Medical Images

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CSE/EE 577, Autumn 2011

Overview

Production of X-ray images

Cross sectional images

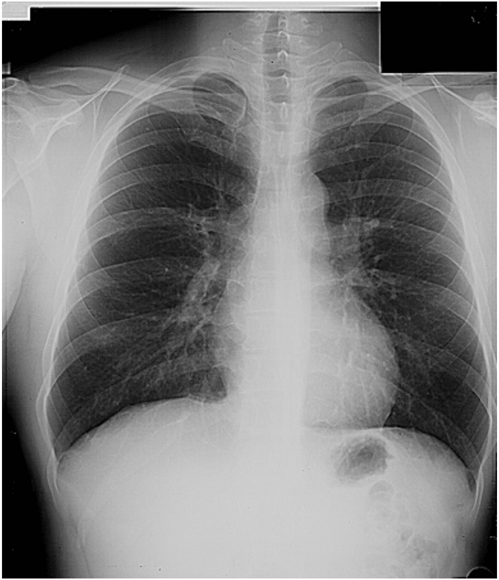
Reformatted images

Other imaging modalities

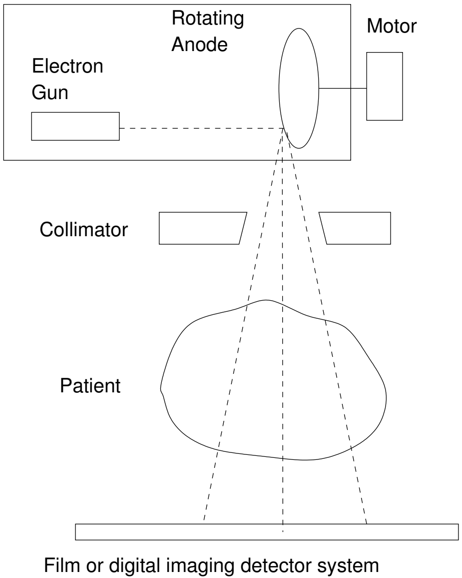
Types of medical images

- X-ray projections (chest X-ray, joints, etc.)
- X-ray cross sectional image (Computed Tomography, or CT)
- Nuclear Medicine scans (bone scans)
- Positron Emission Tomography
- Ultrasound
- Magnetic Resonance Images (MRI)
- other...

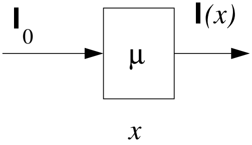
An example X-ray image



The process of X-ray image production



Attenuation



Photons get absorbed in proportion to the number and cross section of atoms in the slab. The intensity change for a small thickness Δx is

$$\Delta I = -\mu I \Delta x$$

where μ is the "attenuation coefficient". For continuous media, this gives

$$\frac{dI}{dx} = -\mu I$$

so

$$I(x) = I_0 e^{-\mu x}$$

Attenuation with varying density

If μ varies as the X-rays traverse a thick body, projecting from three dimensions to two (y and z) for an image,

$$I(y,z) = I_0 e^{-\int_0^w \mu(x,y,z)dx}$$

The intensity of X-rays that reach the film or detector will depend on the total material along each ray from the source to the image, so objects will appear as *overlapped* or obscured.

Scattering and absorption of photons (X-rays)

- Photoelectric effect - a photon is absorbed by an atom and an electron is emitted:

$$\mu_p \propto \rho Z^3 E^{-3}$$

- Compton scattering - a photon collides with an orbital electron and is deflected:

$$\mu_c \propto \rho \left(\frac{Z}{A} \right) E^{-1}$$

- Pair production - a photon is absorbed in a collision with a nucleus and an electron-positron pair is produced.

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- At 70 KeV the probability of Photoelectric absorption is much greater than that of Compton scattering, and is highly dependent on the kind of atom (actually atomic number)
- At 160 KeV Compton scattering predominates and is not dependent on composition, only tissue density.

