Mutual Information Based Registration of Medical Images

Pluim et al: Survey

Mattes et al: CT/PET Registration

Background

- Mutual information-based registration was proposed by Viola and Wells (MIT) in 1994-5.

- It has become commonplace in many clinical applications.

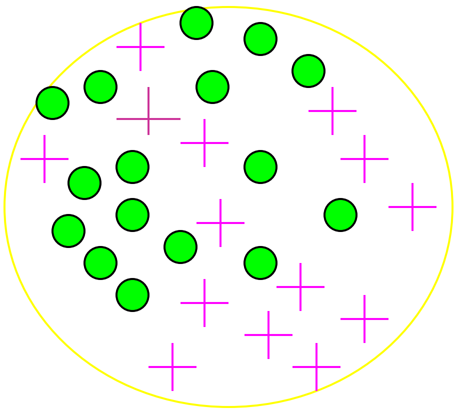
- It comes from information theory: the Shannon entropy

$$H = \sum p_i \log (1/p_i) = -\sum p_i \log p_i$$

- The more rare an event, the more meaning is associated with its occurrence

ENTROPY

- Entropy comes from information theory. The higher the entropy the more the information content.



- Entropy = $$\sum_i - p_i \log_2 p_i$$

p_i is the probability of event i

Compute it as the proportion of event i in the set.

16/30 are green circles; 14/30 are pink crosses

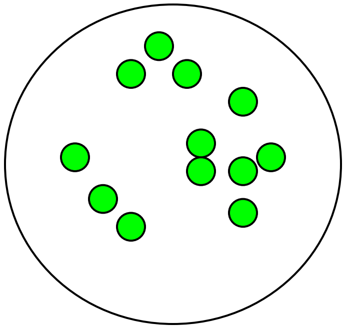
$\log_2(16/30)$ = -.9; $\log_2(14/30)$ = -1.1

Entropy = -(16/30)(-.9) –(14/30)(-1.1) = .99

2-Class Case:

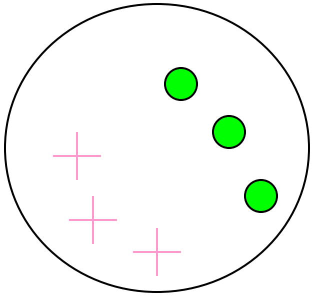
- What is the entropy of a group in which all examples belong to the same class?
  - entropy = $- 1 \log_2 1 = 0$

minimum entropy



- What is the entropy of a group with 50% in either class?
  - entropy = $-0.5 \log_2 0.5 - 0.5 \log_2 0.5 = 1$

