# CSE 582 – Compilers

LR Parser Construction
Hal Perkins
Autumn 2002



## Agenda

- LR(0) state construction
- FIRST, FOLLOW, and nullable
- Variations SLR, LR(1), LALR



## LR State Machine

- Idea: Build a DFA that recognizes handles
  - Language generated by a CFG is generally not regular, but
  - Language of handles for a CFG is regular
  - Parser reduces when DFA accepts



## Prefixes, Handles, &c (review)

- If *S* is the start symbol of a grammar *G*,
  - If *S* =>* $\alpha$ then $\alpha$ is a *sentential form* of *G*
  - $\gamma$ is a *viable prefix* of *G* if there is some derivation S $=>^*_{rm}$ $\alpha A$w $=>^*_{rm}$ $\alpha\beta$w and $\gamma$ is a prefix of $\alpha\beta$.
  - The occurrence of $\beta$ in $\alpha\beta$w is a *handle* of $\alpha\beta$w
- An *item* is a marked production (a . at some position in the right hand side)
  - [*A* ::= . *X Y*]   [*A* ::= *X* . *Y*]   [*A* ::= *X Y* . ]



## Building the LR(0) States

- Example grammar
  - *S′* ::= *S* $
  - *S* ::= ( *L* )
  - *S* ::= x
  - *L* ::= *S*
  - *L* ::= *L* , *S*
  - We add a production S′ with the original start symbol followed by end of file ($)
  - Question: What language does this grammar generate?



## Start of LR Parse

*S′* ::= *S* $
*S* ::= ( *L* )
*S* ::= x
*L* ::= *S*
*L* ::= *L* , *S*

- Initially
  - Stack is empty
  - Input is the right hand side of *S′*, i.e., *S* $
  - Initial configuration is [*S′* ::= . *S* $]
  - But, since position is just before *S*, we are also just before anything that can be derived from *S*

## Initial state

*S′* ::= *S* $
*S* ::= ( *L* )
*S* ::= x
*L* ::= *S*
*L* ::= *L* , *S*

*S′* ::= . *S* $ ← start
*S* ::= . ( *L* ) ← completion
*S* ::= . x ← completion

- A state is just a set of items
  - Start: an initial set of items
  - Completion (or closure): additional productions whose left hand side appears to the right of the dot in some item already in the state



## Shift Actions (1)

*S′* ::= *S* $
*S* ::= ( *L* )
*S* ::= x
*L* ::= *S*
*L* ::= *L* , *S*

*S′* ::= . *S* $
*S* ::= . ( *L* )
*S* ::= . x
—x→ *S* ::= x .

- To shift past the x, add a new state with the appropriate item(s)
  - In this case, a single item; the closure adds nothing
  - This state will lead to a reduction since no further shift is possible



## Shift Actions (2)

*S′* ::= *S* $
*S* ::= ( *L* )
*S* ::= x
*L* ::= *S*
*L* ::= *L* , *S*

*S′* ::= . *S* $
*S* ::= . ( *L* )
*S* ::= . x
—(→
*S* ::= ( . *L* )
*L* ::= . *L* , *S*
*L* ::= . *S*
*S* ::= . ( *L* )
*S* ::= . x

- If we shift past the ( , we are at the beginning of *L*
- the closure adds all productions that start with *L*, which requires adding all productions starting with *S*



## Goto Actions

*S′* ::= *S* $
*S* ::= ( *L* )
*S* ::= x
*L* ::= *S*
*L* ::= *L* , *S*

*S′* ::= . *S* $
*S* ::= . ( *L* )
*S* ::= . x
—*S*→ *S′* ::= *S* . $

- Once we reduce *S*, we'll pop the rhs from the stack exposing the first state. Add a *goto* transition on *S* for this.



## Basic Operations

- *Closure* ( *S* )
  - Adds all items implied by items already in *S*
- *Goto* ( *I*, *X* )
  - *I* is a set of items
  - *X* is a grammar symbol (terminal or non-terminal)
  - *Goto* moves the dot past the symbol *X* in all appropriate items in set *I*



## Closure Algorithm

- *Closure* ( *S* ) =

  repeat
    for any item [A ::= $\alpha$ . *X* $\beta$] in *S*
      for all productions *X* ::= $\gamma$
        add [*X* ::= . $\gamma$] to *S*
  until *S* does not change
  return *S*

