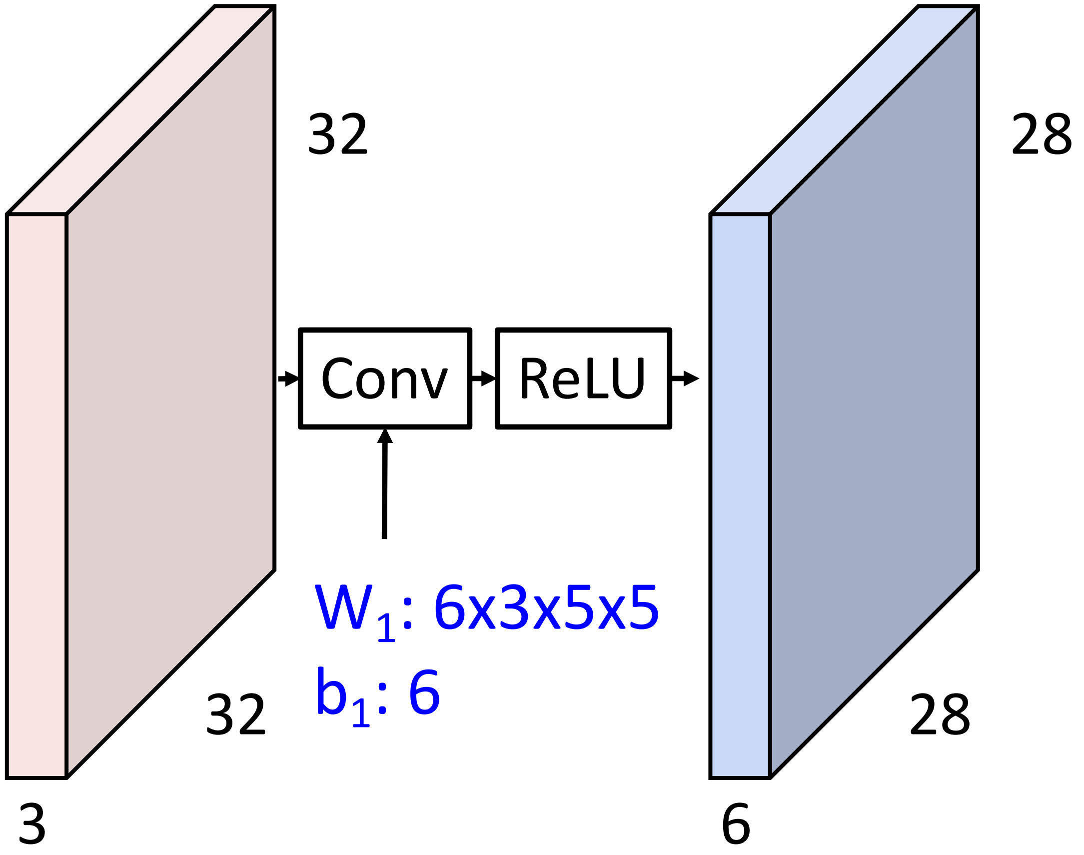
First hidden layer:
N x 6 x 28 x 28

First-layer conv filters: local image templates
(Often learns oriented edges, opposing colors)