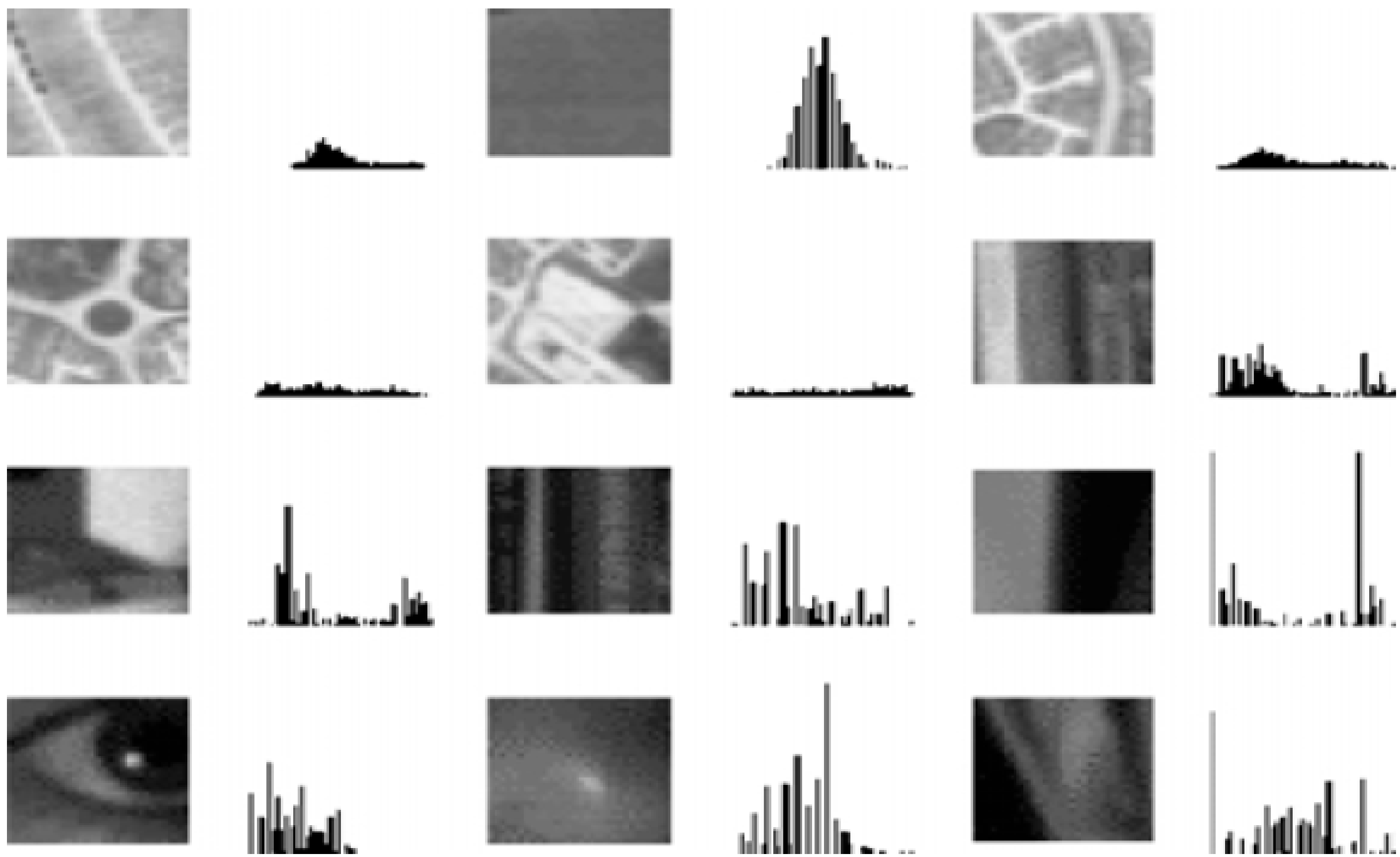
maximum entropy



Entropy for Images

- Shannon entropy can be computed for a gray-tone image.
- It then focuses on the distribution of the gray tones.
- An image consisting of almost a single intensity will have low entropy.
- An image with roughly equal quantities of different gray tones will have high entropy.

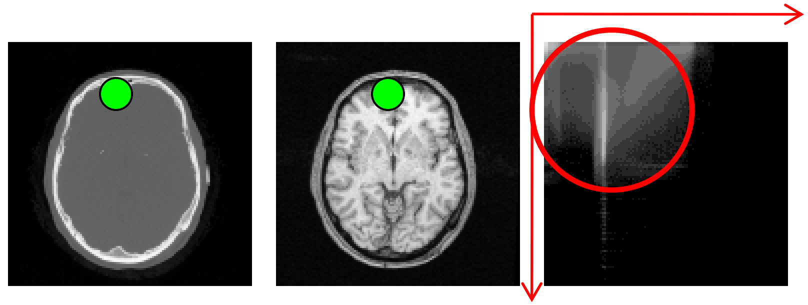
Histograms of Image Intensity



Mutual Information

- Woods introduced a registration measure for multimodality images in 1992.
- The measure was based on the assumption that regions of similar tissue (and similar gray tones) in one image would correspond to regions in the other image that also consist of similar gray values (but not the same as in the first image).
- Instead of defining regions of similar tissue in the image, they defined the regions in a feature space.
- When the images are correctly registered, the **joint histogram** of the two images will show certain clusters for gray tones of matching structures.

