CSE P 501 – Compilers

LR Parser Construction
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Agenda

- LR(0) state construction
- FIRST, FOLLOW, and nullable
- Variations SLR, LR(1), LALR

LR State Machine

- Idea: Build a DFA that recognizes handles
  - Language generated by a CFG is generally not regular, but
  - Language of handles for a CFG is regular
    - So a DFA can be used to recognize handles
  - Parser reduces when DFA accepts

Prefixes, Handles, &c (review)

- If *S* is the start symbol of a grammar *G*,
  - If $S \Rightarrow^* \alpha$ then α is a *sentential form* of *G*
  - γ is a *viable prefix* of *G* if there is some derivation $S \Rightarrow^*_{rm} \alpha A w \Rightarrow^*_{rm} \alpha\beta w$ and γ is a prefix of $\alpha\beta$.
  - The occurrence of β in $\alpha\beta w$ is a *handle* of $\alpha\beta w$
- An *item* is a marked production (a . at some position in the right hand side)
  - [*A* ::= . *X Y*]   [*A* ::= *X* . *Y*]   [*A* ::= *X Y* .]

Building the LR(0) States

- Example grammar

  S′ ::= *S* $
  S ::= (*L*)
  S ::= x
  L ::= *S*
  L ::= *L* , *S*
  - We add a production S′ with the original start symbol followed by end of file ($)
  - Question: What language does this grammar generate?

S′ ::= *S* $
S ::= (*L*)
S ::= x
L ::= *S*
L ::= *L* , *S*

Start of LR Parse

- Initially
  - Stack is empty
  - Input is the right hand side of *S′*, i.e., *S* $
  - Initial configuration is [*S′* ::= . *S* $]
  - But, since position is just before *S*, we are also just before anything that can be derived from *S*

Initial state

$S' ::= S\$$
$S ::= (L)$
$S ::= x$
$L ::= S$
$L ::= L , S$

$S' ::= . S\$$ ← start
$S ::= . (L)$
$S ::= . x$
completion

- A state is just a set of items
  - Start: an initial set of items
  - Completion (or closure): additional productions whose left hand side appears to the right of the dot in some item already in the state

Shift Actions (1)

$S' ::= S\$$
$S ::= (L)$
$S ::= x$
$L ::= S$
$L ::= L , S$

$S' ::= . S\$$
$S ::= . (L)$
$S ::= . x$

$\xrightarrow{x}$ $S ::= x .$

- To shift past the x, add a new state with the appropriate item(s)
  - In this case, a single item; the closure adds nothing
  - This state will lead to a reduction since no further shift is possible

Shift Actions (2)

$S' ::= S\$$
$S ::= (L)$
$S ::= x$
$L ::= S$
$L ::= L , S$

$S' ::= . S\$$
$S ::= . (L)$
$S ::= . x$

$\xrightarrow{(}$

$S ::= (. L)$
$L ::= . L , S$
$L ::= . S$
$S ::= . (L)$
$S ::= . x$

- If we shift past the (, we are at the beginning of L
- the closure adds all productions that start with L, which requires adding all productions starting with S

Goto Actions

$S' ::= S\$$
$S ::= (L)$
$S ::= x$
$L ::= S$
$L ::= L , S$

$S' ::= . S\$$
$S ::= . (L)$
$S ::= . x$

$\xrightarrow{S}$ $S' ::= S . \$$

- Once we reduce S, we'll pop the rhs from the stack exposing the first state. Add a *goto* transition on S for this.

Basic Operations

- *Closure* (S)
  - Adds all items implied by items already in S
- *Goto* (I, X)
  - I is a set of items
  - X is a grammar symbol (terminal or non-terminal)
  - *Goto* moves the dot past the symbol X in all appropriate items in set I

Closure Algorithm

- *Closure* (S) =

  repeat
    for any item [A ::= α . $X\beta$] in S
      for all productions $X ::= \gamma$
        add [$X ::= . \gamma$] to S
  until S does not change
  return S

Goto Algorithm

- *Goto* (I, X) =

  set *new* to the empty set
  for each item [A ::= α . X β] in I
  add [A ::= α X . β] to *new*
  return *Closure* (*new*)

  - This may create a new state, or may return an existing one

LR(0) Construction

- First, augment the grammar with an extra start production S' ::= S $
- Let T be the set of states
- Let E be the set of edges
- Initialize T to *Closure* ([S' ::= . S $])
- Initialize E to empty

LR(0) Construction Algorithm

repeat
for each state I in T
for each item [A ::= α . X β] in I
Let *new* be *Goto* (I, X)
Add *new* to T if not present
Add $I \xrightarrow{X}$ *new* to E if not present
until E and T do not change in this iteration

- Footnote: For symbol $, we don't compute *goto* (I, $); instead, we make this an *accept* action.

LR(0) Reduce Actions

- Algorithm:

  Initialize R to empty
  for each state I in T
  for each item [A ::= α .] in I
  add (I, A ::= α) to R

Building the Parse Tables (1)

- For each edge $I \xrightarrow{X} J$
  - if X is a terminal, put sj in column X, row I of the action table (shift to state j)
  - If X is a non-terminal, put gj in column X, row I of the goto table

Building the Parse Tables (2)

- For each state I containing an item [S' ::= S . $], put *accept* in column $ of row I
- Finally, for any state containing [A ::= γ .] put action rn in every column of row I in the table, where n is the production number

Example: States for

$S' ::= S\$$
$S ::= (L)$
$S ::= x$
$L ::= S$
$L ::= L , S$

Example: Tables for

$S' ::= S\$$
$S ::= (L)$
$S ::= x$
$L ::= S$
$L ::= L , S$

Where Do We Stand?