AlexNet: 64 filters, each 3x11x11

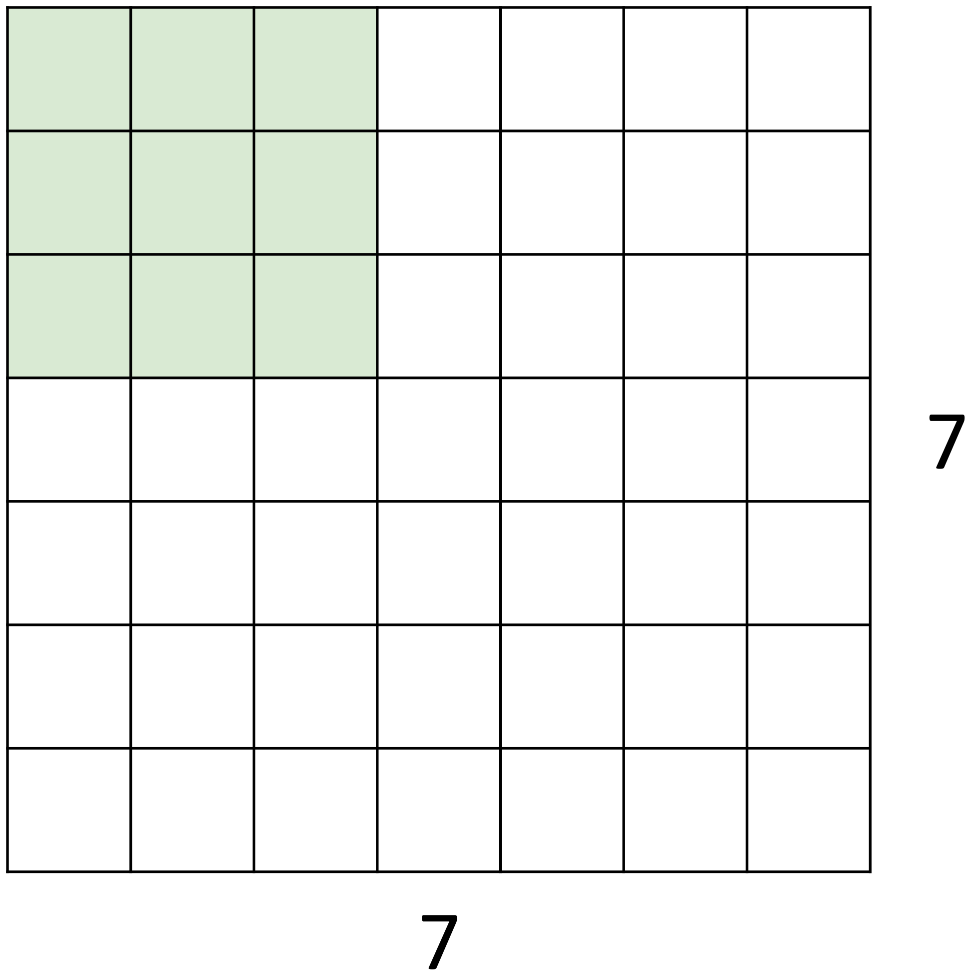
A closer look at spatial dimensions



Input:
N x 3 x 32 x 32

First hidden layer:
N x 6 x 28 x 28

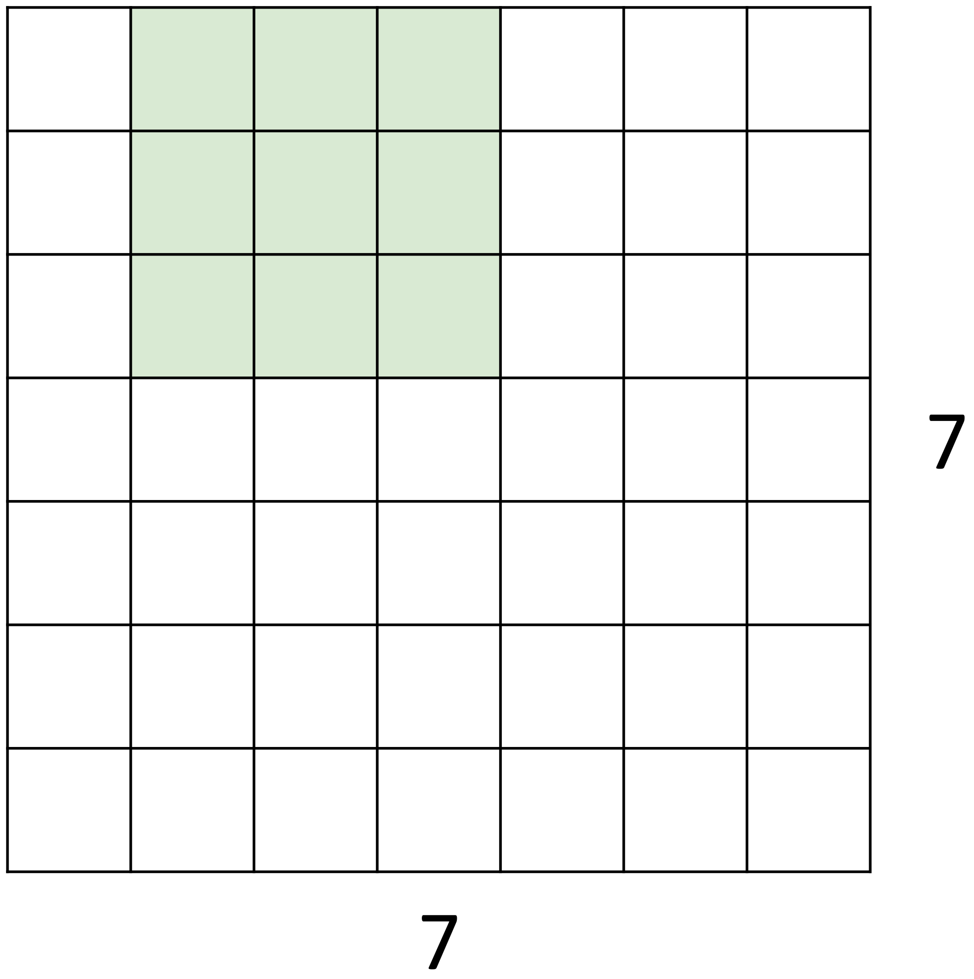
A closer look at spatial dimensions



Input: 7x7

Filter: 3x3

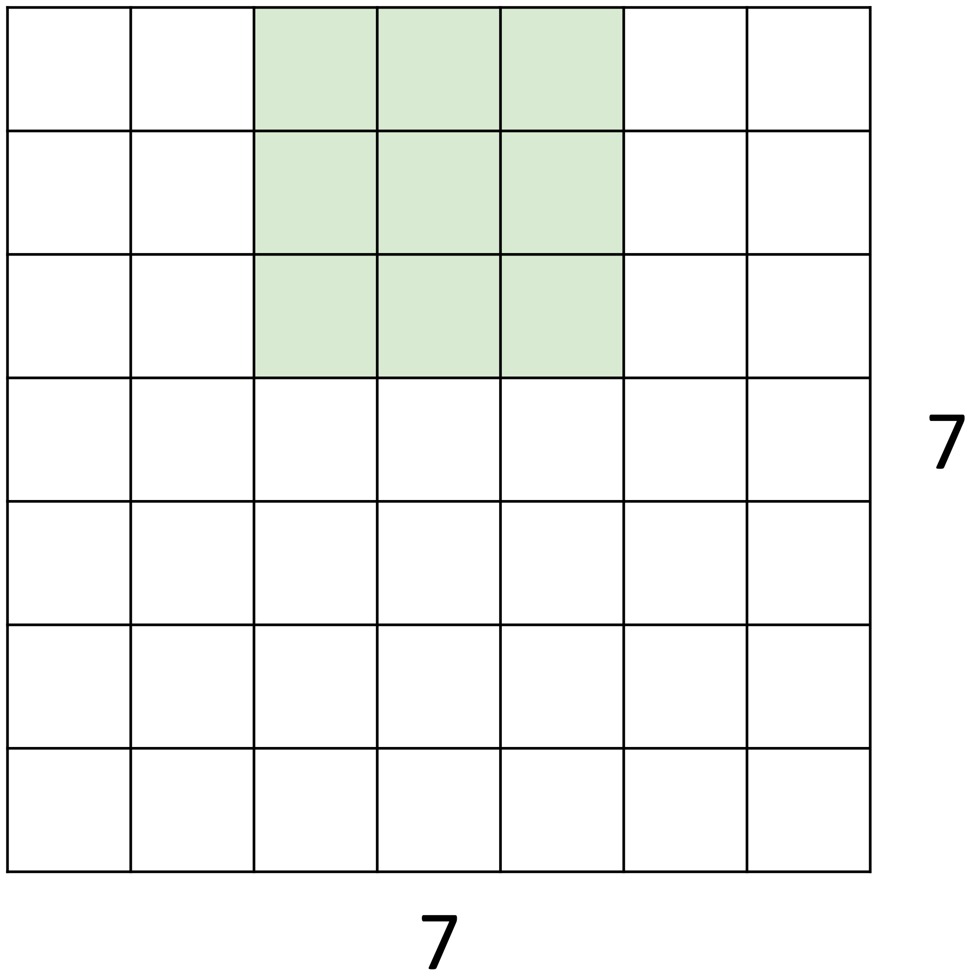
A closer look at spatial dimensions



Input: 7x7

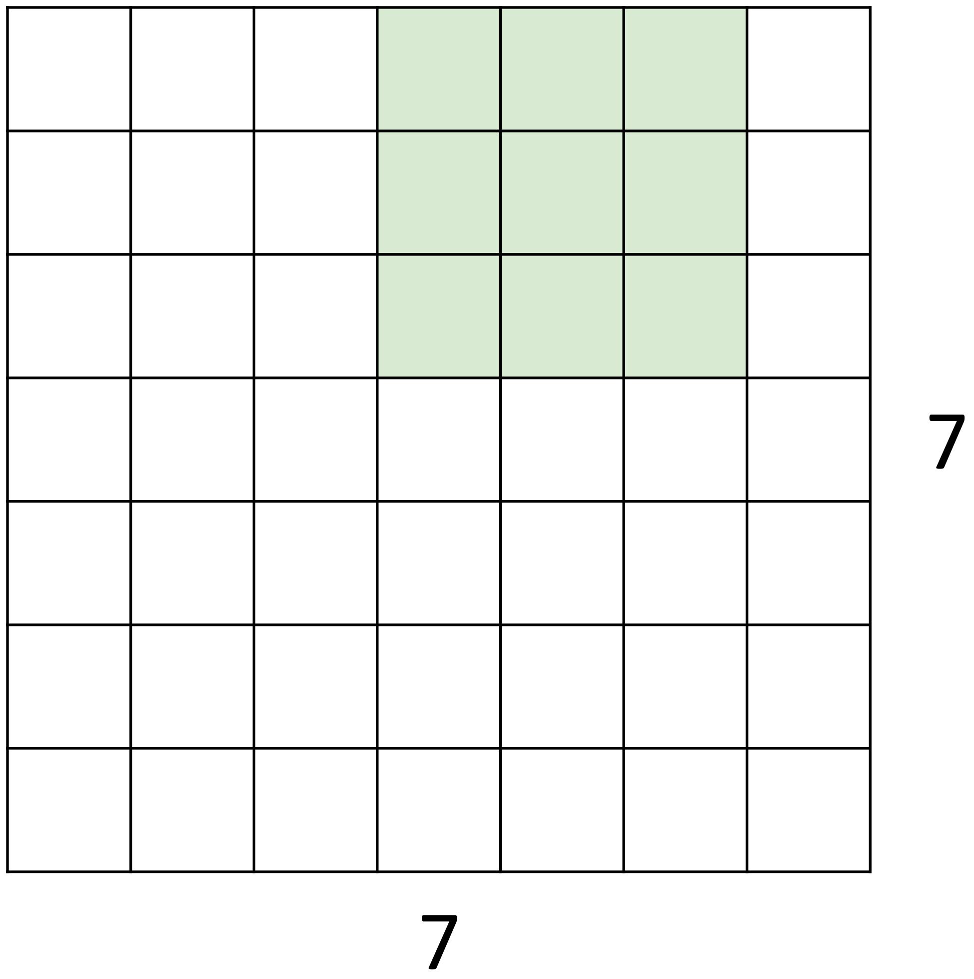
Filter: 3x3

A closer look at spatial dimensions



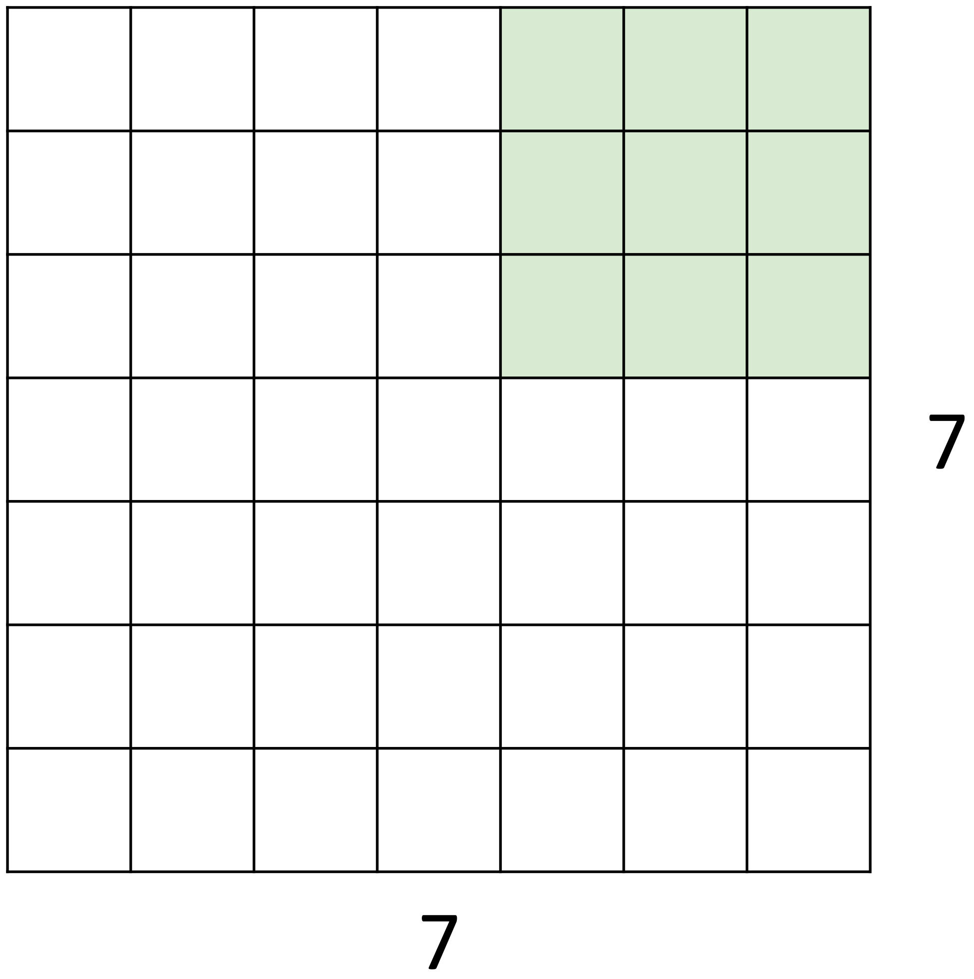
Input: 7x7
Filter: 3x3

A closer look at spatial dimensions



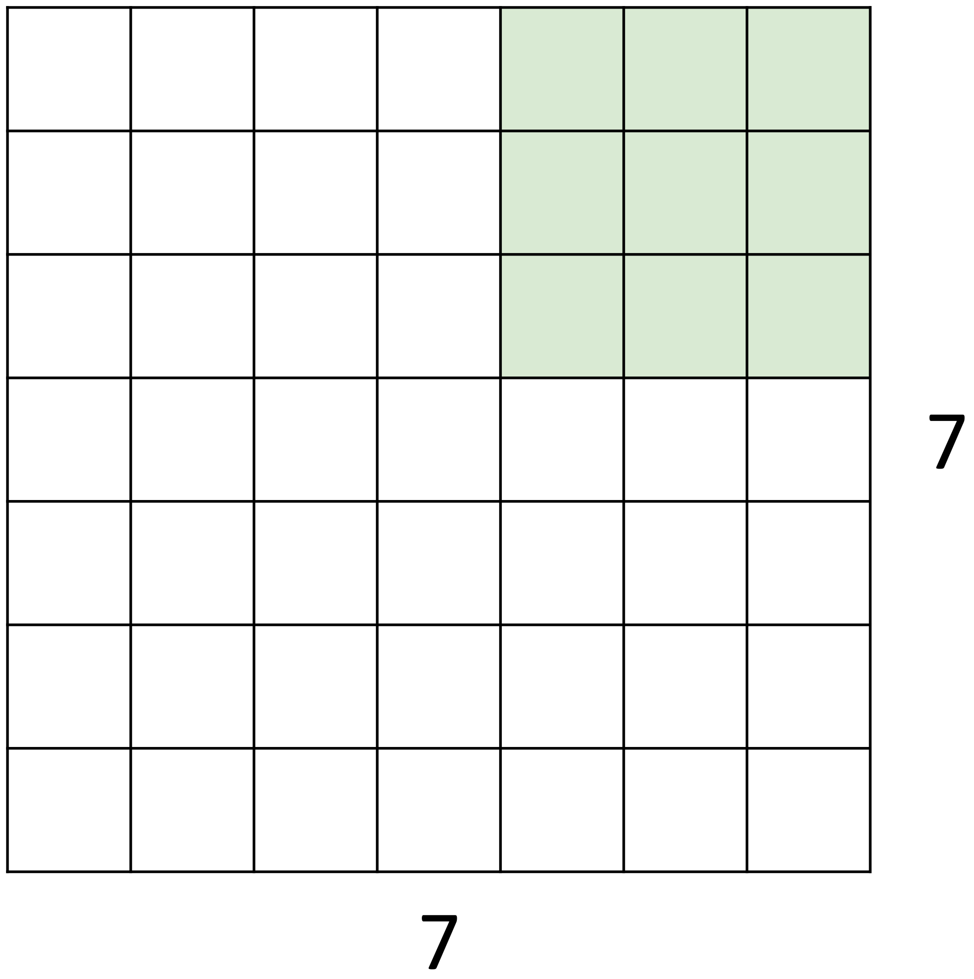
Input: 7x7
Filter: 3x3

A closer look at spatial dimensions



Input: 7x7
Filter: 3x3
Output: 5x5

A closer look at spatial dimensions



Input: 7x7
Filter: 3x3
Output: 5x5

In general:
Input: W
Filter: K
Output: W – K + 1

Problem: Feature maps "shrink" with each layer!

A closer look at spatial dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

Input: 7x7
Filter: 3x3
Output: 5x5

In general:
Input: W
Filter: K
Output: W – K + 1

Problem: Feature maps "shrink" with each layer!

Solution: **padding**
Add zeros around the input

A closer look at spatial dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

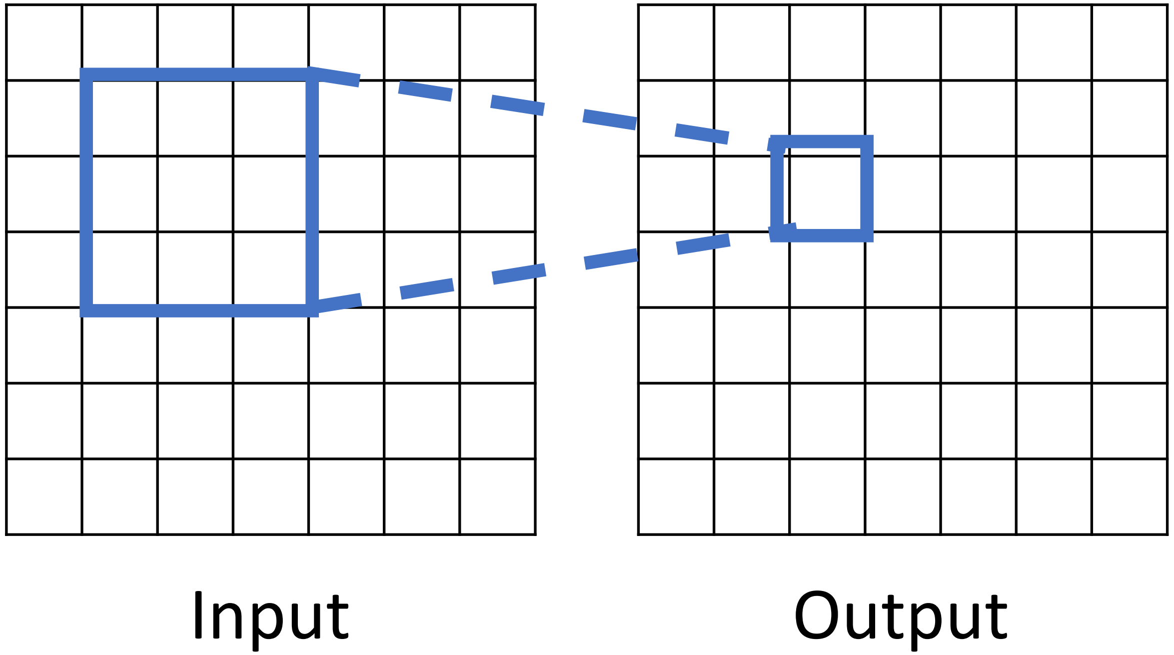
Input: 7x7
Filter: 3x3
Output: 5x5

In general:
Input: W
Filter: K
Padding: P
Output: W – K + 1 + 2P

Very common:
Set P = (K – 1) / 2 to make output have same size as input!

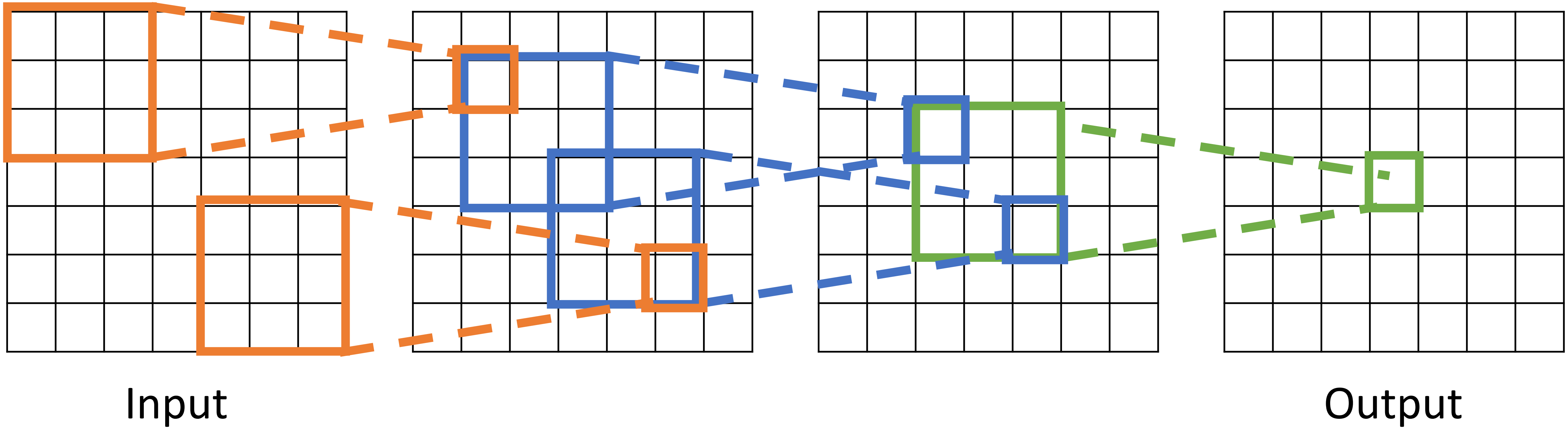
Receptive Fields

For convolution with kernel size K, each element in the output depends on a K x K **receptive field** in the input



Receptive Fields

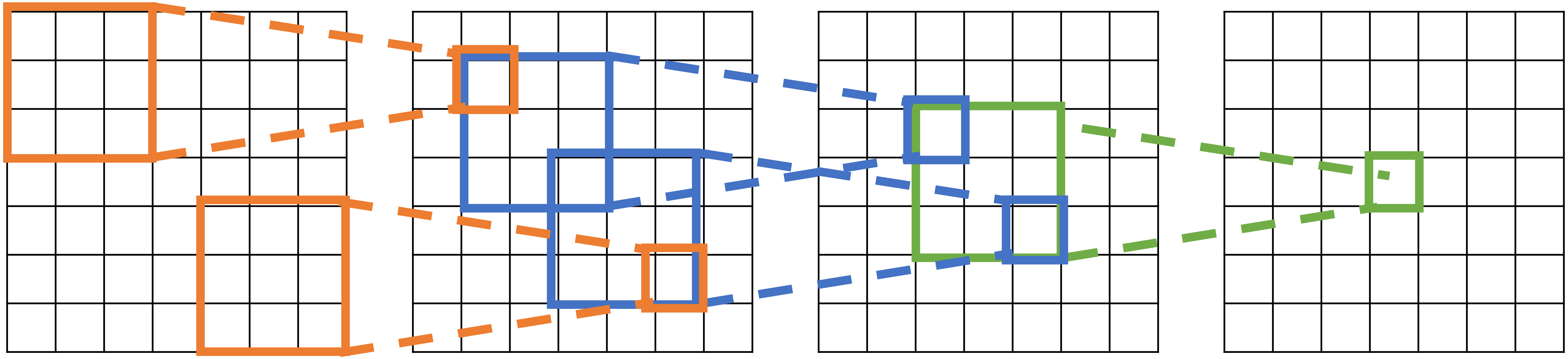
Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



Be careful – ”receptive field in the input” vs “receptive field in the previous layer”
Hopefully clear from context!

Receptive Fields

Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



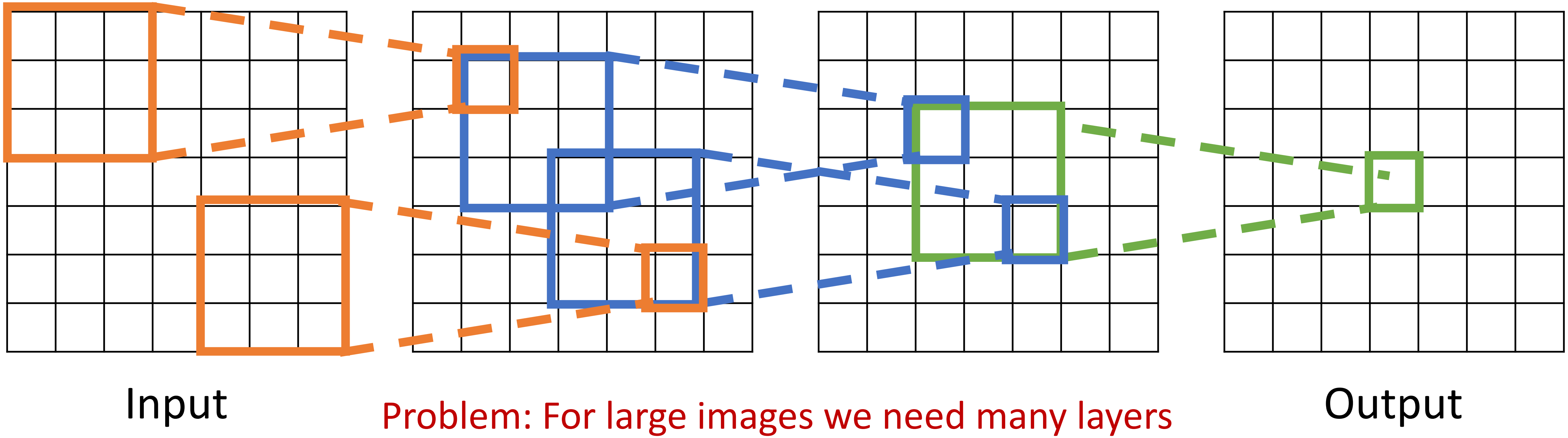
Input

Output

Problem: For large images we need many layers for each output to "see" the whole image image

Receptive Fields

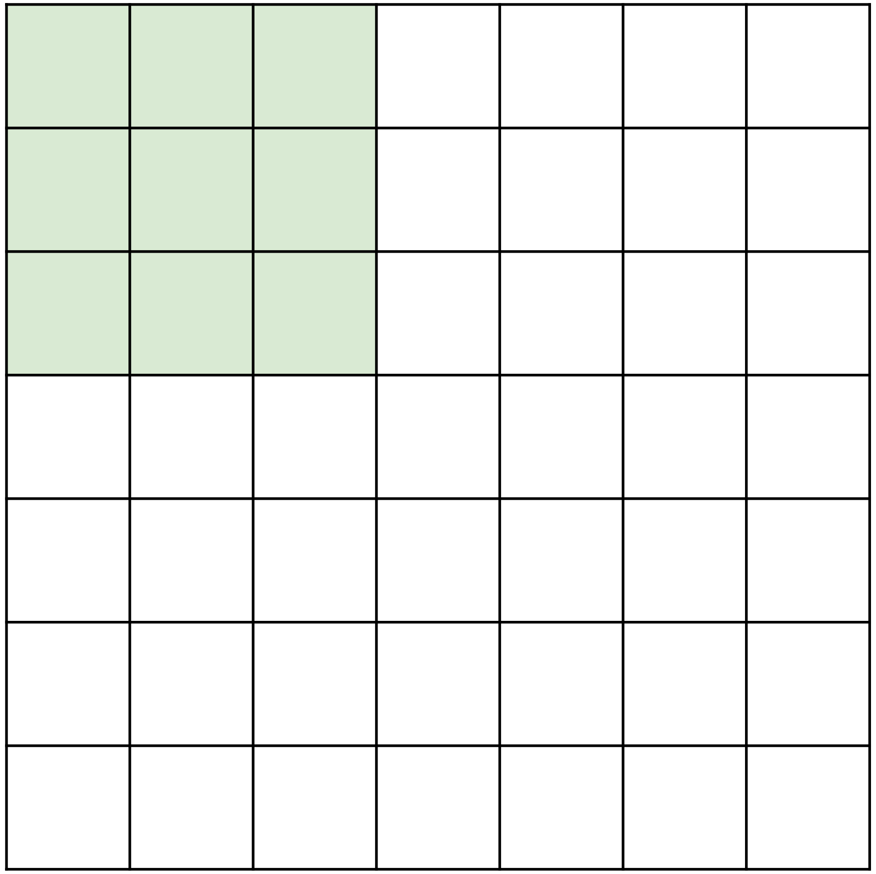
Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



Problem: For large images we need many layers for each output to "see" the whole image image