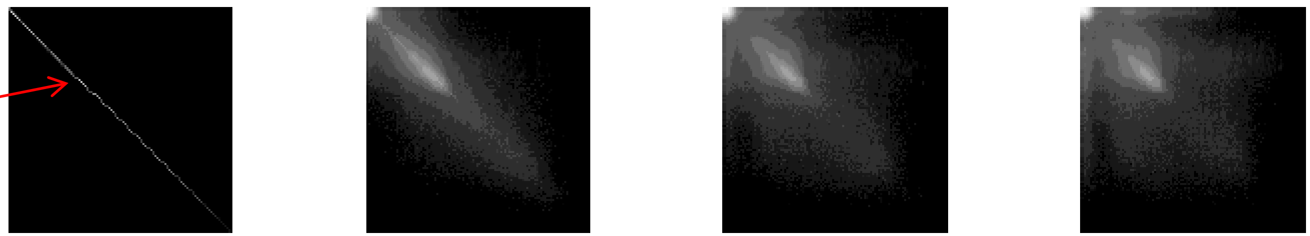
CT/MRI Example



CT Image MR Image Joint Histogram

- For each pair of corresponding points (**x**,**y**) with **x** in the CT image and **y** in the MR image, there is a gray tone correspondence (gx,gy).
- The joint histogram counts how many times each gray tone correspondence occurs.

Joint Gray-tone Histograms of an MR Image with itself at Different Rotations



3.82 | 6.79 | 6.98 | 7.15 joint entropy

0 degrees | 2 degrees | 5 degrees | 10 degrees

Because the images are identical, all gray-tone correspondences lie on the diagonal of the histogram matrix.

Measures of Mutual Information

- $H = -\sum p_{i,j} \log p_{i,j}$ is the Shannon entropy for a joint distribution; p_{ij} is probability of co-occurence of i and j.

  - Def. 1: I(A,B) = H(B) – H(B|A)

  - Def. 2: I(A,B) = H(A) + H(B) – H(A,B)

  - Def. 3: Kullback-Leibler distance

$$I(A,B) = \sum_{a,b} p(a,b) \log \frac{p(a,b)}{p(a)p(b)}.$$

joint gray values

joint in case of independent images

Different Aspects of Mutual Information Procedures

- Preprocessing (ie. filtering)
- Measures (entropy measure, normalization measures)
- Spatial Information (not just gray tones, but where)
- Transformation (applied to register images)
  - rigid
  - affine
  - deformable
- Implementation
  - interpolation
  - probability distribution estimation
  - optimization

Modalities

- MR with CT, PET, SPECT, US
- CT with PET, SPECT, other (video, fluoroscopy)
- Microscopy with other

Anatomical Entities

- brain
- thorax/lungs
- spine
- heart
- breast
- abdomen/liver
- pelvis
- tissue
- other