- We have built the LR(0) state machine and parser tables
  - No lookahead yet
  - Different variations of LR parsers add lookahead information, but basic idea of states, closures, and edges remains the same

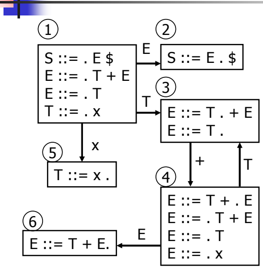
A Grammar that is not LR(0)

- Build the state machine and parse tables for a simple expression grammar

  $S ::= E\$$
  $E ::= T + E$
  $E ::= T$
  $T ::= x$

LR(0) Parser for

0. $S ::= E\$$
1. $E ::= T + E$
2. $E ::= T$
3. $T ::= x$



	x	+	$	E	T
1	s5			g2	G3
2			acc		
3	r2	s4,r2	r2		
4	s5			g6	G3
5	r3	r3	r3		
6	r1	r1	r1		

- State 3 is has two possible actions on +
  - shift 4, or reduce 2
- ∴ Grammar is not LR(0)

SLR Parsers

- Idea: Use information about what can follow a non-terminal to decide if we should perform a reduction
- Easiest form is SLR – Simple LR
- So we need to be able to compute FOLLOW(A) – the set of symbols that can follow A in any possible derivation
  - But to do this, we need to compute FIRST(γ) for strings γ that can follow A

Calculating FIRST(γ)

- Sounds easy… If $\gamma = X\,Y\,Z$, then FIRST(γ) is FIRST(X), right?
  - But what if we have the rule X ::= ε?
  - In that case, FIRST(γ) includes anything that can follow an X – i.e. FOLLOW(X)

FIRST, FOLLOW, and nullable

- nullable(X) is true if X can derive the empty string
- Given a string γ of terminals and non-terminals, FIRST(γ) is the set of terminals that can begin strings derived from γ.
- FOLLOW(X) is the set of terminals that can immediately follow X in some derivation
- All three of these are computed together

Computing FIRST, FOLLOW, and nullable (1)

- Initialization
  - set FIRST and FOLLOW to be empty sets
  - set nullable to false for all non-terminals
  - set FIRST[a] to a for all terminal symbols a

Computing FIRST, FOLLOW, and nullable (2)

```
repeat
  for each production X := Y1 Y2 … Yk
      if Y1 … Yk are all nullable (or if k = 0)
         set nullable[X] = true
      for each i from 1 to k and each j from i+1 to k
         if Y1 … Yi-1 are all nullable (or if i = 1)
             add FIRST[Yi] to FIRST[X]
         if Yi+1 … Yk are all nullable (or if i = k)
             add FOLLOW[X] to FOLLOW[Yi]
         if Yi+1 … Yj-1 are all nullable (or if i+1=j)
             add FIRST[Yj] to FOLLOW[Yi]
Until FIRST, FOLLOW, and nullable do not change
```


Example

- Grammar
  - Z ::= d
  - Z ::= $X\,Y\,Z$
  - Y ::= ε
  - Y ::= c
  - X ::= Y
  - X ::= a

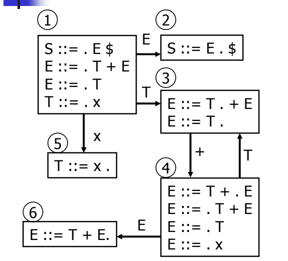
	nullable	FIRST	FOLLOW
X			
Y			
Z			

SLR Construction

- This is identical to LR(0) – states, etc., except for the calculation of reduce actions
- Algorithm:
  - Initialize R to empty
  - for each state I in T
    - for each item [A ::= α .] in I
      - for each terminal a in FOLLOW(A)
        - add (I, a, A ::= α) to R
  - i.e., reduce α to A in state I only on lookahead a

SLR Parser for

0. S ::= E $
1. E ::= T + E
2. E ::= T
3. T ::= x



	x	+	$	E	T
1	s5			g2	g3
2			acc		
3	~~r2~~	s4,~~r2~~	r2		
4	s5			g6	g3
5	~~r3~~	r3	r3		
6	~~r1~~	~~r1~~	r1		

On To LR(1)

- Many practical grammars are SLR
- LR(1) is more powerful yet
- Similar construction, but notion of an item is more complex, incorporating lookahead information

LR(1) Items

- An LR(1) item [A ::= α . β, a] is
  - A grammar production (A ::= $\alpha\beta$)
  - A right hand side position (the dot)
  - A lookahead symbol (a)
- Idea: This item indicates that α is the top of the stack and the next input is derivable from βa.
- Full construction: see the book

LR(1) Tradeoffs

- LR(1)
  - Pro: extremely precise; largest set of grammars
  - Con: potentially very large parse tables with many states

LALR(1)

- Variation of LR(1), but merge any two states that differ only in lookahead
  - Example: these two would be merged

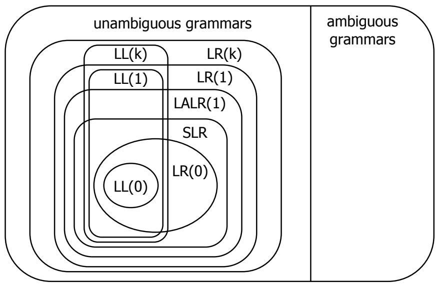
    [A ::= x . , a]

    [A ::= x . , b]

LALR(1) vs LR(1)

- LALR(1) tables can have many fewer states than LR(1)
- LALR(1) may have reduce conflicts where LR(1) would not (but in practice this doesn't happen often)

Language Heirarchies



Coming Attractions

- LL(k) Parsing – Top-Down
- Recursive Descent Parsers
  - What to do if you need a parser in a hurry