Solution: Downsample inside the network

Strided Convolution

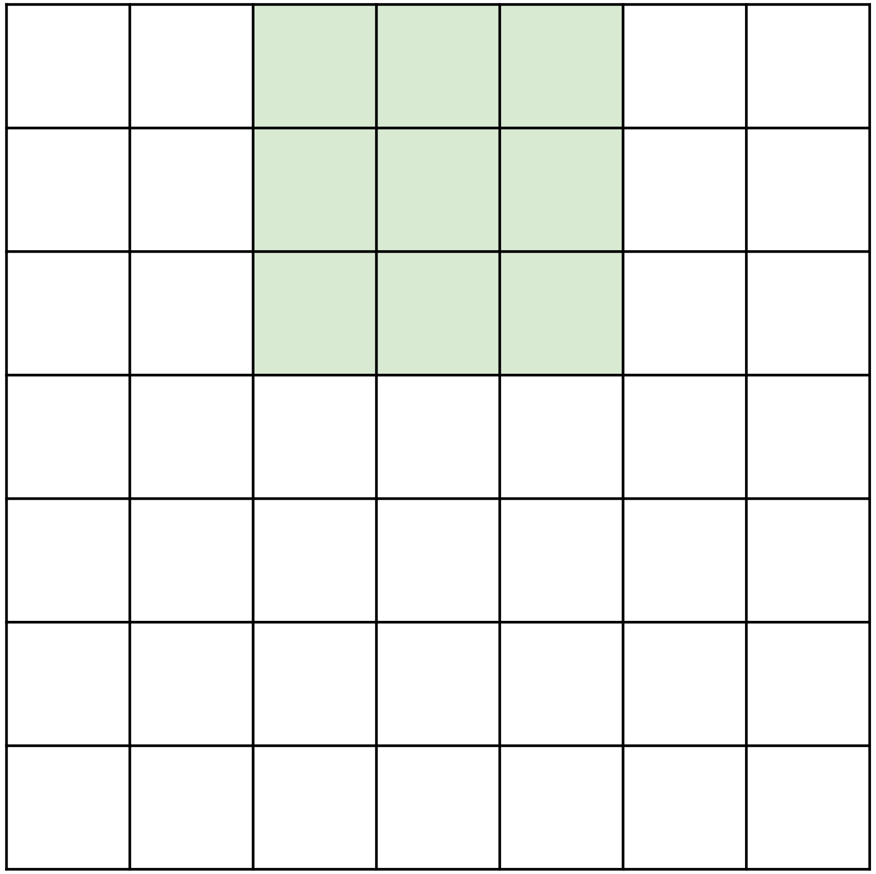


Input: 7x7

Filter: 3x3

Stride: 2

Strided Convolution

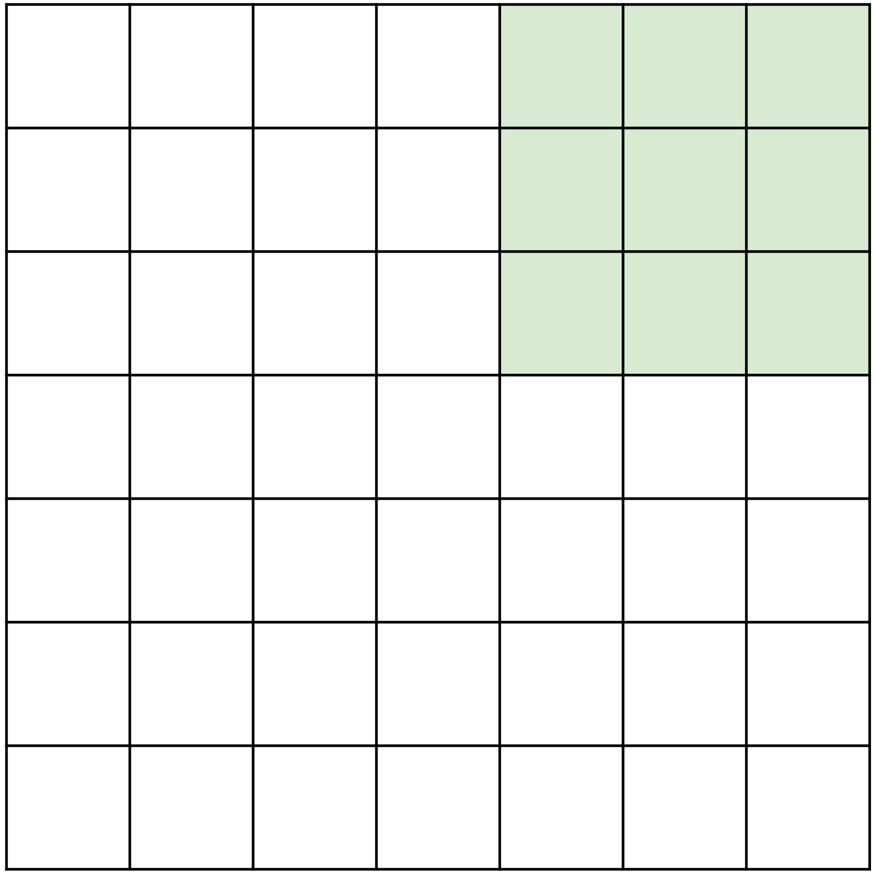


Input: 7x7

Filter: 3x3

Stride: 2

Strided Convolution



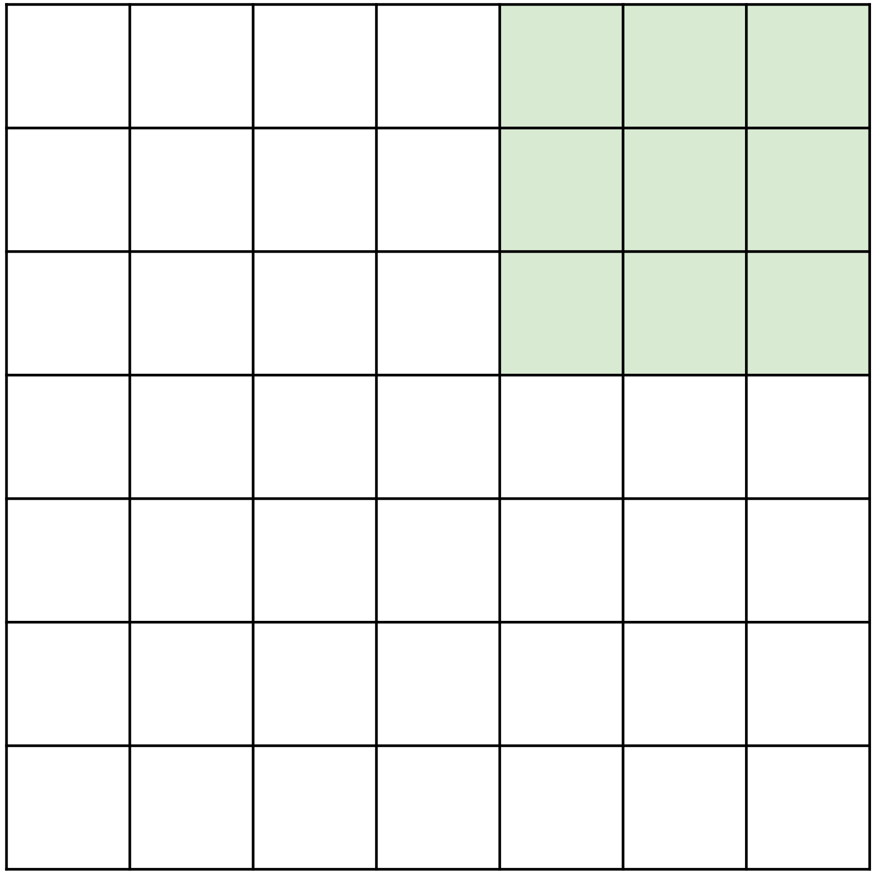
Input: 7x7

Filter: 3x3

Stride: 2

Output: 3x3

Strided Convolution



Input: 7x7

Filter: 3x3

Stride: 2

Output: 3x3

In general:

Input: W

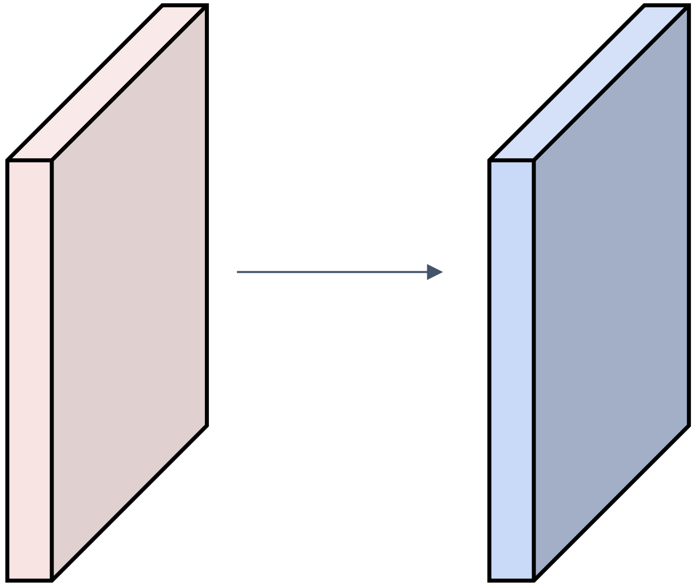
Filter: K

Padding: P

Stride: S

Output: (W – K + 2P) / S + 1

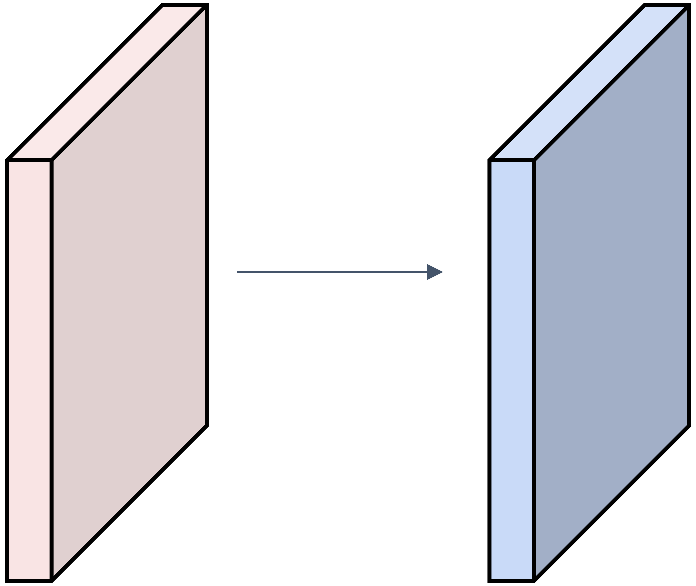
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: ?

Convolution Example



Input volume: 3 x **32** x **32**

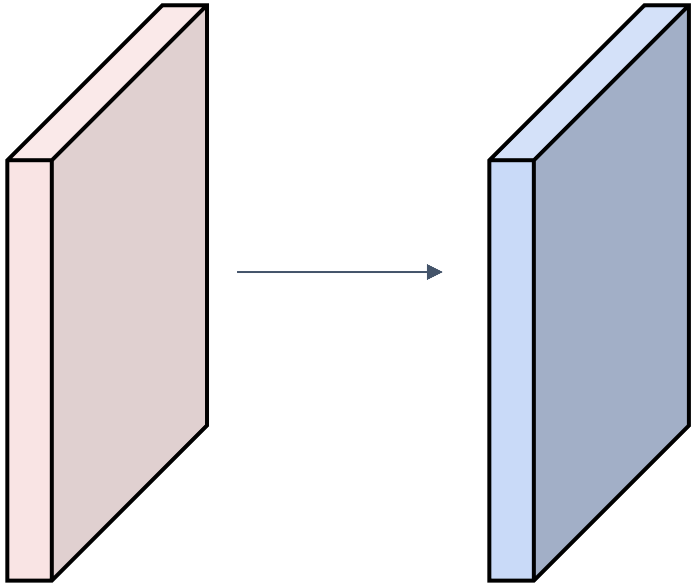
10 5x5 filters with stride **1**, pad **2**

Output volume size:

(**32**+2***2**-**5**)/**1**+1 = 32 spatially, so

10 x 32 x 32

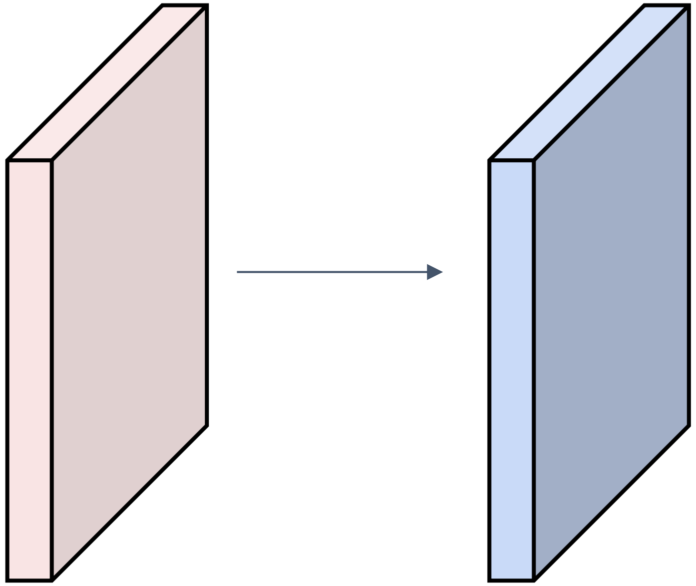
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: ?

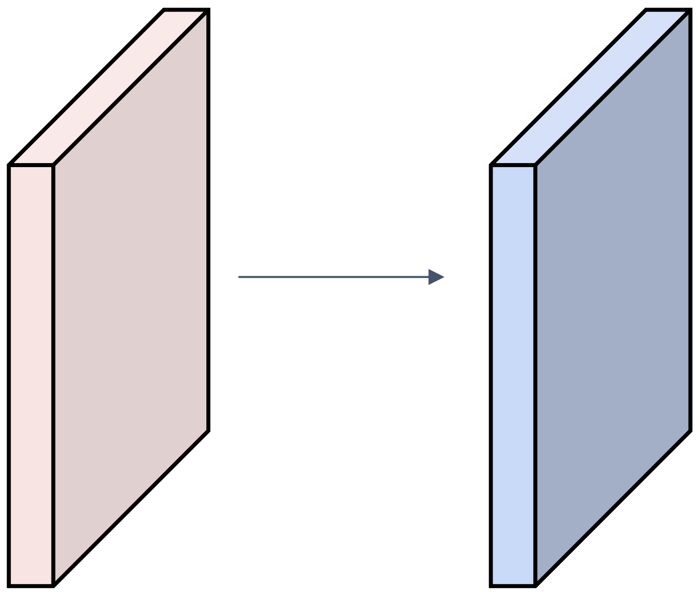
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: **760**
Parameters per filter: 3*5*5 + 1 (for bias) = **76**
10 filters, so total is 10 * **76** = **760**

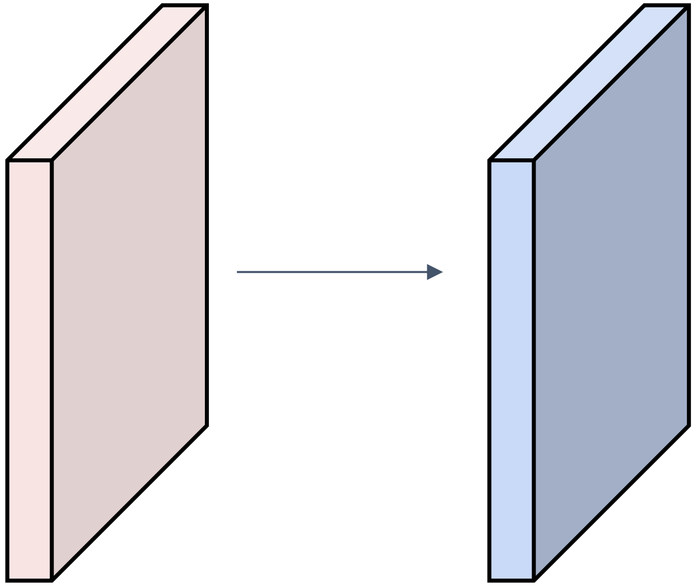
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: 760
Number of multiply-add operations: ?