PET-CT Image Registration in the Chest Using Free-Form Deformations

David Mattes, David Haynor, et al
UW Medical Center

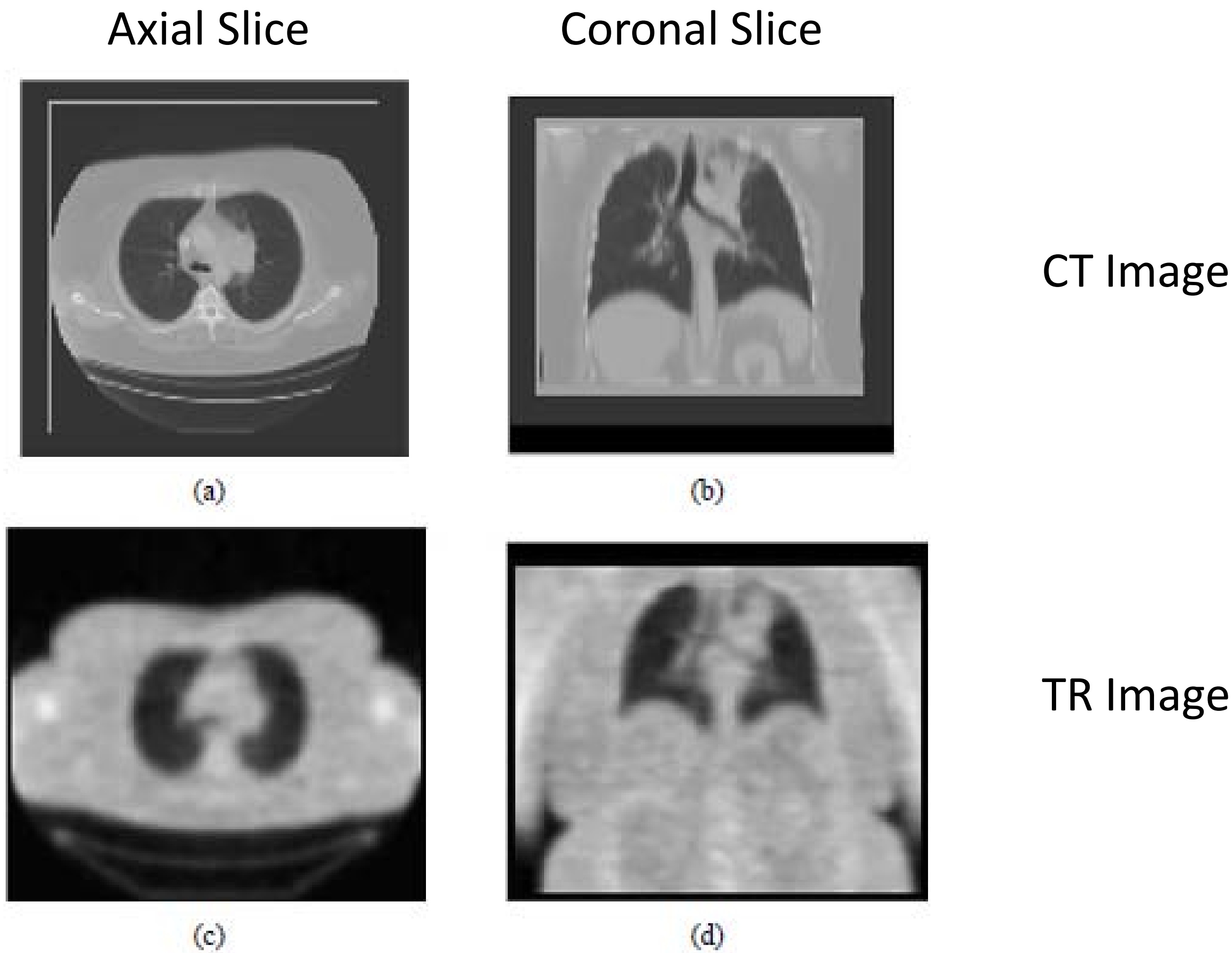
- Popular implementation of mutual information registration
- Available in ITK package
- We use it in our research.

Application

- PET-to-CT image registration in the chest
- Fuse images from a modality with high anatomic detail (CT)
- With images from a modality delineating biological function (PET)
- Producing a nonparametric deformation that registers them.

Overall Method

- The PET image has a corresponding transmission image (TR)
- The TR image is similar to a CT attenuation map with a higher energy radiation beam, resulting in less soft-tissue detail and limited resolution
- Once the TR and CT images are registered, the resulting transformation can be applied to the emission image for improved PET image interpretation.
- GOAL: find a deformation map that aligns the TR image with the CT image and evaluate the accuracy.