## Goto Algorithm

- *Goto* (*I*, *X* ) =

  set *new* to the empty set
  for each item [A ::= α . *X* β] in *I*
      add [A ::= α *X* . β] to *new*
  return *Closure* (*new* )

  - This may create a new state, or may return an existing one

10/14/2002 © 2002 Hal Perkins & UW CSE E-13

## LR(0) Construction

- First, augment the grammar with an extra start production *S′* ::= *S* $
- Let *T* be the set of states
- Let *E* be the set of edges
- Initialize *T* to *Closure* ( [*S′* ::= . *S* $] )
- Initialize *E* to empty

10/14/2002 © 2002 Hal Perkins & UW CSE E-14

## LR(0) Algorithm

repeat
  for each state *I* in *T*
    for each item [*A* ::= α . *X* β] in *I*
      Let *new* be *Goto* ( *I*, *X* )
      Add *new* to *T* if not present
      Add $I \xrightarrow{X} new$ to *E* if not present
until *E* and *T* do not change in this iteration

- Footnote: For symbol $, we don't compute *goto* (*I*, $); instead, we make this an *accept* action.

10/14/2002 © 2002 Hal Perkins & UW CSE E-15

## LR(0) Reduce Actions

- Algorithm:

  Initialize *R* to empty
  for each state *I* in *T*
    for each item [*A* ::= α .] in *I*
      add (*I*, *A* ::= α) to *R*

10/14/2002 © 2002 Hal Perkins & UW CSE E-16

## Building the Parse Tables (1)

- For each edge $I \xrightarrow{X} J$
  - if X is a terminal, put s*j* in column X, row *I* of the action table (shift to state *j* )
  - If X is a non-terminal, put g*j* in column X, row *I* of the goto table

10/14/2002 © 2002 Hal Perkins & UW CSE E-17

## Building the Parse Tables (2)

- For each state *I* containing an item [*S′* ::= *S* . $], put *accept* in column $ of row *I*
- Finally, for any state containing [*A* ::= γ .] put action r*n* in every column of row *n* in the table, where *n* is the production number

10/14/2002 © 2002 Hal Perkins & UW CSE E-18

## Example: States for

*S′* ::= *S* $
*S* ::= ( *L* )
*S* ::= x
*L* ::= *S*
*L* ::= *L* , *S*





## Example: Tables for

*S′* ::= *S* $
*S* ::= ( *L* )
*S* ::= x
*L* ::= *S*
*L* ::= *L* , *S*





## Where Do We Stand?

- We have built the LR(0) state machine and parser tables
  - No lookahead yet
  - Different variations of LR parsers add lookahead information, but basic idea of states, closures, and edges remains the same





## A Grammar that is not LR(0)

- Build the state machine and parse tables for a simple expression grammar

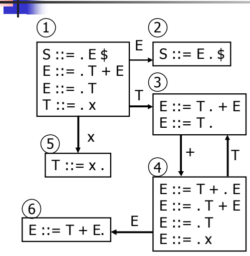
  *S* ::= *E* $
  *E* ::= *T* + *E*
  *E* ::= *T*
  *T* ::= x





## LR(0) Parser for

0. *S* ::= *E* $
1. *E* ::= *T* + *E*
2. *E* ::= *T*
3. *T* ::= x




| | x | + | $ | E | T |
|---|---|---|---|---|---|
| 1 | s5 | | | g2 | G3 |
| 2 | | | acc | | |
| 3 | r2 | s4,r2 | r2 | | |
| 4 | s5 | | | g6 | G3 |
| 5 | r3 | r3 | r3 | | |
| 6 | r1 | r1 | r1 | | |

- State 3 is has two possible actions on +
  - shift 4, or reduce 2
- ∴ Grammar is not LR(0)





## SLR Parsers

- Idea: Use information about what can follow a non-terminal to decide if we should perform a reduction
- Easiest form is SLR – Simple LR
- So we need to be able to compute FOLLOW(*A* ) – the set of symbols that can follow *A* in any possible derivation
  - But to do this, we need to compute FIRST($\gamma$) for strings $\gamma$ that can follow *A*

## Calculating FIRST($\gamma$)

- Sounds easy… If $\gamma = X\,Y\,Z$, then FIRST($\gamma$) is FIRST($X$), right?
  - But what if we have the rule $X ::= \varepsilon$?
  - In that case, FIRST($\gamma$) includes anything that can follow an $X$ – FOLLOW($X$)