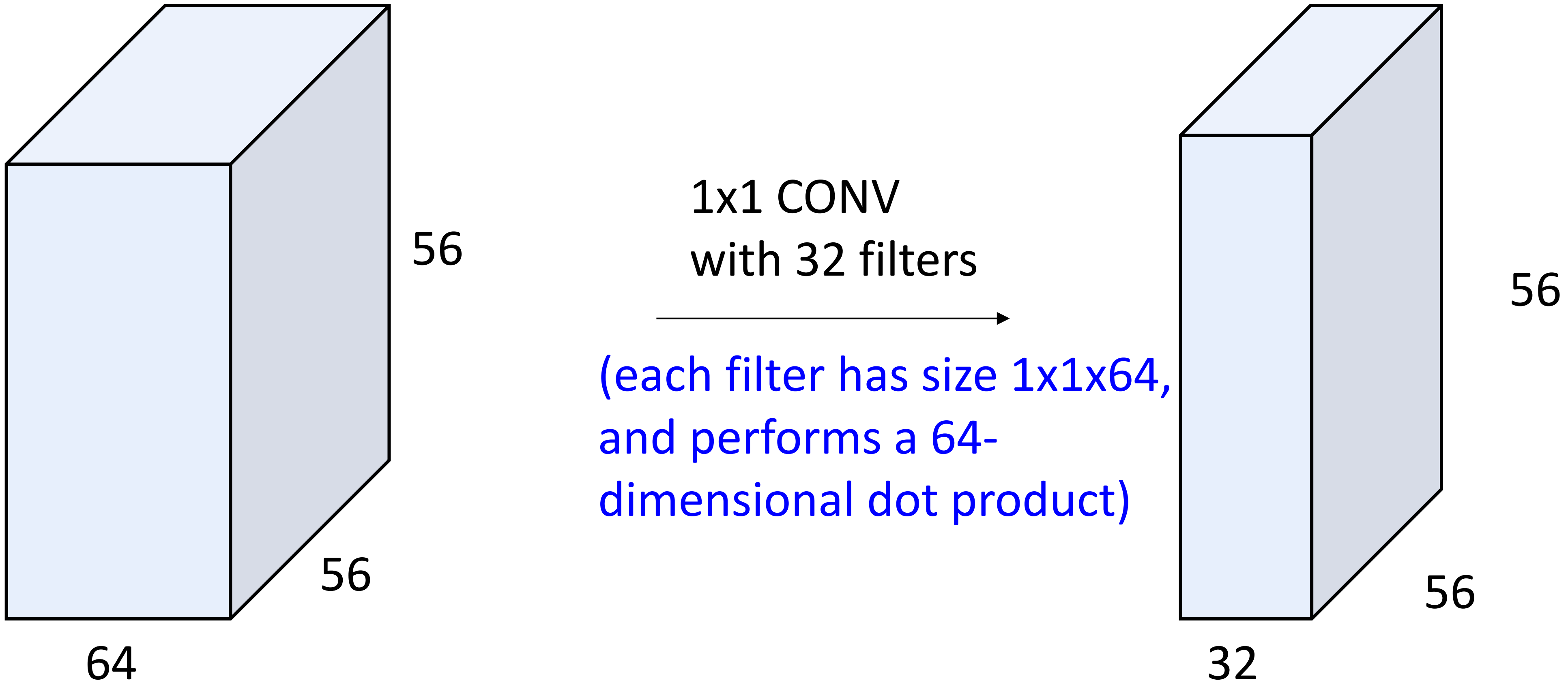
Convolution Example



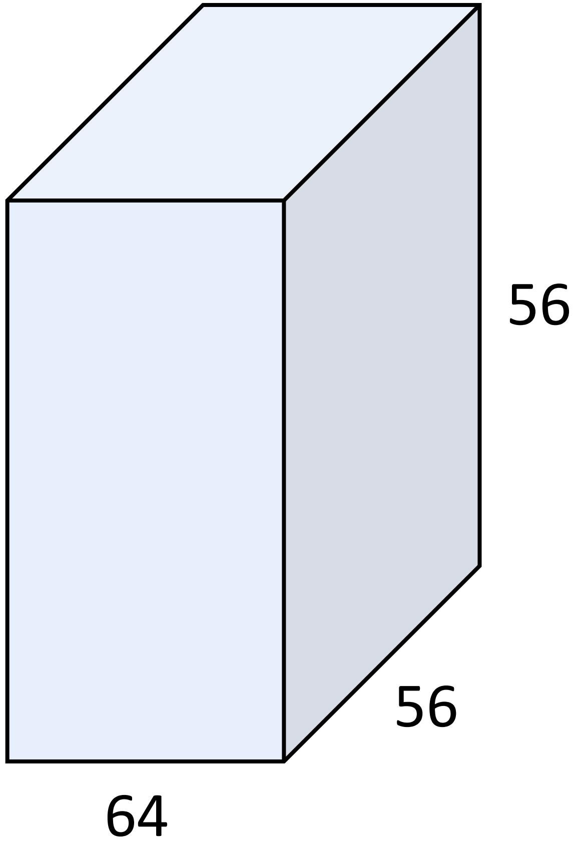
Input volume: **3** x 32 x 32
10 **5x5** filters with stride 1, pad 2

Output volume size: **10 x 32 x 32**
Number of learnable parameters: 760
Number of multiply-add operations: **768,000**
10*32*32 = 10,240 outputs; each output is the inner product of two **3**x**5x5** tensors (75 elems); total = 75*10240 = **768K**

Example: 1x1 Convolution

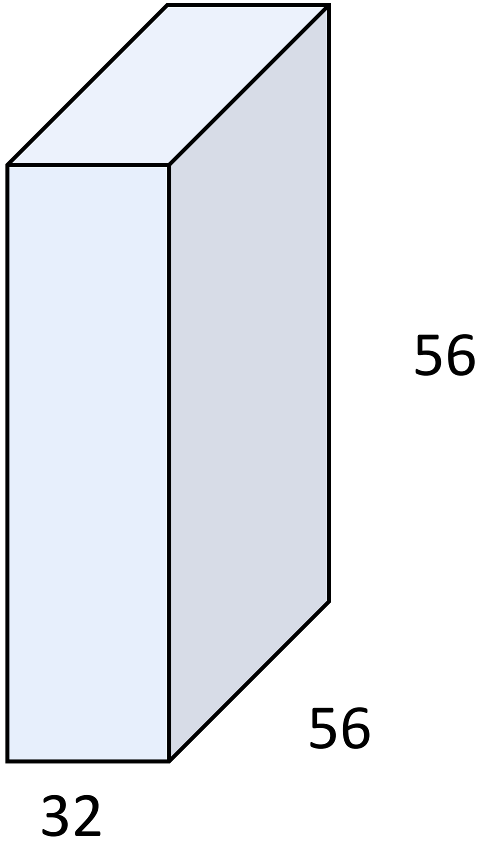


Example: 1x1 Convolution



1x1 CONV
with 32 filters

(each filter has size 1x1x64, and performs a 64-dimensional dot product)



Stacking 1x1 conv layers gives MLP operating on each input position

Lin et al, “Network in Network”, ICLR 2014

Convolution Summary

Input: C_{in} x H x W

Hyperparameters:

- **Kernel size**: K_H x K_W
- **Number filters**: C_{out}
- **Padding**: P
- **Stride**: S

Weight matrix: C_{out} x C_{in} x K_H x K_W
giving C_{out} filters of size C_{in} x K_H x K_W

Bias vector: C_{out}

Output size: C_{out} x H' x W' where:

- H' = (H – K + 2P) / S + 1
- W' = (W – K + 2P) / S + 1

Convolution Summary

Input: C_{in} x H x W

Hyperparameters:

- **Kernel size**: K_H x K_W
- **Number filters**: C_{out}
- **Padding**: P
- **Stride**: S

Weight matrix: C_{out} x C_{in} x K_H x K_W giving C_{out} filters of size C_{in} x K_H x K_W

Bias vector: C_{out}

Output size: C_{out} x H' x W' where:

- H' = (H – K + 2P) / S + 1
- W' = (W – K + 2P) / S + 1

Common settings:

$K_H = K_W$ (Small square filters)

P = (K – 1) / 2 ("Same" padding)

C_{in}, C_{out} = 32, 64, 128, 256 (powers of 2)

K = 3, P = 1, S = 1 (3x3 conv)

K = 5, P = 2, S = 1 (5x5 conv)

K = 1, P = 0, S = 1 (1x1 conv)

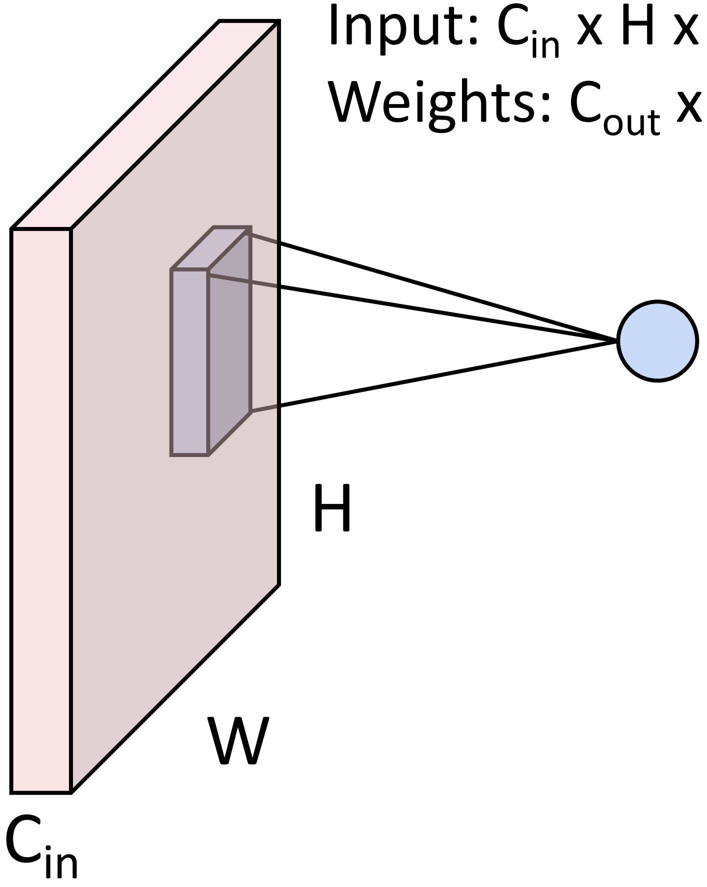
K = 3, P = 1, S = 2 (Downsample by 2)

Other types of convolution

So far: 2D Convolution

Input: C_{in} x H x W

Weights: C_{out} x C_{in} x K x K

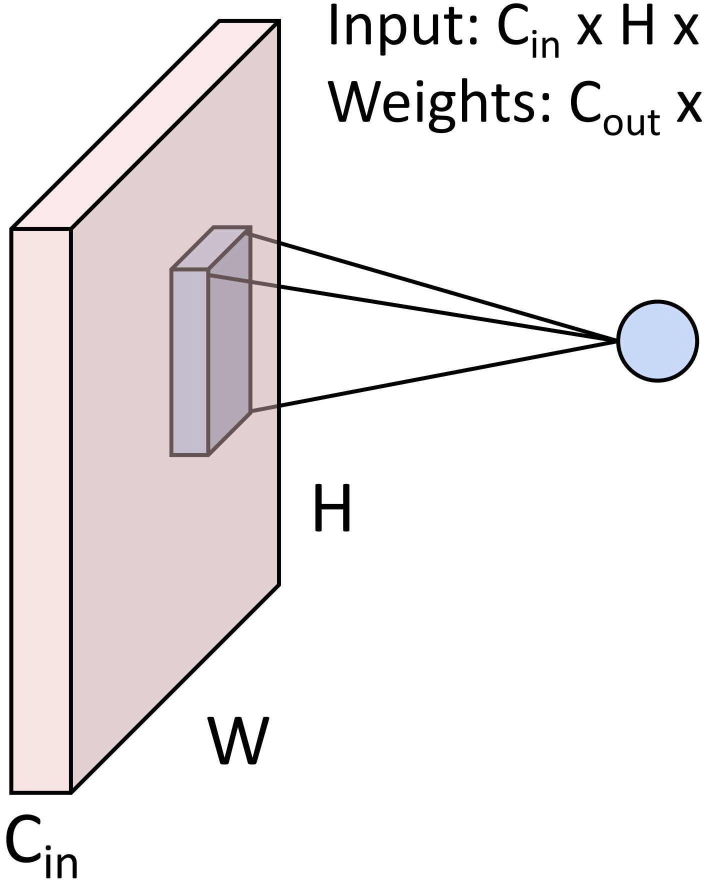


Other types of convolution

So far: 2D Convolution

Input: C_{in} x H x W

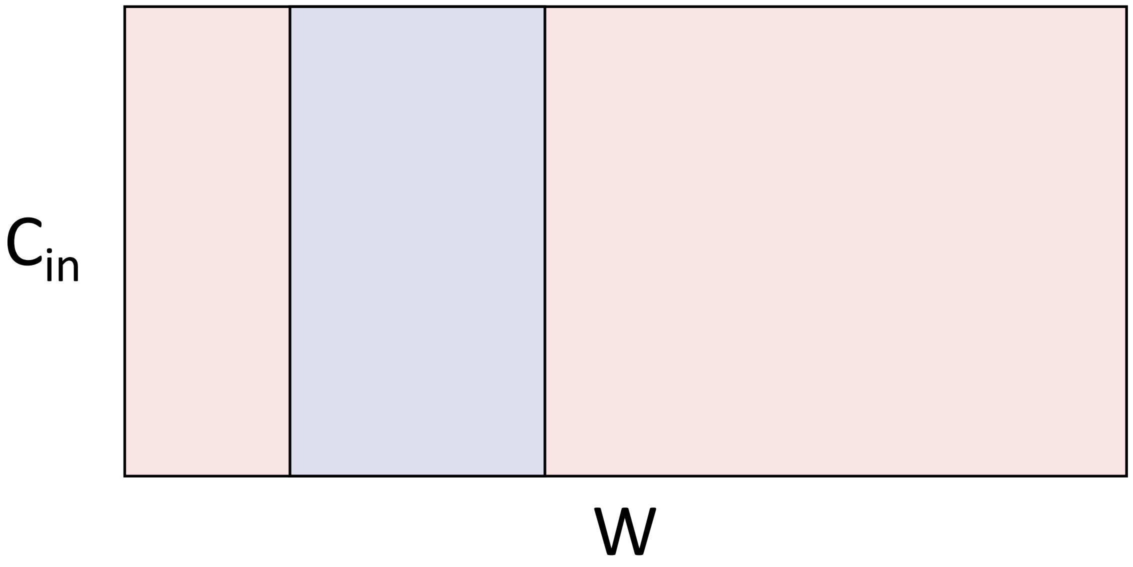
Weights: C_{out} x C_{in} x K x K



1D Convolution

Input: C_{in} x W

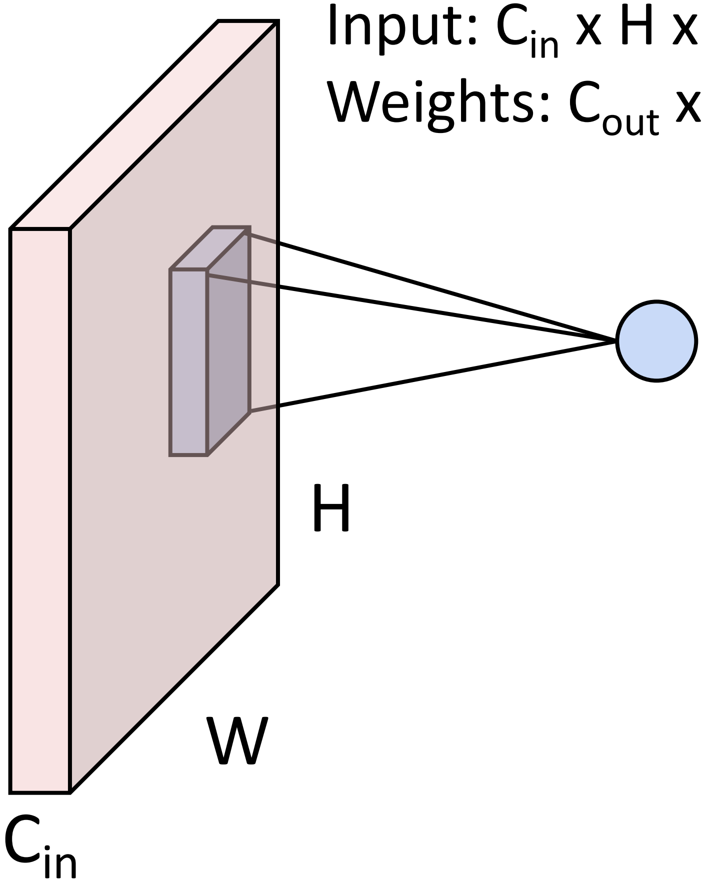
Weights: C_{out} x C_{in} x K



Other types of convolution

So far: 2D Convolution

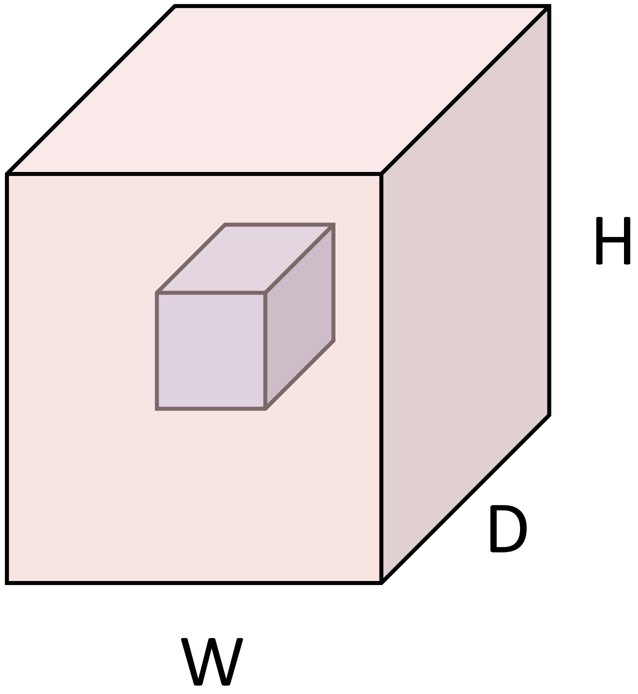
Input: C_{in} x H x W
Weights: C_{out} x C_{in} x K x K



3D Convolution

Input: C_{in} x H x W x D
Weights: C_{out} x C_{in} x K x K x K

C_{in}-dim vector
at each point
in the volume



PyTorch Convolution Layer

Conv2d

CLASS `torch.nn.Conv2d(`*`in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros'`*`)` [SOURCE]

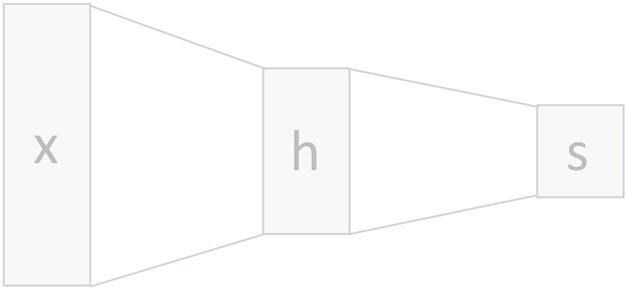
Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size (N, C_{in}, H, W) and output $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$ can be precisely described as:

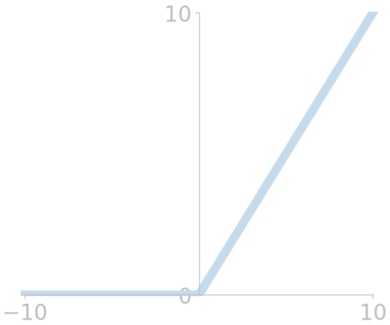
$$\text{out}(N_i, C_{\text{out}_j}) = \text{bias}(C_{\text{out}_j}) + \sum_{k=0}^{C_{\text{in}}-1} \text{weight}(C_{\text{out}_j}, k) \star \text{input}(N_i, k)$$

Components of a Convolutional Network

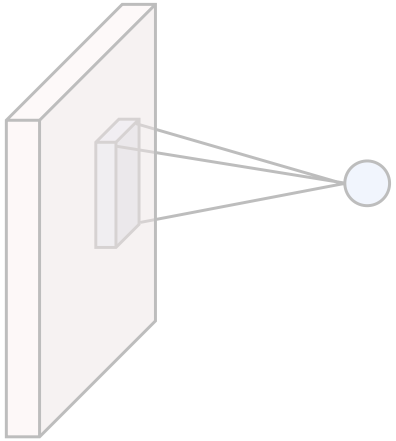
Fully-Connected Layers



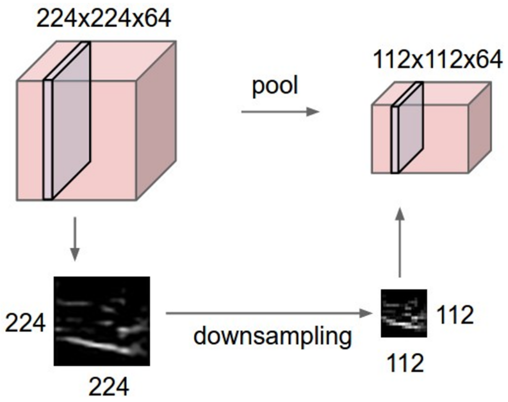
Activation Function



Convolution Layers



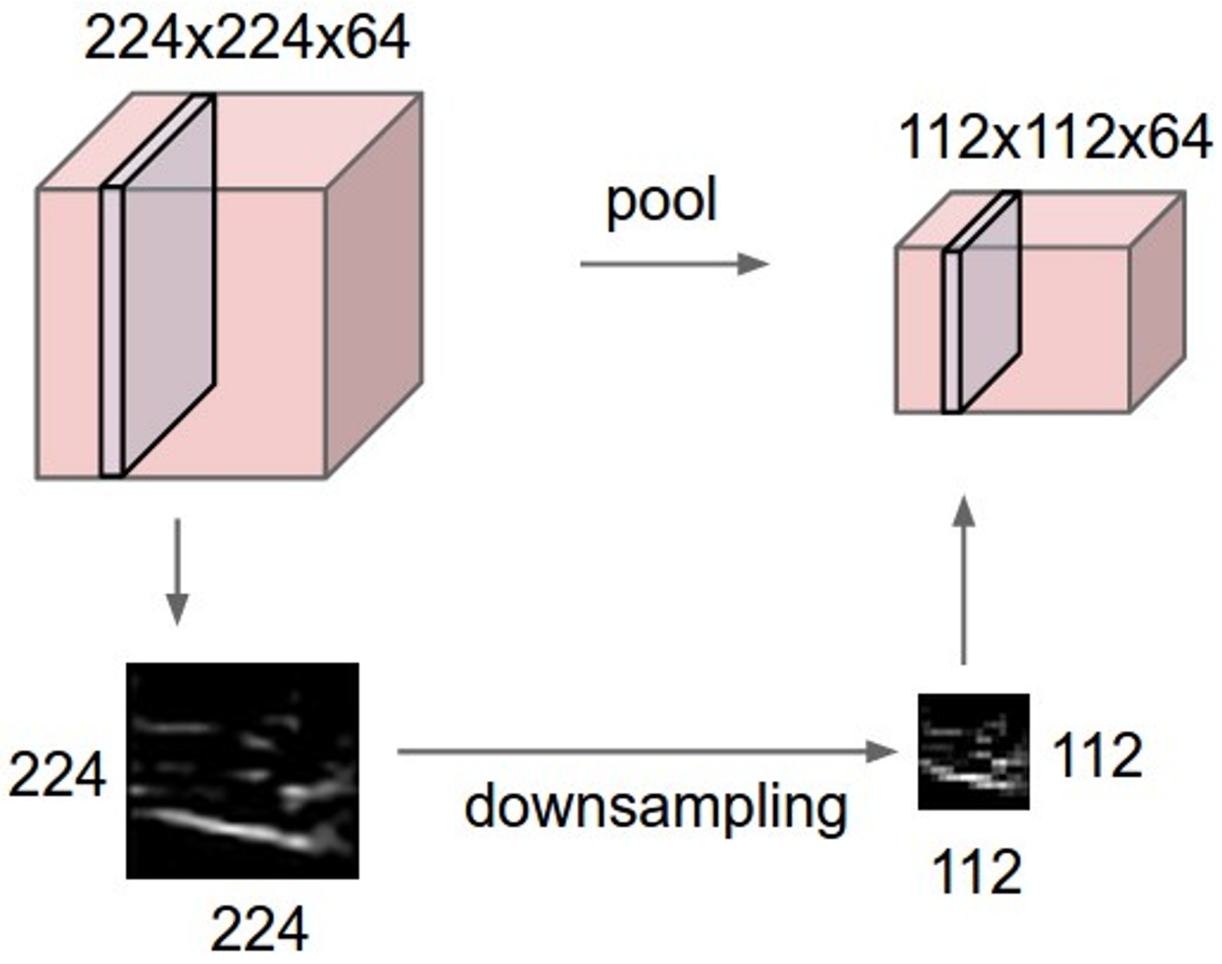
Pooling Layers



Normalization

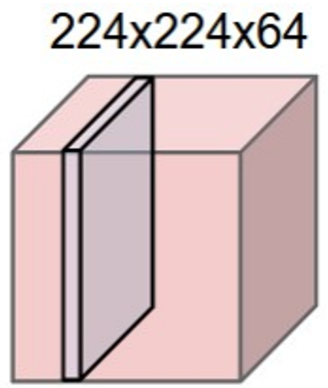
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Pooling Layers: Another way to downsample



Hyperparameters:
Kernel Size
Stride
Pooling function

Max Pooling



Single depth slice

x

1	1	2	4
5	**6**	7	**8**
3	2	1	0
1	2	3	**4**

y

Max pooling with 2x2 kernel size and stride 2

6	8
3	4

Introduces **invariance** to small spatial shifts

No learnable parameters!

Pooling Summary

Input: C x H x W

Hyperparameters:

- Kernel size: K
- Stride: S
- Pooling function (max, avg)

Output: C x H' x W' where

- $H' = (H - K) / S + 1$
- $W' = (W - K) / S + 1$

Learnable parameters: None!

Common settings:

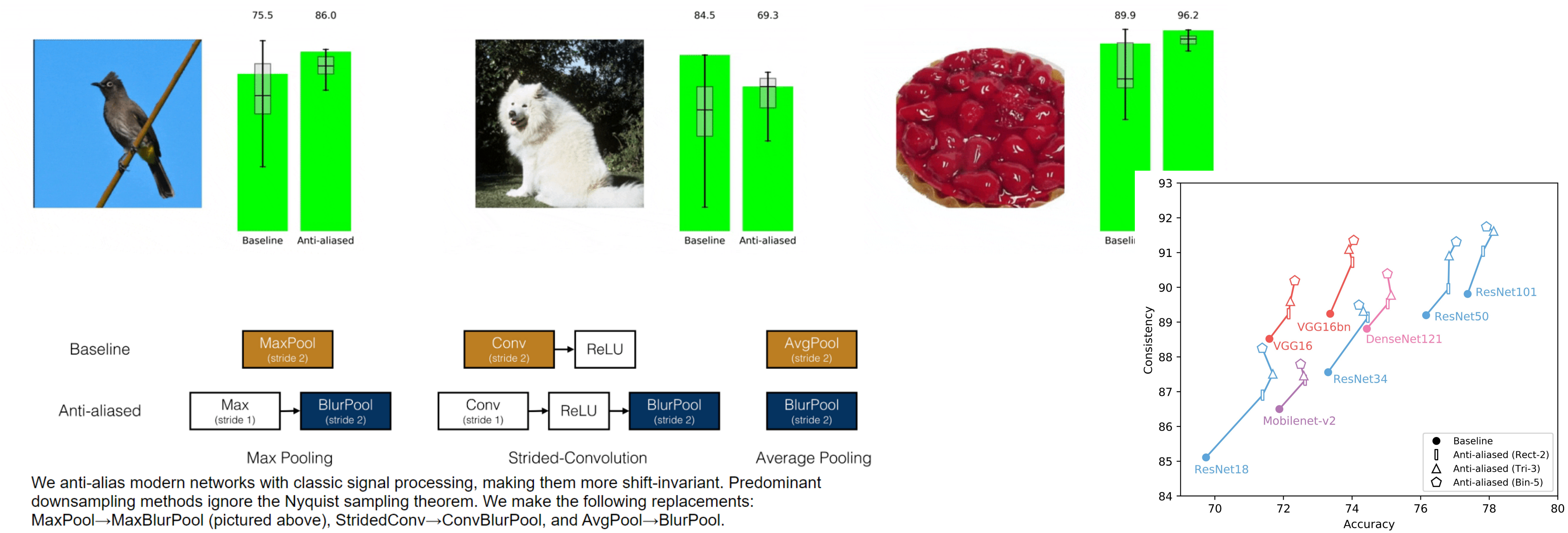
max, K = 2, S = 2

max, K = 3, S = 2 (AlexNet)

What about shift invariance?

Making Convolutional Networks Shift-Invariant Again

Richard Zhang

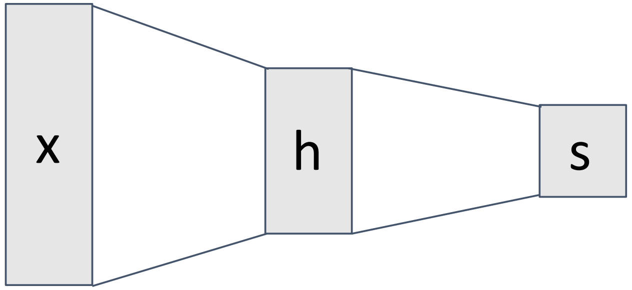


We anti-alias modern networks with classic signal processing, making them more shift-invariant. Predominant downsampling methods ignore the Nyquist sampling theorem. We make the following replacements: MaxPool→MaxBlurPool (pictured above), StridedConv→ConvBlurPool, and AvgPool→BlurPool.

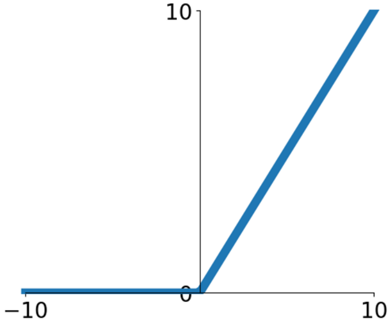
https://richzhang.github.io/antialiased-cnns/

Components of a Convolutional Network

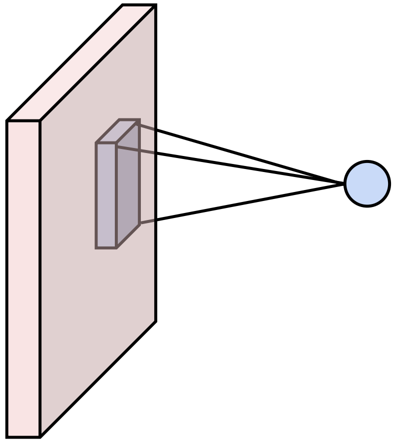
Fully-Connected Layers



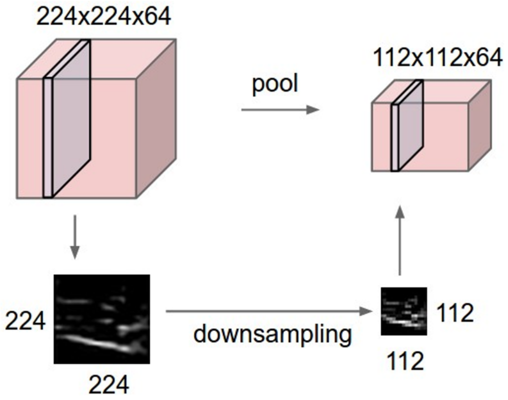
Activation Function



Convolution Layers



Pooling Layers



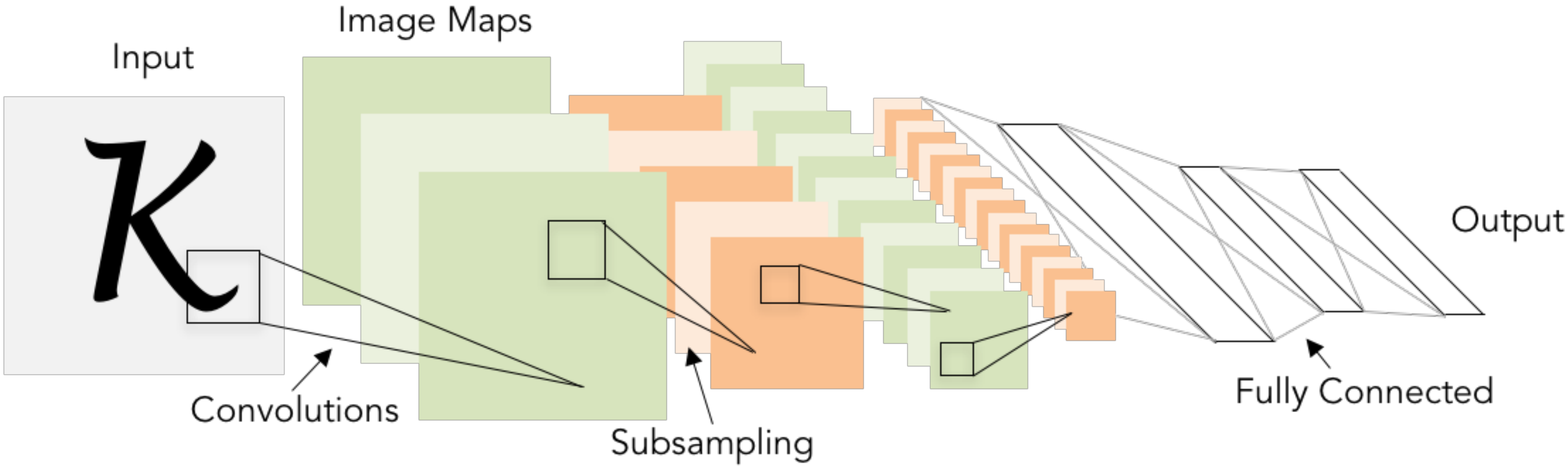
Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Convolutional Networks

Classic architecture: [Conv, ReLU, Pool] x N, flatten, [FC, ReLU] x N, FC

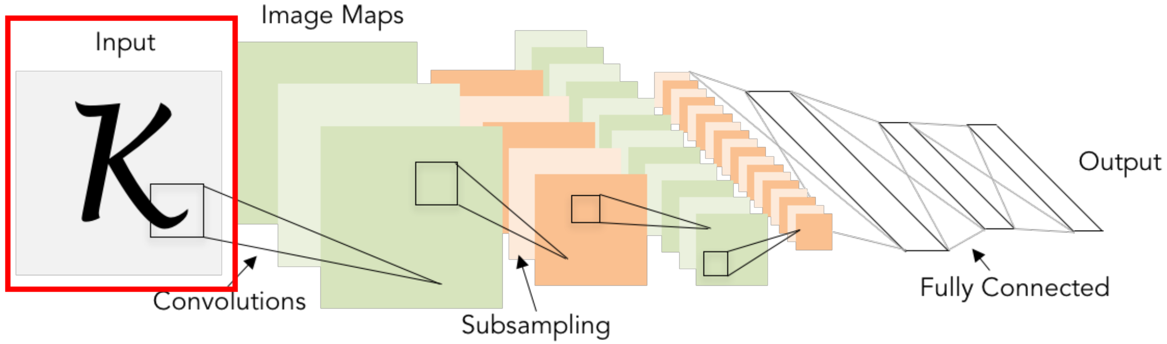
Example: LeNet-5



Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

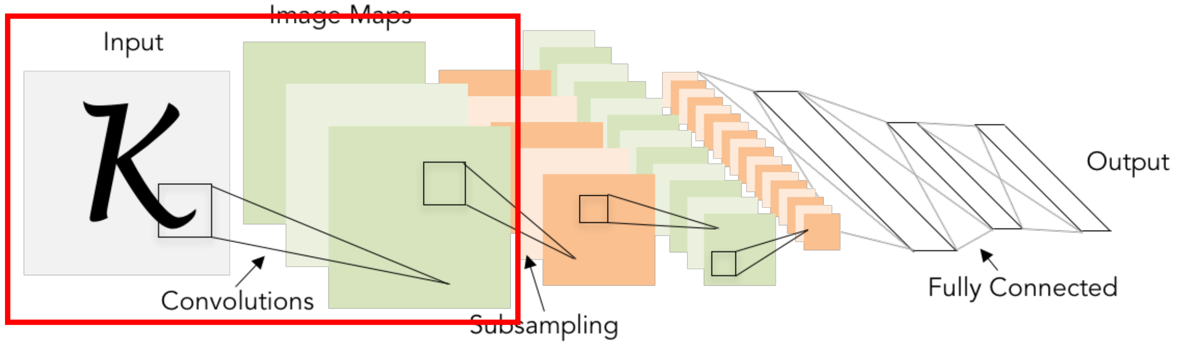
Layer	Output Size	Weight Size
Input	1 x 28 x 28	