Axial Slice
Coronal Slice
CT Image
TR Image
(a)
(b)
(c)
(d)

Methodology

Notation

- $f_T(x)$ is a test image over domain V_T
- $f_R(x)$ is a reference image over domain V_R
- $g(x \mid \mu)$ is a deformation from V_T to V_R
- μ is the set of parameters of the transformation
- We want to find the set of parameters μ that minimizes an image discrepancy function S

$$\boxed{\mu = \arg\min \; S(f_R, f_T \circ g(\bullet \mid \mu))}$$

- They hypothesize that the set of transformation parameters that minimizes S brings the transformed test image into best registration with the reference image.

Image Representation

- Optimizing a function requires taking derivatives.
- Thus it is easier if the function can be represented in a form that is explicitly differentiable.
- This means that both the deformations and the similarity criterion must be differentiable.
- So images are represented using a B-Spline basis.
- Parzen windows are used instead of simple, bilinear interpolation.

SOME of the Math

- An image *f(x)*, coming in as a set of sampled values, is represented by a cubic spline function that can be interpolated at any between-pixel position.

$$f(\mathbf{x}) = \sum_i c_i \beta^{(3)}(\mathbf{x} - \mathbf{x}_i)$$

- The spline function is differentiable.

- The smoothed joint histogram of $(f_R, f_T \circ g(\bullet | \mu))$ is defined as a cross product of the two spline functions.

- Computation of mutual information requires
  - the smoothed joint histogram
  - the marginal smoothed histogram for the test image
  - the marginal probability for the reference image, which is independent of the transformation parameters

- The image discrepancy measure is the negative of mutual information S between the reference image and the transformed test image expressed as a function of the transformation parameters μ.

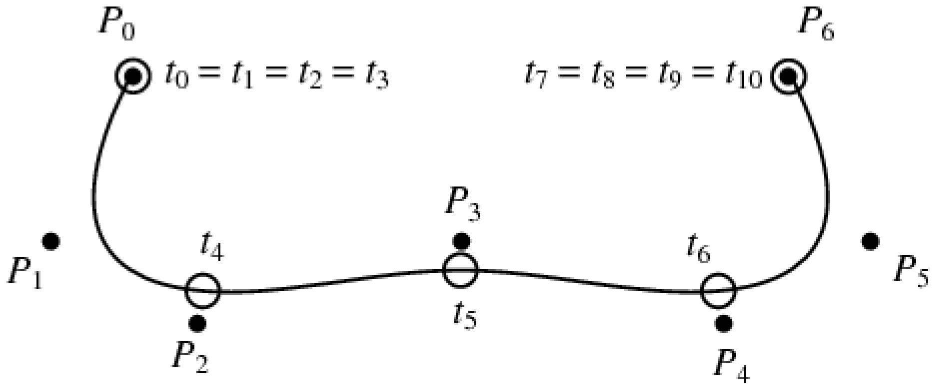
$$S(\mu) = -\sum_{\iota}\sum_{\kappa} p(\iota, \kappa|\mu) \log \frac{p(\iota, \kappa|\mu)}{p_T(\iota|\mu)p_R(\kappa)}$$

where p, p_T, and p_R are the joint, marginal test, and marginal reference probability distributions, respectively.

- The variables τ and κ are the histogram bin indexes for the reference and test images, respectively.

Deformations

- Deformations are also modeled as cubic B-splines.
- They are defined on a much coarser grid.
- A deformation is defined on a sparse, regular grid of control points placed over the test image.
- A deformation is varied by defining the motion $g(\lambda_j)$ at each control point λ_j.



Transformation

- The transformation of the test image is specified by mapping reference image coordinates according to a locally perturbed rigid body transformation.