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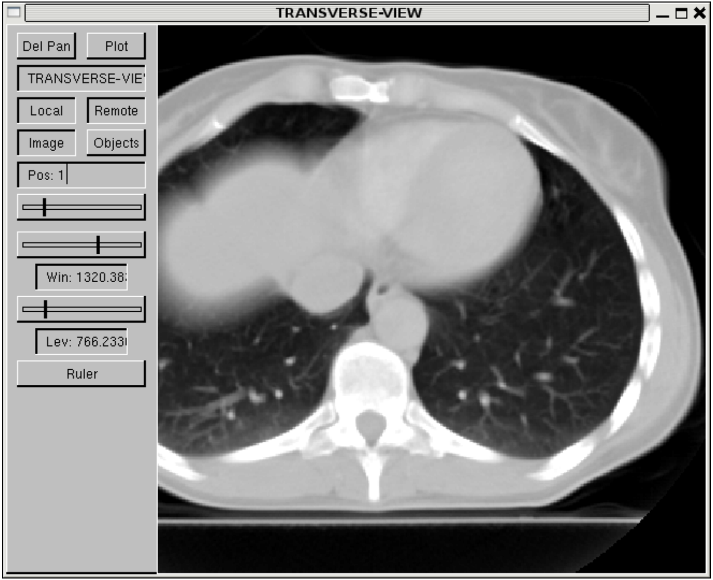
- These ideas were first used for radio astronomy by Bracewell and others to map the Sun.
- Later, Cormack, Hounsfield and others showed how to use them for medical X-ray images.

A CT cross sectional image of the chest



The Fourier Transform

Every sufficiently smooth function can be considered as made up of sine (or cosine) waves with varying weights. The weights as a function of the wave frequency define a function called the Fourier transform.

- If f is a function of x, the Fourier transform of f is

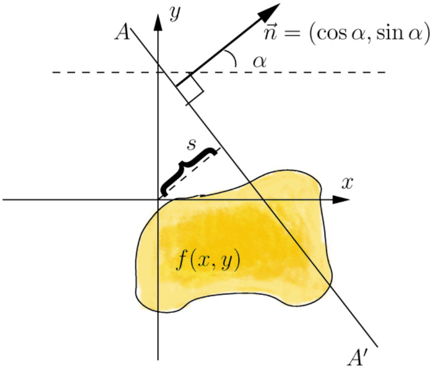
$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(x)e^{-ix\omega}dx$$

- The two dimensional version is just a double integral with x and y, giving a function of ω_x and ω_y

$$\hat{f}(\omega_x, \omega_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)e^{-ix\omega_x}e^{-iy\omega_y}dxdy$$

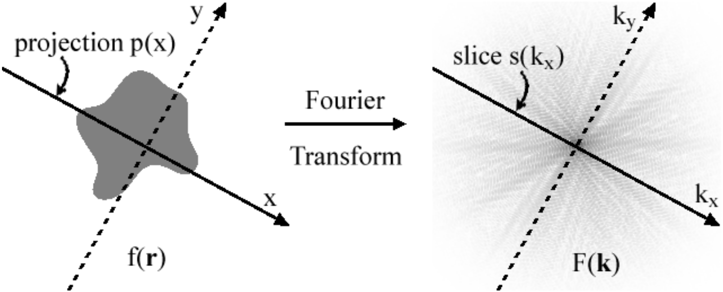
- The inverse transform looks similar, except the sign in the exponent is changed.

The Radon Transform



The Radon Transform of a function of two variables is the integral of that function along a line in the two-dimensional plane. It is a projection, something like the X-ray formula.

The Projection Slice Theorem



For simplicity just project onto the x axis, giving

$$p(x) = \int_{-\infty}^{\infty} f(x, y) dy$$

The Projection Slice Theorem, continued

The slice through the Fourier transform of f corresponding to this is the function of ω_x obtained by setting $\omega_y = 0$.

$$s(\omega_x) = \hat{f}(\omega_x, 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-ix\omega_x} dxdy$$

This is just the one-dimensional Fourier transform of the projection.

$$\hat{p} = \int_{-\infty}^{\infty} p(x) e^{-ix\omega_x} dx$$

To get the entire $\hat{f}$, just rotate the projection, which also rotates the Fourier space coordinates, and then transform back.

Image Reconstruction

A more complete description of the steps:

- collect all the projections (Radon transforms) at all the different angles (only 180 degrees are needed),

$$R[f(x,y)](p,\theta) = \int_{-\infty}^{\infty} f(p\cos\theta - q\sin\theta, p\sin\theta + q\cos\theta)dq$$

  where p, q are the rotated (linear) coordinate system,

- Fourier transform each one (one dimensional transform) in the corresponding p direction,

$$\begin{aligned} F[R](\omega,\theta) &= \int_{-\infty}^{\infty} R(p,\theta)e^{-i\omega p}dp \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-i\omega p}dpdq \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-i\omega(x\cos\theta + y\sin\theta)}dxdy \end{aligned}$$

Image Reconstruction continued

- The collection of 1-D Fourier transforms is (by inspection) the two dimensional transform with respect to new variables, $u = \omega \cos\theta$ and $v = \omega \sin\theta$, i.e.,