Lecun et al, "Gradient-based learning applied to document recognition", 1998

Example: LeNet-5*

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20*,K=5,P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU**	20 x 28 x 28	

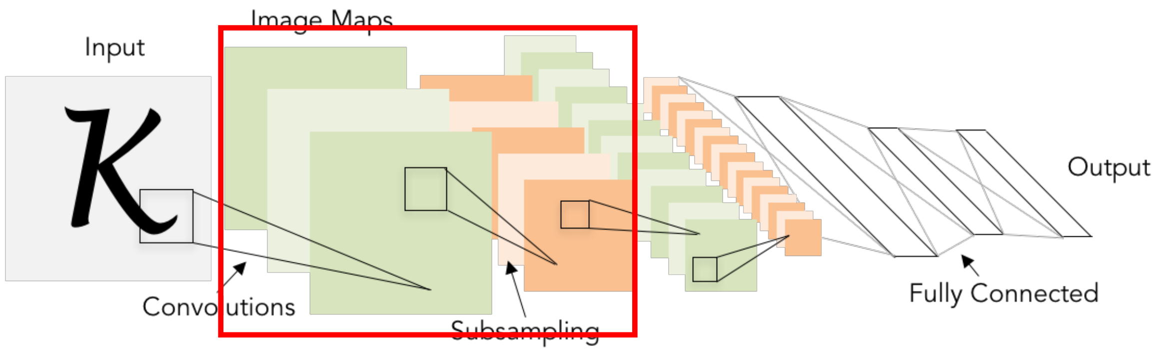


* Original paper: C_{out} = 6
** Original paper: sigmoid

Lecun et al, "Gradient-based learning applied to document recognition", 1998

Example: LeNet-5

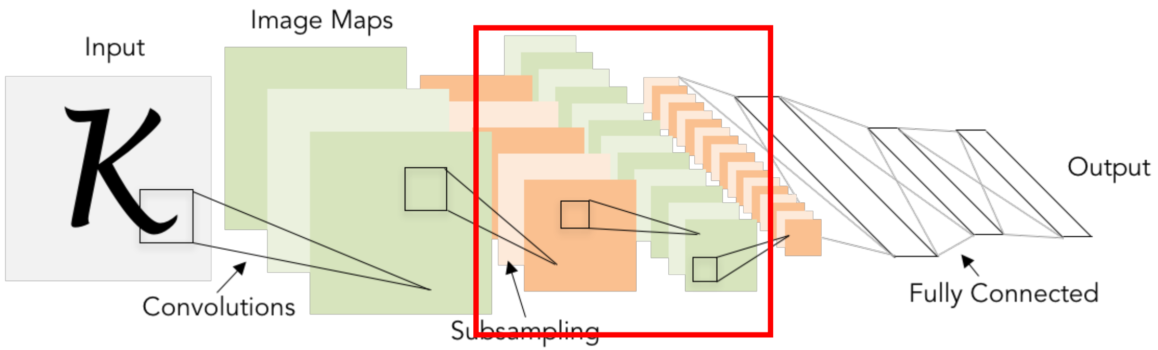
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)*	20 x 14 x 14	



* 2x2 strided convolution

Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5



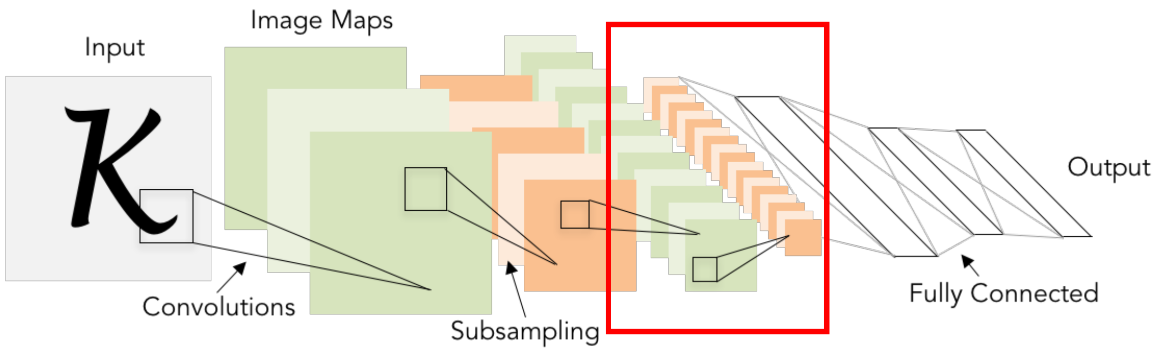
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50*,K=5,P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU**	50 x 14 x 14	

* Original paper: C_{out} = 16, grouped convolutions
** Original paper: sigmoid

Lecun et al, "Gradient-based learning applied to document recognition", 1998

Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)*	50 x 7 x 7	

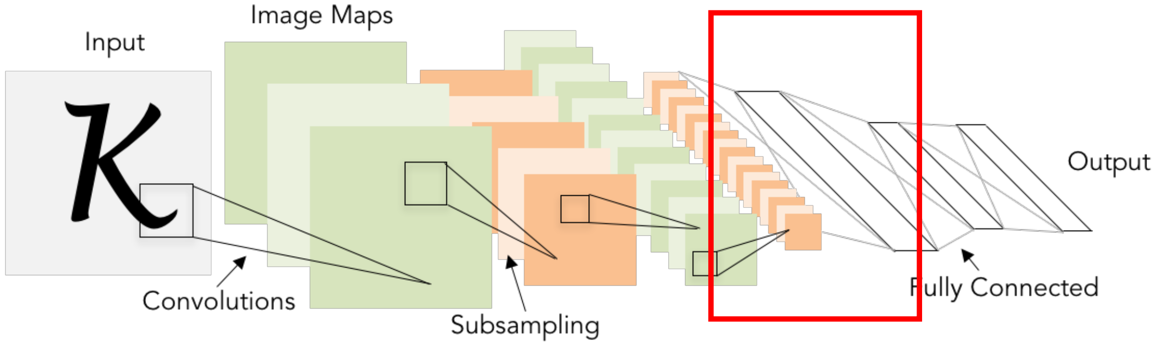


* 2x2 strided convolution

Lecun et al, "Gradient-based learning applied to document recognition", 1998

Example: LeNet-5

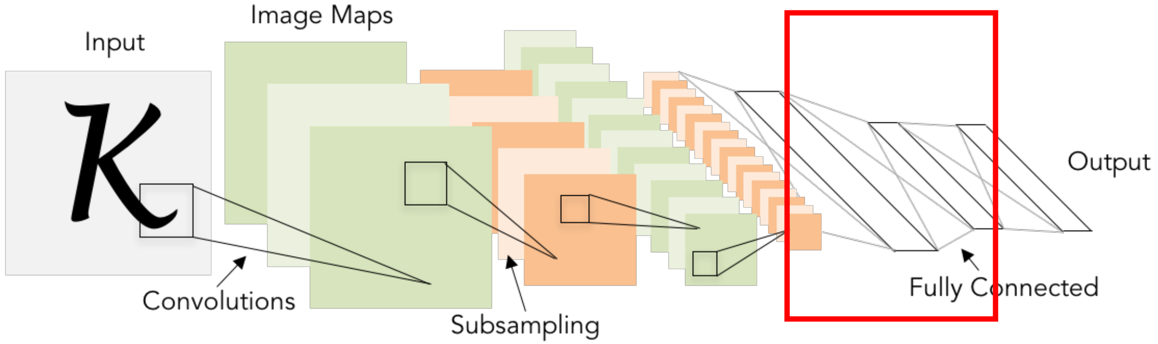
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	



Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU*	500	

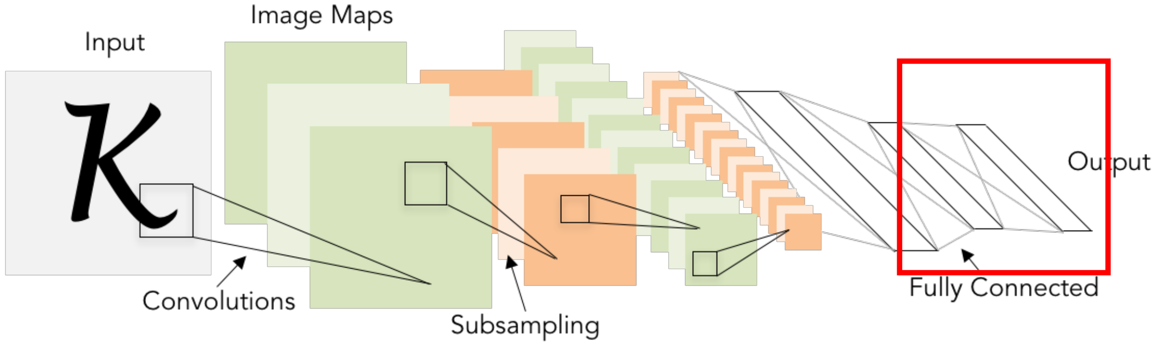


* Original paper has different 1x1 convolutions, sigmoid non-linearities

Lecun et al, "Gradient-based learning applied to document recognition", 1998

Example: LeNet-5*

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	
Linear (500 -> 10)*	10	500 x 10

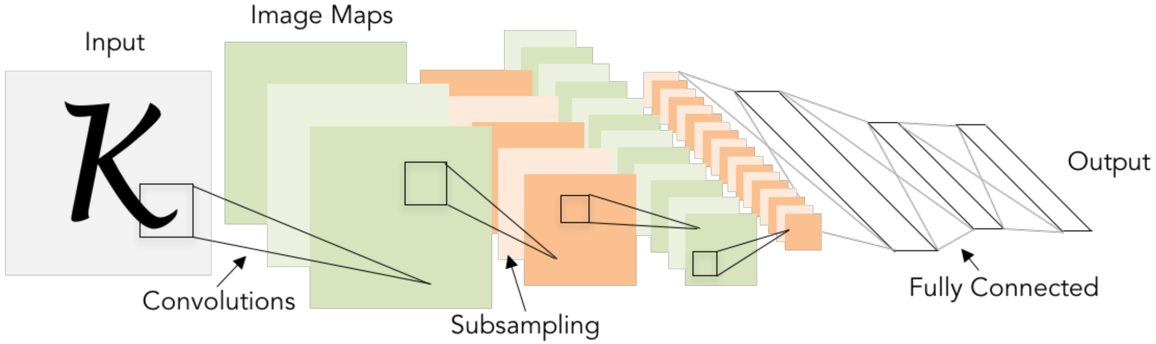


* Original paper uses RBF (radial basis function) kernels instead of a softmax

Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	
Linear (500 -> 10)	10	500 x 10



As we go through the network:

Spatial size **decreases**
(using pooling or strided conv)

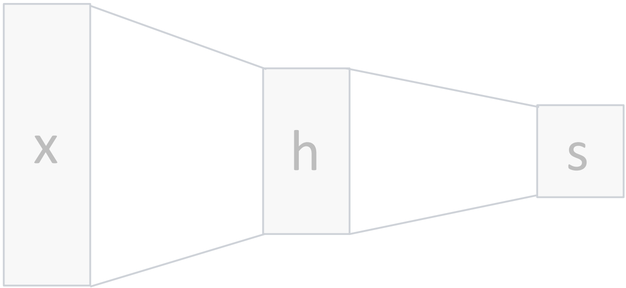
Number of channels **increases**
(total “volume” is preserved!)

Lecun et al, “Gradient-based learning applied to document recognition”, 1998

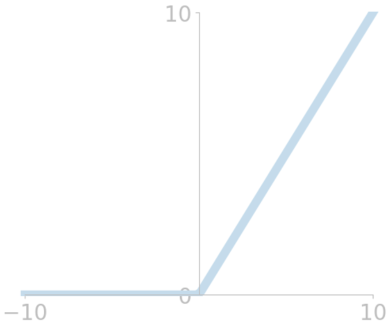
Problem: Deep Networks very hard to train!

Components of a Convolutional Network

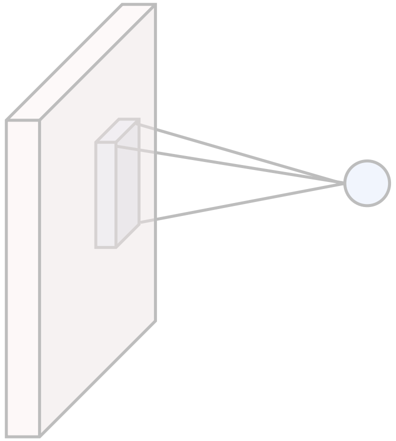
Fully-Connected Layers



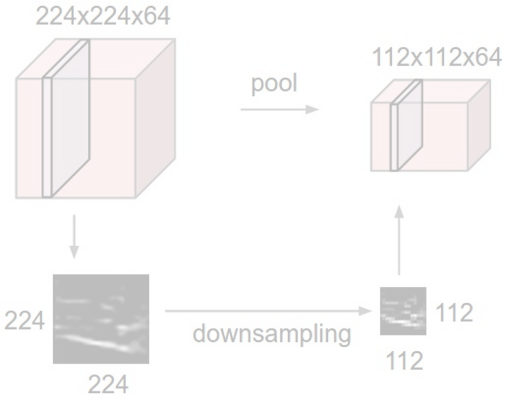
Activation Function



Convolution Layers



Pooling Layers



Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Batch Normalization

Idea: "Normalize" the outputs of a layer so they have zero mean and unit variance. **Why?**

Why? Helps reduce "internal covariate shift", improves optimization

We can normalize a batch of activations like this:

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

This is a **differentiable function**, so we can use it as an operator in our networks and backprop through it!

Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

Activation and weight scaling

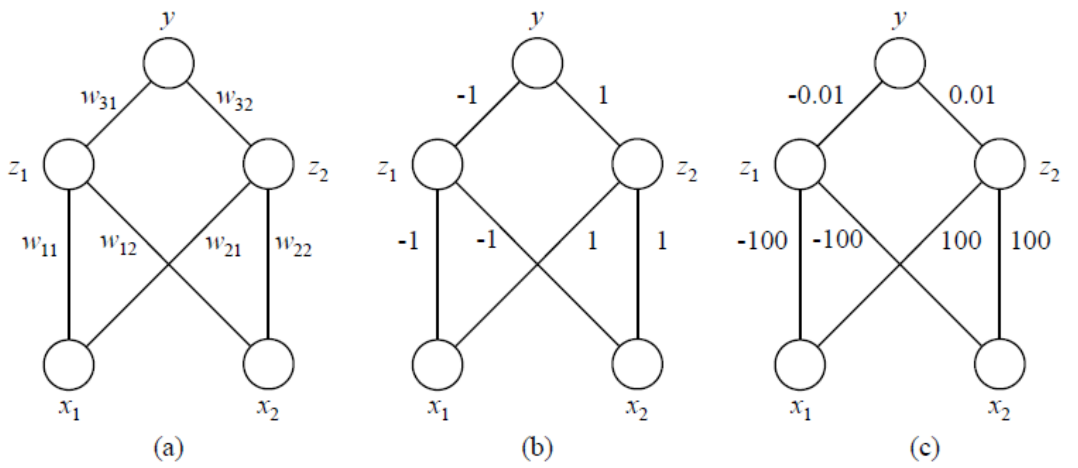


Figure 5.53 *Simple two hidden unit network with a ReLU activation function and no bias parameters for regressing the function $y = |x_1 + 1.1x_2|$: (a) can you guess a set of weights would fit this function? (b) a reasonable set of starting weights; (c) a poorly scaled set of weights.*

2. Starting with the weights shown in column b, compute the activations for the hidden and final units as well as the regression loss for the four input values $(x_1, x_2) \in \{-1, 0, 1\} \times \{-1, 0, 1\}$.

3. Now compute the gradients of the squared loss with respect to all six weights using the backpropagation chain rule equations (5.78–5.81) and sum them up across the training samples to get a final gradient.

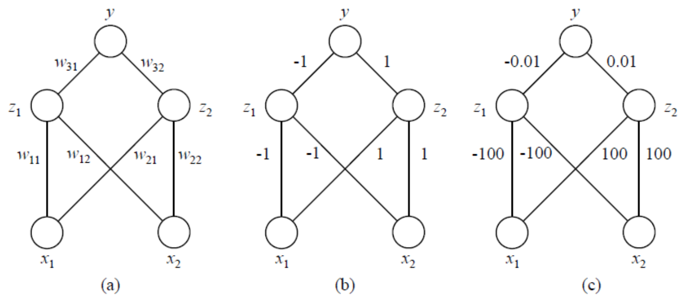
4. What step size should you take in the gradient direction, and what would your update squared loss become?

5. Repeat this exercise for the initial weights in column (c) of Figure 5.52.

6. Given this new set of weights, how much worse is your error decrease, and how many iterations would you expect it to take to achieve a reasonalbe solution?