- The parameters of the transformation are:

$$\mu = \{ \gamma, \theta, \phi, t_x, t_y, t_z; \delta_j \}$$

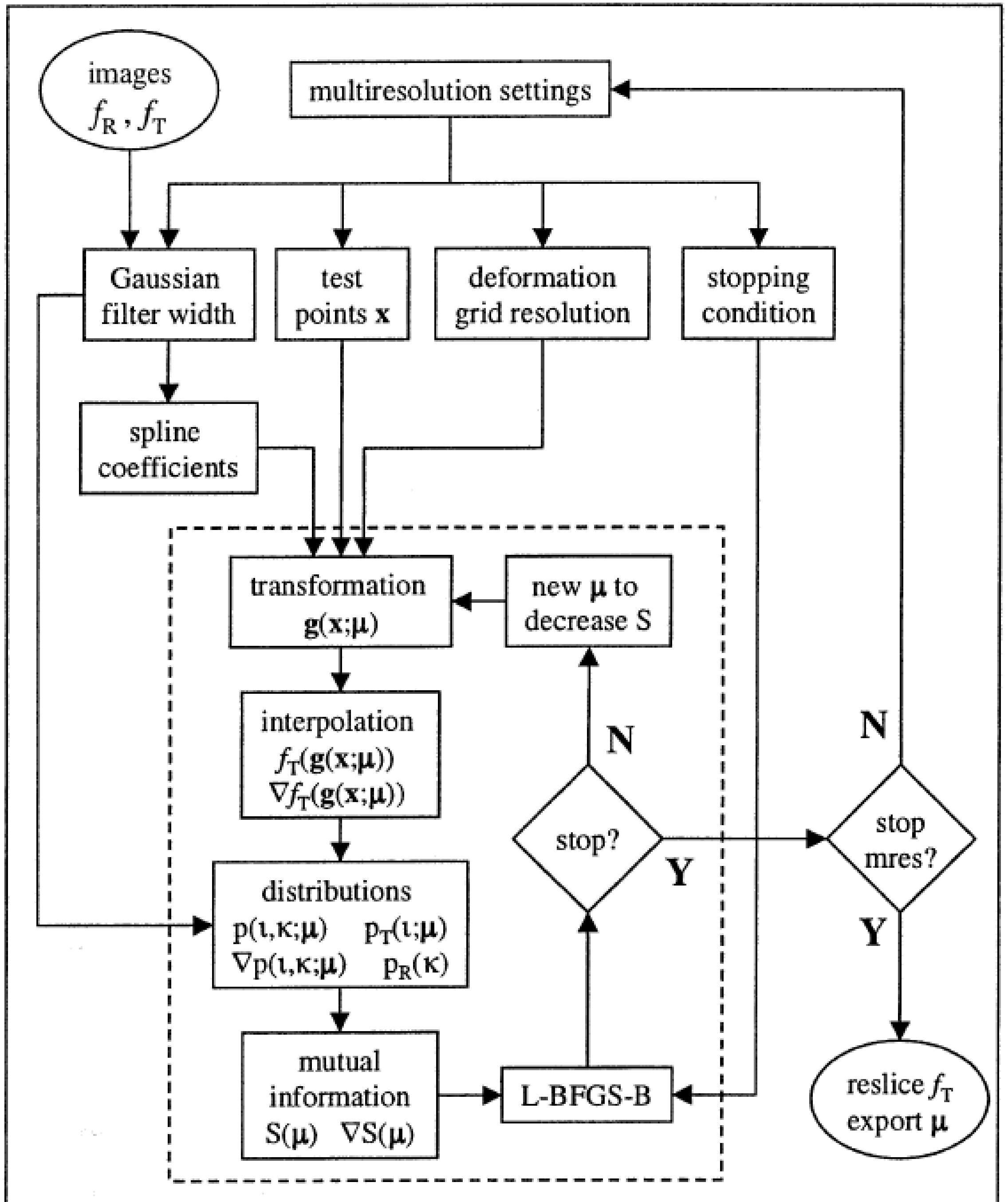
$\{\gamma, \theta, \phi\}$ are the roll-pitch-yaw Euler angles,