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-i(xu+yv)} dxdy$$

- so, Fourier transform back the entire collection (two dimensional transform) in terms of u, v,

$$f_{recon}(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{i(xu+yv)} dudv$$

Filtered Back-Projection

Instead of all the Fourier transforms, each projection can be "back-projected" as it is obtained. To do this, rewrite in terms of integration over ω and θ. This change of variables gives a factor of $|\omega|$,

$$f_{recon}(r,\theta) = \int_0^{\pi} \int_{-\infty}^{\infty} |\omega| F(\omega,\theta) e^{-i\omega p} d\omega d\theta$$

The integrand is the (inverse) Fourier transform of $|\omega| F(\omega,\theta)$ which can be rewritten as the convolution of the individual (inverse) Fourier transforms, i.e.,

$$f_{recon} = \int_0^{\pi} F^{-1}[|\omega|] \otimes R(p,\theta) d\theta$$

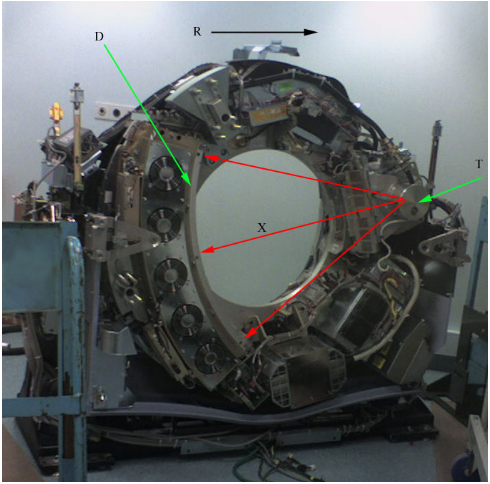
The convolution of each projection can be computed while the machine collects the data for the next angle. The function $F^{-1}[|\omega|]$ is called the *filter* function, and this version is called *filtered back-projection*.

Other Reconstruction Algorithms

Many different filter functions have been tried in addition to the mathematically implied one. They can be adjusted to correct for finite, limited data sets, and other problems.
Another method is to treat the problem as an algebraic inverse problem.

1. Start with a uniform image.
2. Compute its projections, and the difference between them and the measured projections.
3. Adjust the image points in proportion to the difference.
4. Iterate until the error is low enough to accept.

CT Image Data Collection



CT scanner with cover removed to show the X-ray tube, the detectors and the ring on which both rotate synchronously.

Hounsfield Units

The numbers computed for each pixel are normalized to a scale known as “Hounsfield Units,” after Godfrey Hounsfield. The Hounsfield scale is defined so that the Hounsfield number of water at standard temperature and pressure is 0, and the Hounsfield number of air is -1000. The formula is:

$$H(x) = \frac{\mu(x) - \mu_{water}}{\mu_{water} - \mu_{air}} \times 1000$$

Typical bone values range from 400 to 2000 HU.
Although most articles and texts refer to μ as a “linear attenuation coefficient”, the CT reconstruction does not exactly produce such numbers, despite the theory in the preceding slides.

Window and Level

The range of Hounsfield Units from air to hard bone is over 2,000 units (bone is around 1,000 H). This is too many gray levels to be useful. Typical gray scale displays offer only 256 gray levels. Thus, a subset of the full range is usually selected to map between 0 and 255 in display intensities. The *window* refers to the width of this subset and the *level* refers to its center.

$$H_l = L - W/2$$

$$H_u = L + W/2$$

$$G(H) = \begin{cases} 0 & \text{if } H \leq H_l \\ (H - H_l)/W \times 255 & \text{if } H_l < H < H_u \\ 255 & \text{if } H \geq H_u \end{cases}$$

Gray scale mapping

A more flexible approach to mapping from image pixels to display values uses a map array and lookups. It is faster than repeating the interpolation for many identically valued pixels.

```
(defun make-graymap (window level range-top)
  (let* ((map (make-array (1+ range-top)))
         (low-end (- level (truncate (/ window 2))))
         (high-end (+ low-end window)))
    (do ((i 0 (1+ i)))
        ((= i low-end))
      (setf (aref map i) 0)) ;; black
    (do ((i low-end (1+ i)))
        ((= i high-end))
      (setf (aref map i)
        (round (/ (* 255 (- i low-end)) window))))
    (do ((i high-end (1+ i)))
        ((> i range-top))
      (setf (aref map i) 255))
    map))
```

Applying the gray scale map

```
(defun map-image (raw-image window level range)
  (let* ((x-dim (array-dimension raw-image 1))
         (y-dim (array-dimension raw-image 0))
         (new-image (make-array (list y-dim x-dim)))
         (map (make-graymap window level range)))
    (dotimes (i y-dim)
      (dotimes (j x-dim)
        (setf (aref new-image i j)
          (aref map (aref raw-image i j)))))
    new-image))
```

An advantage of this is that by redefining `make-graymap` you can use any transformation function you want, e.g., a logarithmic, bilinear, trillinear or other scale.