7. Would batch normalization help in this case?

Activation and weight scaling

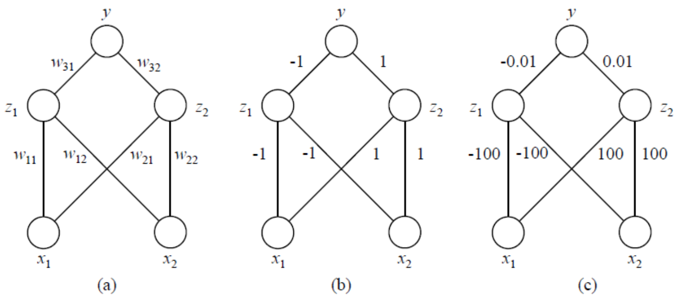


x1	x2	w11	w12	w21	w22	z1	z2	w31	w32	y	target	L2 loss
-1	-1	-1	-1	1	1	2	0	1	1	2	2.1	0.01
-1	0					1	0			1	1	0
-1	1					0	0			0	0.1	0.01
0	-1					1	0			1	1.1	0.01
0	0					0	0			0	0	0
0	1					0	1			1	1.1	0.01
1	-1					0	0			0	0.1	0.01
1	0					0	1			1	1	0
1	1					0	2			2	2.1	0.01
												0.06

x1	x2	d_w11	d_w12	d_w21	d_w22	d_z1	d_z2	d_w31	d_w32	g_y
-1	-1	0.1	0.1	0	0	-0.1	-0.1	-0.2	0	-0.1
-1	0	0	0	0	0	0	0	0	0	0
-1	1	0	0	0	0	0	0	0	0	-0.1
0	-1	0	0.1	0	0	-0.1	-0.1	-0.1	0	-0.1
0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	-0.1	-0.1	-0.1	0	-0.1	-0.1
1	-1	0	0	0	0	0	0	0	0	-0.1
1	0	0	0	0	0	0	0	0	0	0
1	1	0	0	-0.1	-0.1	-0.1	-0.1	0	-0.2	-0.1
grad-:		-0.1	-0.2	0.1	0.2			0.3	0.3	

Balanced | Imbalanced

Activation and weight scaling



x1	x2	w11	w12	w21	w22	z1	z2	w31	w32	y	target	L2 loss
-1	-1	-1	-1	1	1	2	0	1	1	2	2.1	0.01
-1	0											
-1	1											
0	-1											
0	0											
0	1											
1	-1											
1	0											
1	1											

x1	x2	d_w11	d_w12	d_w21	d_w22	d_z1
-1	-1	0.1	0.1	0	0	
-1	0	0	0	0	0	
-1	1	0	0	0	0	
0	-1	0	0.1	0	0	
0	0	0	0	0	0	
0	1	0	0	0	-0.1	
1	-1	0	0	0	0	
1	0	0	0	0	0	
1	1	0	0	-0.1	-0.1	
grad-:		-0.1	-0.2	0.1	0.2	

Balanced | Imbalanced

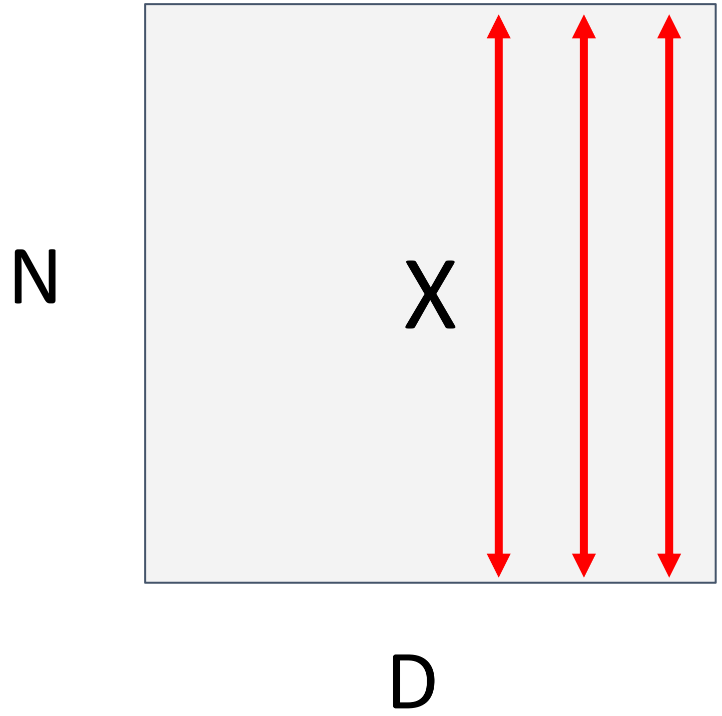
x1	x2	w11	w12	w21	w22	z1	z2	w31	w32	y	target	L2 loss
-1	-1	-100	-100	100	100	200	0	0.01	0.01	2	2.1	0.01
-1	0					100	0			1	1	0
-1	1					0	0			0	0.1	0.01
0	-1					100	0			1	1.1	0.01
0	0					0	0			0	0	0
0	1					0	100			1	1.1	0.01
1	-1					0	0			0	0.1	0.01
1	0					0	100			1	1	0
1	1					0	200			2	2.1	0.01
												0.06

x1	x2	d_w11	d_w12	d_w21	d_w22	d_z1	d_z2	d_w31	d_w32	g_y
-1	-1	0.001	0.001	0	0	-0.001	-0.001	-20	0	-0.1
-1	0	0	0	0	0	0	0	0	0	0
-1	1	0	0	0	0	0	0	0	0	-0.1
0	-1	0	0.001	0	0	-0.001	-0.001	-10	0	-0.1
0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	-0.001	-0.001	-0.001	0	-10	-0.1
1	-1	0	0	0	0	0	0	0	0	-0.1
1	0	0	0	0	0	0	0	0	0	0
1	1	0	0	-0.001	-0.001	-0.001	-0.001	0	-20	-0.1
grad-:		-0.001	-0.002	0.001	0.002			30	30	

Balanced | Imbalanced

Batch Normalization

Input: $x : N \times D$



$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N} (x_{i,j} - \mu_j)^2$$

Per-channel std, shape is D

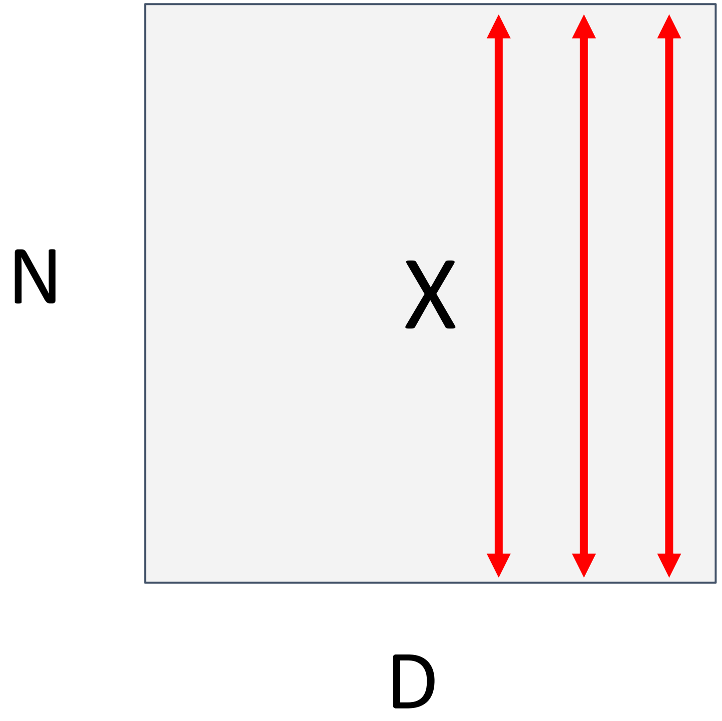
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

Ioffe and Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift”, ICML 2015

Batch Normalization

Input: $x : N \times D$



$$\mu_j = \frac{1}{N} \sum_{i=1}^{N} x_{i,j}$$ Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^{N} (x_{i,j} - \mu_j)^2$$ Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$ Normalized x, Shape is N x D

Problem: What if zero-mean, unit variance is too hard of a constraint?

Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

Batch Normalization

Input: $x : N \times D$

Learnable scale and shift parameters:

$\gamma, \beta : D$

Learning $\gamma = \sigma$, $\beta = \mu$ will recover the identity function!

$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N}(x_{i,j} - \mu_j)^2$$

Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization: Test-Time

Problem: Estimates depend on minibatch; can't do this at test-time!

Input: $x : N \times D$

Learnable scale and shift parameters:

$$\gamma, \beta : D$$

Learning $\gamma = \sigma$, $\beta = \mu$ will recover the identity function!

$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N}(x_{i,j} - \mu_j)^2$$

Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization: Test-Time

Input: $x : N \times D$

Learnable scale and shift parameters:

$\gamma, \beta : D$

Learning $\gamma = \sigma$, $\beta = \mu$ will recover the identity function!

$\mu_j =$ (Running) average of values seen during training — Per-channel mean, shape is D

$\sigma_j^2 =$ (Running) average of values seen during training — Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization: Test-Time

Input: $x : N \times D$

Learnable scale and shift parameters:

$\gamma, \beta : D$

During testing batchnorm becomes a linear operator! Can be fused with the previous fully-connected or conv layer

$\mu_j =$ (Running) average of values seen during training — Per-channel mean, shape is D

$\sigma_j^2 =$ (Running) average of values seen during training — Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization for ConvNets

Batch Normalization for **fully-connected** networks

$$\mathbf{x}: \; N \times D$$

Normalize

$$\mu, \sigma: \; 1 \times D$$

$$\gamma, \beta: \; 1 \times D$$

$$y = \gamma(x-\mu)/\sigma+\beta$$

Batch Normalization for **convolutional** networks (Spatial Batchnorm, BatchNorm2D)

$$\mathbf{x}: \; N \times C \times H \times W$$

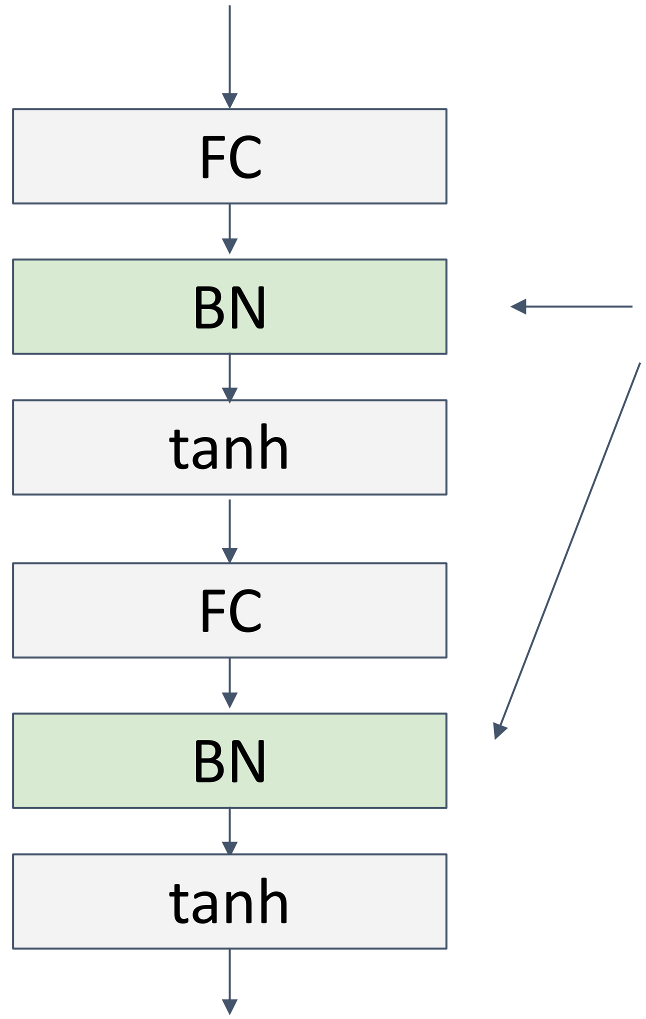
Normalize

$$\mu, \sigma: \; 1 \times C \times 1 \times 1$$

$$\gamma, \beta: \; 1 \times C \times 1 \times 1$$

$$y = \gamma(x-\mu)/\sigma+\beta$$

Batch Normalization

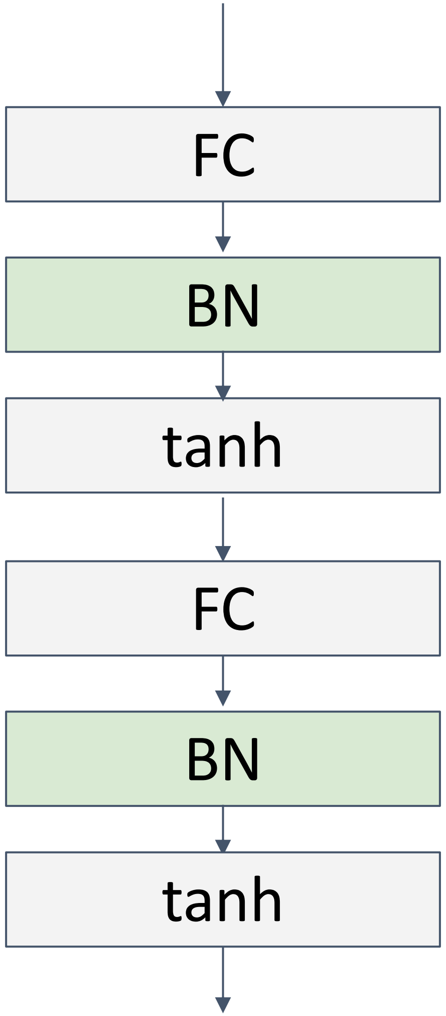


Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

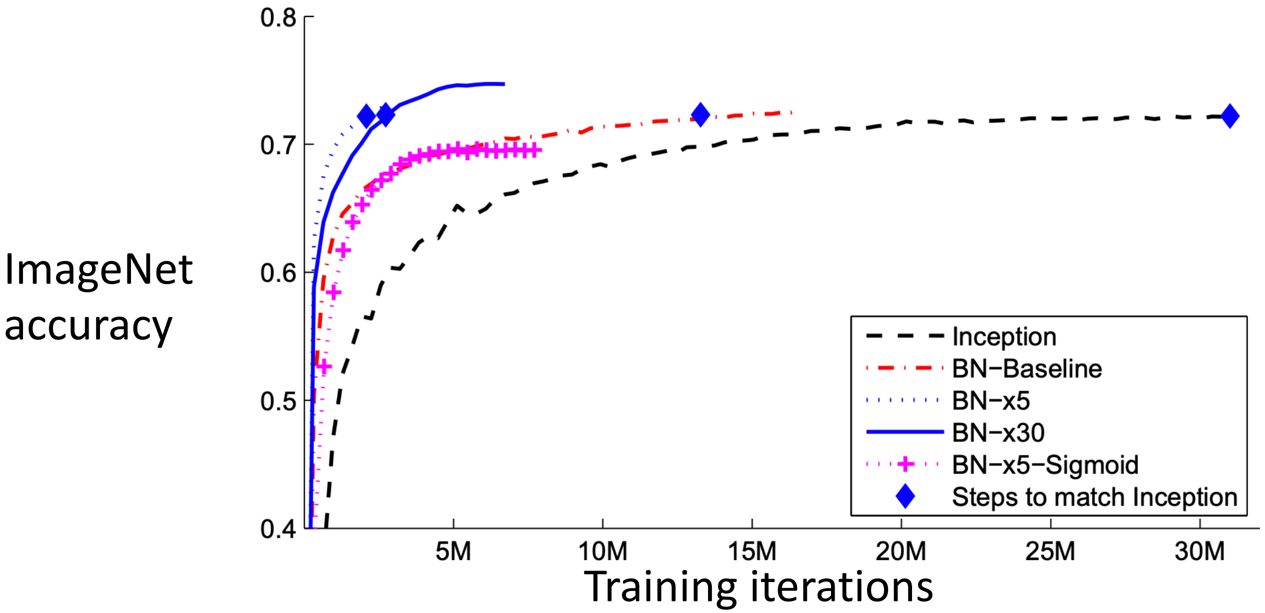
$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

Ioffe and Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift”, ICML 2015

Batch Normalization

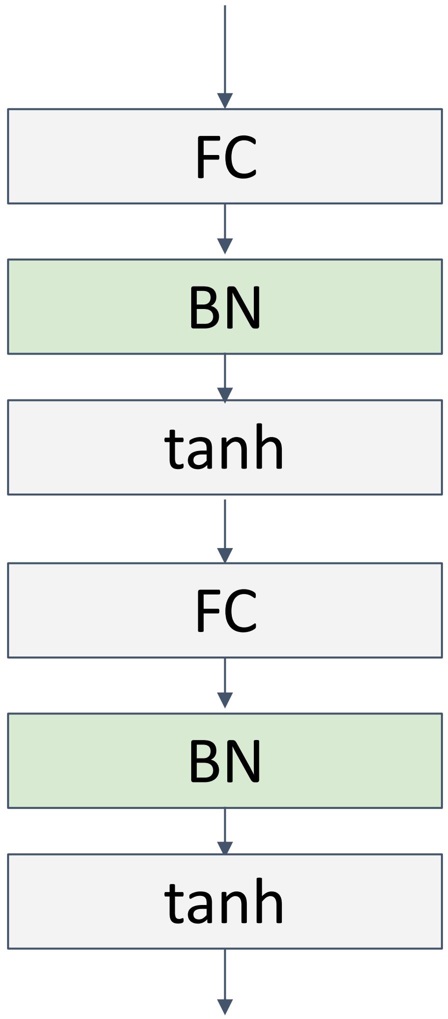


- Makes deep networks **much** easier to train!
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time: can be fused with conv!



Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

Batch Normalization



- Makes deep networks **much** easier to train!
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time: can be fused with conv!
- Not well-understood theoretically (yet)
- Behaves differently during training and testing: this is a very common source of bugs!

Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

Layer Normalization

Batch Normalization for fully-connected networks

$$\mathbf{x}: N \times D$$

Normalize

$$\boldsymbol{\mu}, \boldsymbol{\sigma}: 1 \times D$$

$$\gamma, \beta: 1 \times D$$

$$y = \gamma(x-\mu)/\sigma+\beta$$

Layer Normalization for fully-connected networks

Same behavior at train and test!

Used in RNNs, Transformers

$$\mathbf{x}: N \times D$$

Normalize

$$\boldsymbol{\mu}, \boldsymbol{\sigma}: N \times 1$$

$$\gamma, \beta: 1 \times D$$

$$y = \gamma(x-\mu)/\sigma+\beta$$

Ba, Kiros, and Hinton, “Layer Normalization”, arXiv 2016

Instance Normalization

Batch Normalization for convolutional networks

$$\mathbf{x}: \ N \times C \times H \times W$$

Normalize

$$\boldsymbol{\mu}, \boldsymbol{\sigma}: \ 1 \times C \times 1 \times 1$$

$$\boldsymbol{\gamma}, \boldsymbol{\beta}: \ 1 \times C \times 1 \times 1$$

$$\mathbf{y} = \boldsymbol{\gamma}(\mathbf{x}-\boldsymbol{\mu})/\boldsymbol{\sigma}+\boldsymbol{\beta}$$

Instance Normalization for convolutional networks
Same behavior at train / test!

$$\mathbf{x}: \ N \times C \times H \times W$$

Normalize

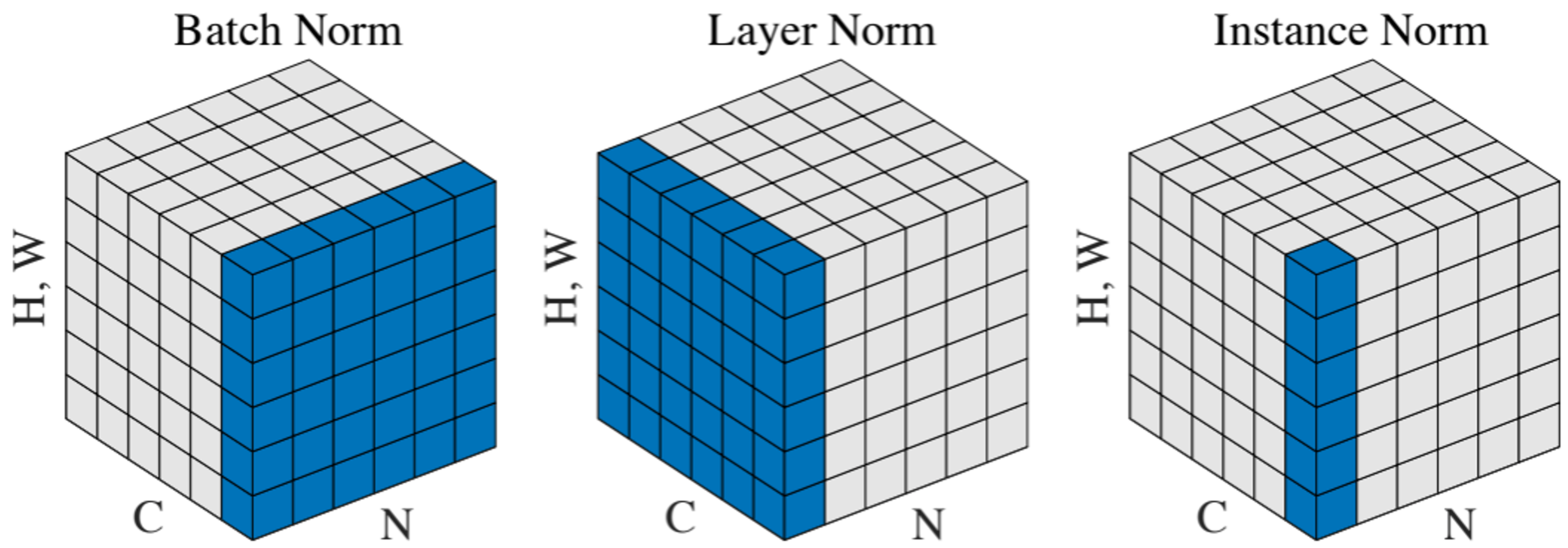
$$\boldsymbol{\mu}, \boldsymbol{\sigma}: \ N \times C \times 1 \times 1$$

$$\boldsymbol{\gamma}, \boldsymbol{\beta}: \ 1 \times C \times 1 \times 1$$

$$\mathbf{y} = \boldsymbol{\gamma}(\mathbf{x}-\boldsymbol{\mu})/\boldsymbol{\sigma}+\boldsymbol{\beta}$$

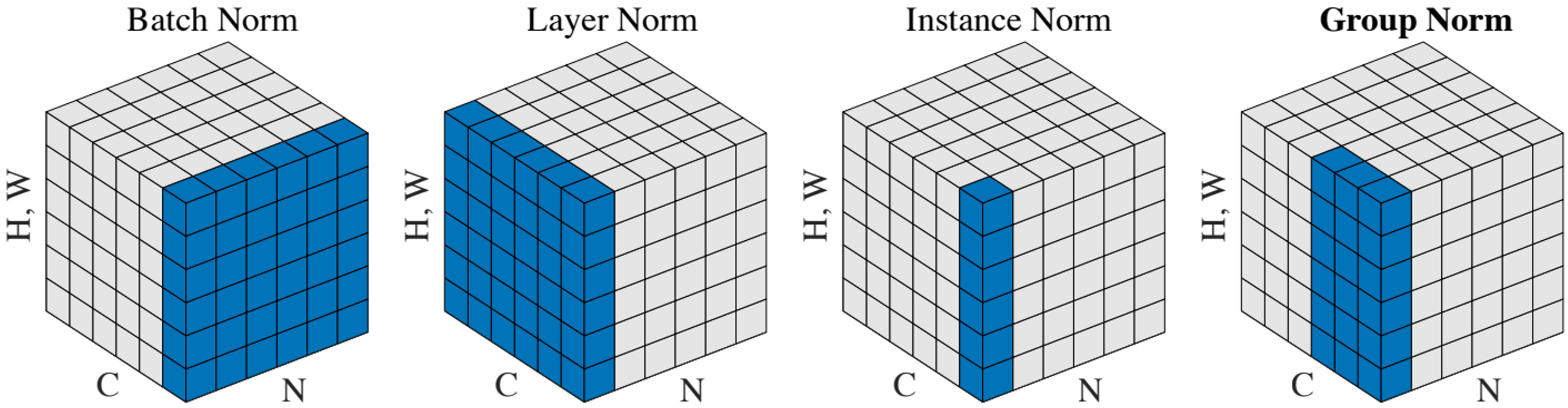
Ulyanov et al, Improved Texture Networks: Maximizing Quality and Diversity in Feed-forward Stylization and Texture Synthesis, CVPR 2017

Comparison of Normalization Layers



Wu and He, “Group Normalization”, ECCV 2018

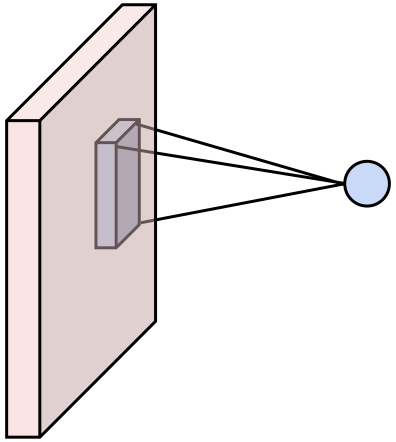
Group Normalization



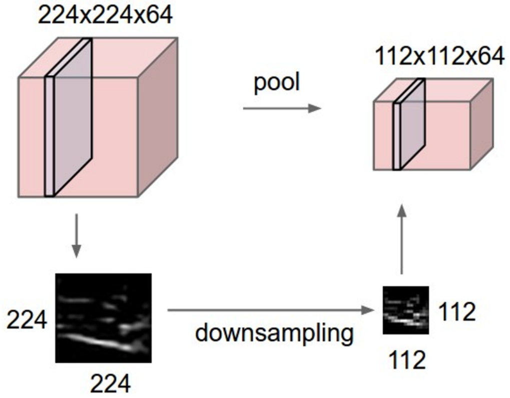
Wu and He, "Group Normalization", ECCV 2018

Components of a Convolutional Network

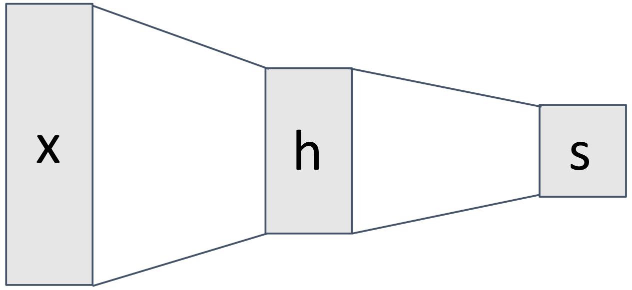
Convolution Layers



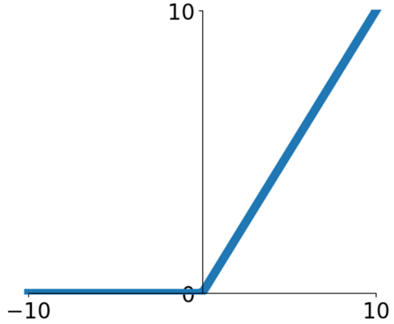
Pooling Layers



Fully-Connected Layers



Activation Function

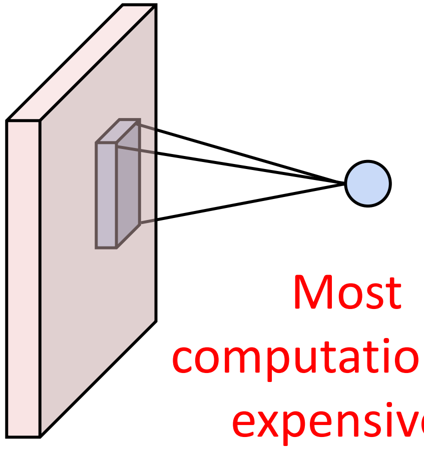


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

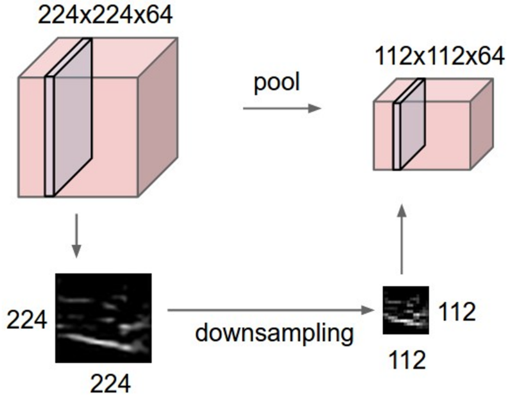
Components of a Convolutional Network

Convolution Layers

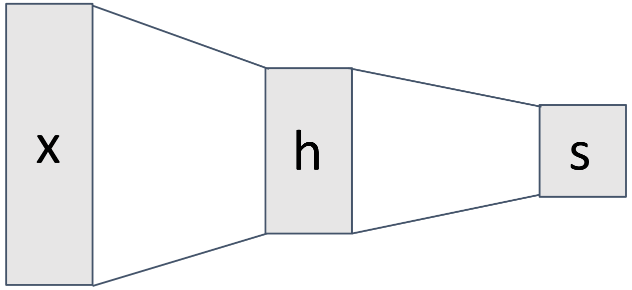


Most computationally expensive!

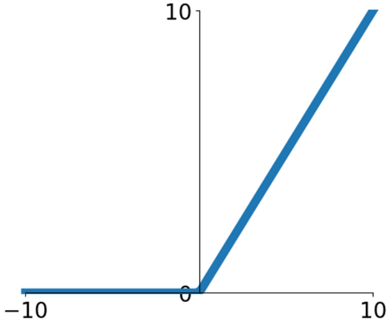
Pooling Layers



Fully-Connected Layers



Activation Function

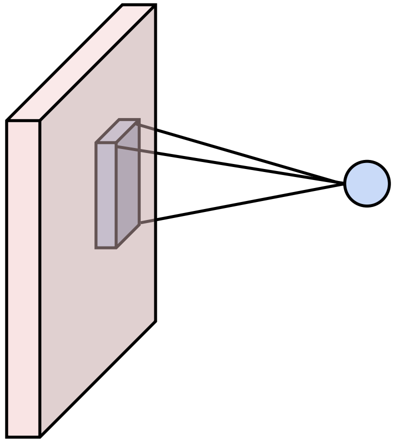


Normalization

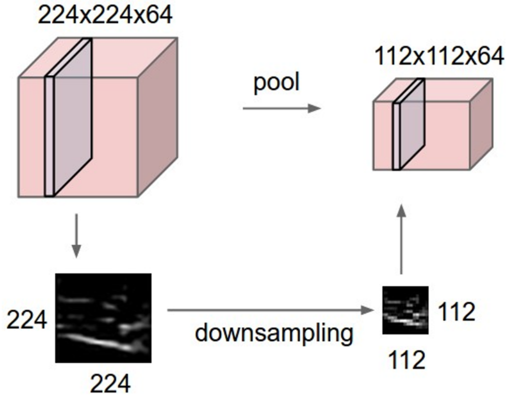
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Summary: Components of a Convolutional Network

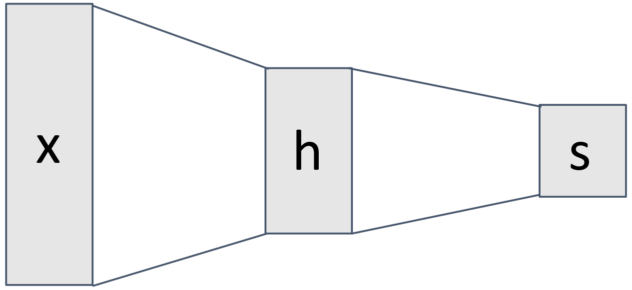
Convolution Layers



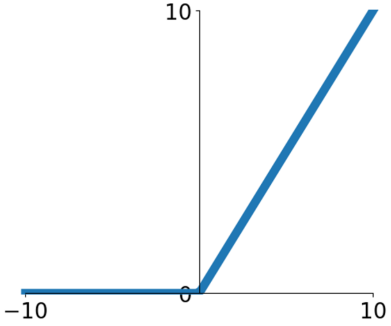
Pooling Layers



Fully-Connected Layers



Activation Function

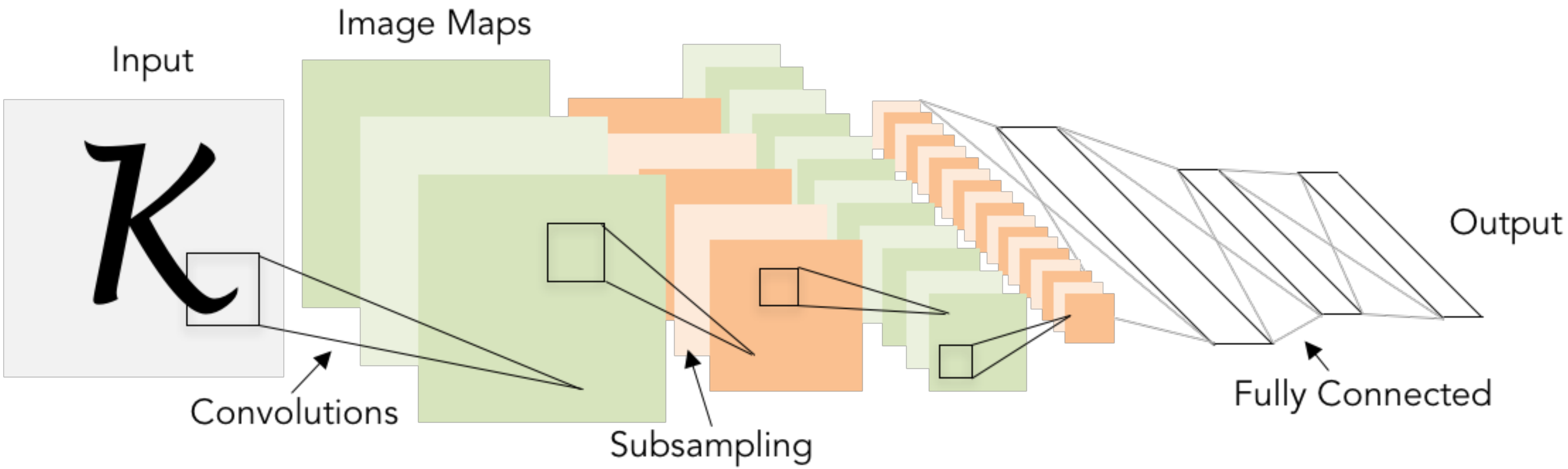


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Summary: Components of a Convolutional Network

Problem: What is the right way to combine all these components?



Convolutional neural networks++

- Training and optimization
- More regularization (dropout, …)
- Convolutional neural networks
- Pooling
- Batch normalization
- CNN architectures