## FIRST, FOLLOW, and nullable

- nullable($X$) is true if $X$ can derive the empty string
- Given a string $\gamma$ of terminals and non-terminals, FIRST($\gamma$) is the set of terminals that can begin strings derived from $\gamma$.
- FOLLOW($X$) is the set of terminals that can immediately follow $X$ in some derivation
- All three of these are computed together



## Computing FIRST, FOLLOW, and nullable (1)

- Initialization

  set FIRST and FOLLOW to be empty sets
  set nullable to false for all non-terminals
  set FIRST[a] to a for all terminal symbols a



## Computing FIRST, FOLLOW, and nullable (2)

repeat
  for each production $X := Y_1\ Y_2 \ldots Y_k$
    if $Y_1 \ldots Y_k$ are all nullable (or if $k = 0$)
      set nullable[$X$] = true
    for each $i$ from 1 to k and each $j$ from $i$+1 to $k$
      if $Y_1 \ldots Y_{i-1}$ are all nullable (or if $i = 1$)
        add FIRST[$Y_i$] to FIRST[$X$]
      if $Y_{i+1} \ldots Y_k$ are all nullable (or if $i = k$)
        add FOLLOW[$X$] to FOLLOW[$Y_i$]
      if $Y_{i+1} \ldots Y_{j-1}$ are all nullable (or if i+1=j)
        add FIRST[$Y_j$] to FOLLOW[$Y_i$]
Until FIRST, FOLLOW, and nullable do not change



## Example

- Grammar

  $Z ::= d$
  $Z ::= X\,Y\,Z$
  $Y ::= \varepsilon$
  $Y ::= c$
  $X ::= Y$
  $X ::= a$

| | nullable | FIRST | FOLLOW |
|---|---|---|---|
| $X$ | | | |
| $Y$ | | | |
| $Z$ | | | |



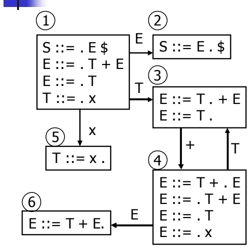
## SLR Construction

- This is identical to LR(0) – states, etc., except for the calculation of reduce actions
- Algorithm:

  Initialize $R$ to empty
  for each state $I$ in $T$
    for each item [$A ::= \alpha$ .] in $I$
      for each terminal a in FOLLOW($A$)
        add ($I$, a, $A ::= \alpha$) to $R$
  - i.e., reduce $\alpha$ to $A$ in state $I$ only on lookahead a

## SLR Parser for

0. S ::= E $
1. E ::= T + E
2. E ::= T
3. T ::= x




| | x | + | $ | E | T |
|---|---|---|---|---|---|
| 1 | s5 | | | g2 | g3 |
| 2 | | | acc | | |
| 3 | r2 | s4,r2 | r2 | | |
| 4 | s5 | | | g6 | g3 |
| 5 | r3 | r3 | r3 | | |
| 6 | r1 | r1 | r1 | | |





## On To LR(1)

- Many practical grammars are SLR
- LR(1) is more powerful yet
- Similar construction, but notion of an item is more complex, incorporating lookahead information





## LR(1) Items

- An LR(1) item [*A* ::= α . β, a] is
  - A grammar production (*A* ::= αβ)
  - A right hand side position (the dot)
  - A lookahead symbol (a)
- Idea: This item indicates that α is the top of the stack and the next input is derivable from βa.
- Full construction: see the book





## LR(1) Tradeoffs

- LR(1)
  - Pro: extremely precise; largest class of grammars
  - Con: potentially very large parse tables with many states





## LALR(1)

- Variation of LR(1), but merge any two states that differ only in lookahead
  - Example: these two would be merged

    [*A* ::= x . , a]

    [*A* ::= x . , b]

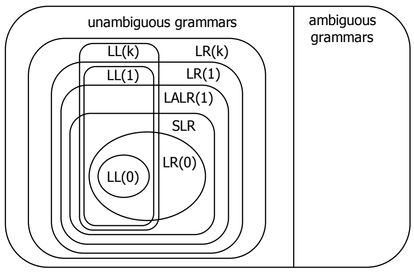




## LALR(1) vs LR(1)

- LALR(1) tables can have many fewer states than LR(1)
- LALR(1) may have reduce conflicts where LR(1) would not (but in practice this doesn't happen often)

## Language Heirarchies




10/14/2002  E-37

## Coming Attractions

- LL(k) Parsing – Top-Down
- Recursive Descent Parsers
  - What to do if you need a parser in a hurry

10/14/2002  E-38