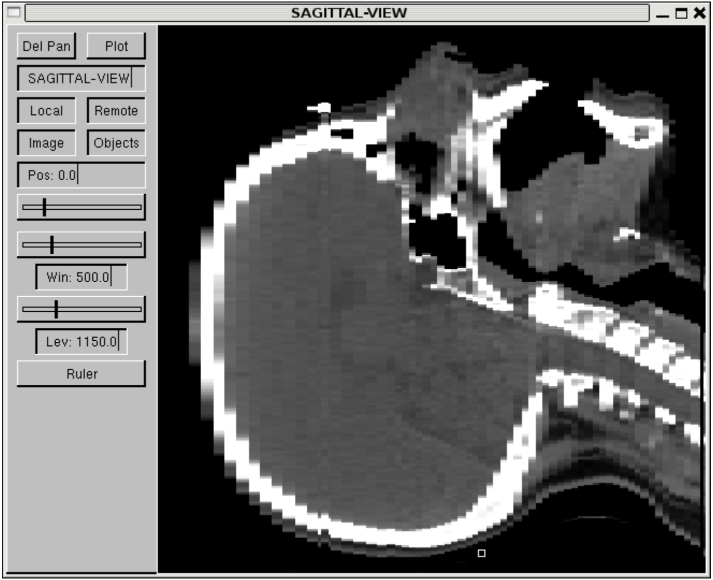
Beam hardening

The equations for attenuation, projection and reconstruction assume that the X-ray beam consists of photons of a single energy. If the beam is not monoenergetic, the transmitted intensity is obtained by integrating over energy.

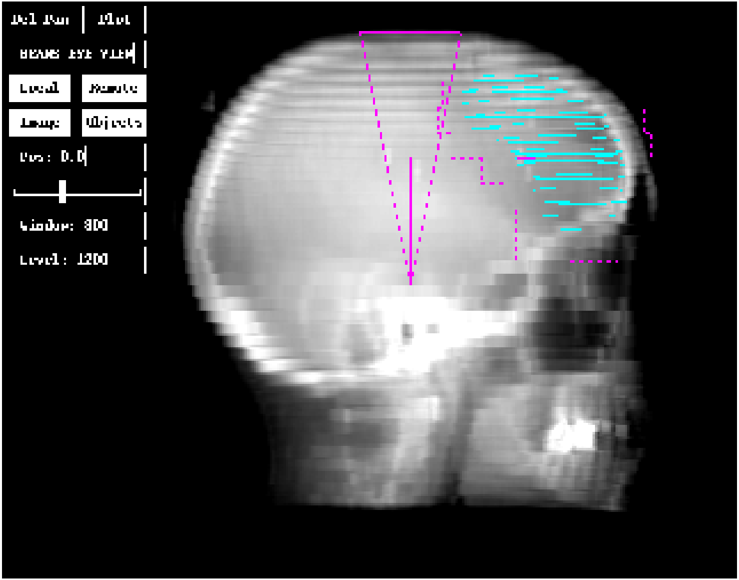
$$I(y) = \int_0^{E_{max}} I_0(E) e^{-\int_0^w \mu(x,y,E)dx} dE$$

Then $\log I$ is no longer the simple line integral or Radon transform of a two dimensional image. Applying the basic reconstruction methods produces artifacts such as streaking. Correcting for this effect is a complex engineering problem, but has largely been solved in modern CT scanners.

A sagittal image computed from cross sectional images



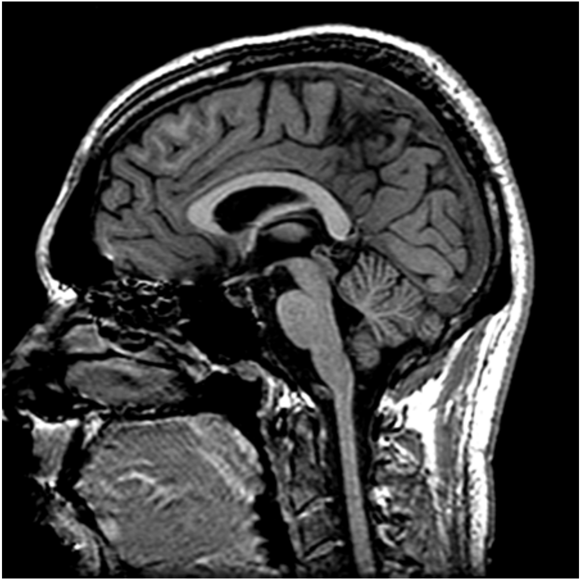
A reprojection from cross sectional images



Prism demo

Live demonstration of window and level controls and reformatting of image data to produce sagittal, coronal and reprojected images (beam's eye views).