$[t_x, t_y, t_z]$ is the translation vector,

and δ_j is the set of deformation coefficients (2200 of them)

Multiresolution Optimization Strategy

- The registration process is automated by varying the deformation in the test image until the discrepancy between the two images is minimized.
- The alignment process is divided into two registrations: one for the rigid body part and one for the deformation
- A limited-memory, quasi-Newton minimization package is used.
- To avoid local minima and decrease computation time, a hierarchical multiresolution optimization scheme is used.


images f_R , f_T
multiresolution settings
Gaussian filter width
test points **x**
deformation grid resolution
stopping condition
spline coefficients
transformation **g**(**x**;**μ**)
new **μ** to decrease S
interpolation
$f_T(\mathbf{g}(\mathbf{x};\boldsymbol{\mu}))$
$\nabla f_T(\mathbf{g}(\mathbf{x};\boldsymbol{\mu}))$
distributions
$p(\iota,\kappa;\boldsymbol{\mu})$ $p_T(\iota;\boldsymbol{\mu})$
$\nabla p(\iota,\kappa;\boldsymbol{\mu})$ $p_R(\kappa)$
mutual information
$S(\boldsymbol{\mu})$ $\nabla S(\boldsymbol{\mu})$
L-BFGS-B
stop?
N
Y
stop mres?
N
Y
reslice f_T export **μ**

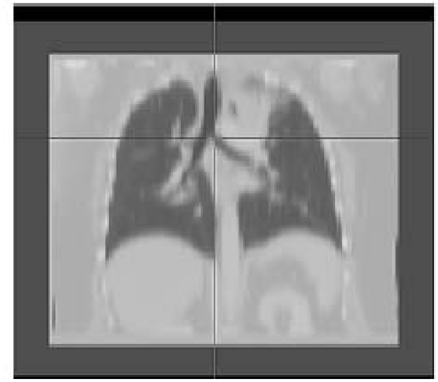
Results

- 28 patients, 27 successful registrations
- 205 slices per image
- average of 100 minutes per registration
- 10 minutes for the rigid body part
- 90 minutes for the deformable part
- error index of .54, which is in the 0 to 6mm error range

axial | coronal

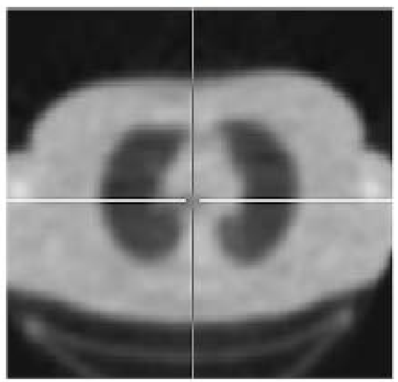
CT

(a)

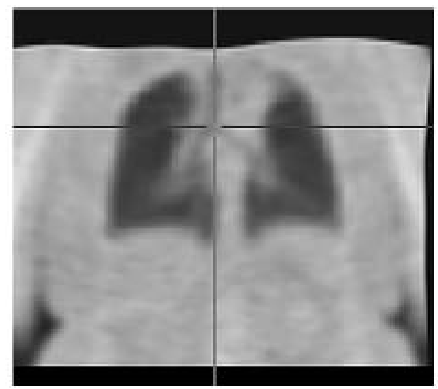


(b)

registered TR

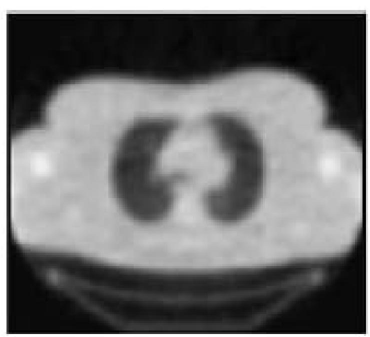


(c)



(d)

unregistered TR



(e)

(f)

Sample Images from 7 Anatomic Locations