Magnetic Resonance Images



Ultrasound


CHOI LUCIA
LC12/5/70
CPMC
RAB 4-8P/Obstetric
11.8cm
14Hz
07-12-2004
10:09:11
Fet.A.CRI
Har-low
Pwr 0
Gn 0
C9 / M7
E0
MI 1.2
CRI

Nuclear Medicine Scans

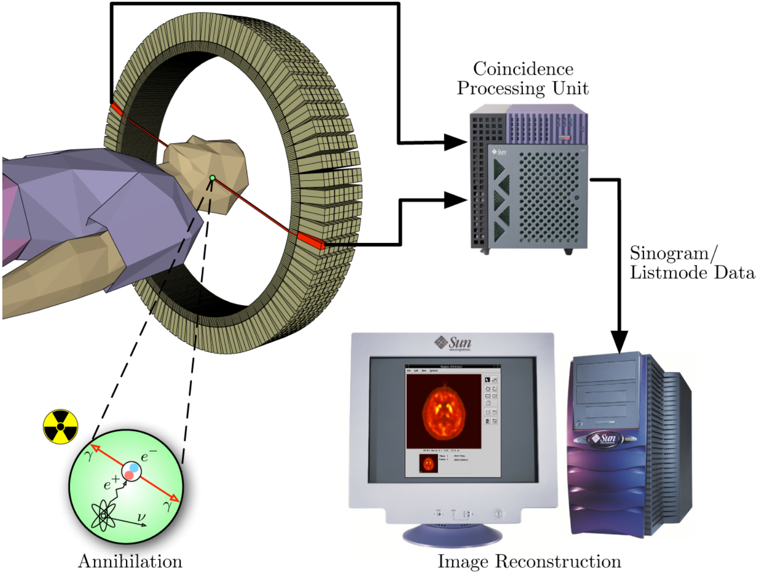
- Sometimes known as “bone scans”
- Involve injection of radio-isotopes in the blood stream
- Similar to X-rays, except the photons are *emitted* rather than transmitted
- The intense areas are those where the radio-isotope was absorbed most
- The brightness of the image indicates *activity*, i.e., uptake.

An example nuclear medicine scan

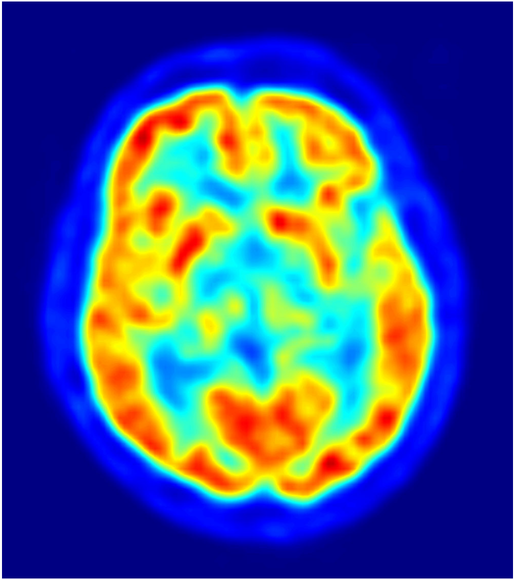
Positron Emission Tomography

- Also uses radio-isotopes (positron emitters)
- Positron annihilation produces *pairs* of photons
- Photons are counted in lines in all directions, similar to CT
- Cross section reconstruction algorithms are similar to CT
- The brightness in these images also indicate metabolic activity

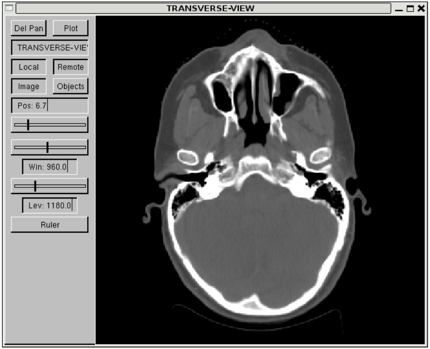
A schematic showing the PET scanner and associated systems



An example PET image



Diagnosis with images



Left: a CT of a person's head, and Right, a PET scan of the same person's head.
What is the diagnosis?