



# CSE P 501 – Compilers

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LR Parsing  
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# Agenda

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- LR Parsing
- Table-driven Parsers
- Parser States
- Shift-Reduce and Reduce-Reduce conflicts



# LR(1) Parsing

- We'll look at LR(1) parsers
  - Left to right scan, Rightmost derivation, 1 symbol lookahead
  - Almost all practical programming languages have an LR(1) grammar
  - LALR(1), SLR(1), etc. – subsets of LR(1)
    - LALR(1) can parse most real languages, is more compact, and is used by YACC/Bison/etc.

GLR(1)



## Bottom-Up Parsing

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- Idea: Read the input left to right
- Whenever we've matched the right hand side of a production, reduce it to the appropriate non-terminal and add that non-terminal to the parse tree
- The upper edge of this partial parse tree is known as the *frontier*

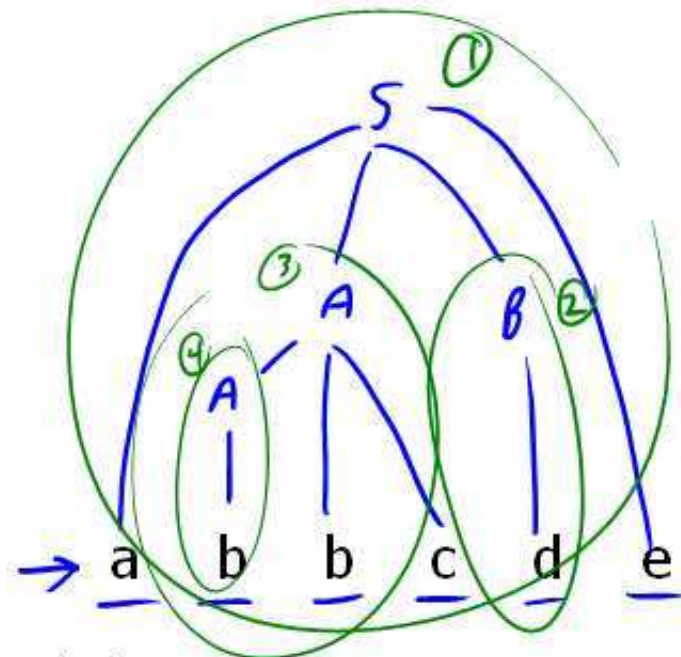


# Example

## ■ Grammar

- $S ::= aABe$
- $A ::= Abc \mid b$
- $B ::= d$

## ■ Bottom-up Parse





## Details

- The bottom-up parser reconstructs a reverse rightmost derivation

- Given the rightmost derivation

$$\underline{S} \Rightarrow \beta_1 \Rightarrow \beta_2 \Rightarrow \dots \Rightarrow \beta_{n-2} \Rightarrow \beta_{n-1} \Rightarrow \beta_n = \underline{w}$$

the parser will first discover  $\beta_{n-1} \Rightarrow \beta_n$ , then  $\beta_{n-2} \Rightarrow \beta_{n-1}$ , etc.

- Parsing terminates when
  - $\beta_1$  reduced to  $S$  (start symbol, success), or
  - No match can be found (syntax error)



## How Do We Parse with This?

- Key: given what we've already seen and the next input symbol, decide what to do.
- Choices:
  - Perform a reduction
  - Look ahead further
- Can reduce  $A \Rightarrow \beta$  if both of these hold:
  - $A \Rightarrow \beta$  is a valid production
  - $A \Rightarrow \beta$  is a step in *this* rightmost derivation
- This is known as a *shift-reduce* parser





## Sentential Forms

- If  $S \Rightarrow^* \alpha$ , the string  $\alpha$  is called a *sentential form* of the grammar
- In the derivation
  - $S \Rightarrow \beta_1 \Rightarrow \beta_2 \Rightarrow \dots \Rightarrow \beta_{n-2} \Rightarrow \beta_{n-1} \Rightarrow \beta_n = W$   
each of the  $\beta_i$  are sentential forms
- A sentential form in a rightmost derivation is called a right-sentential form (similarly for leftmost and left-sentential)





## Handles

- Informally, a substring of the tree frontier that matches the right side of a production
  - Even if  $A ::= \beta$  is a production,  $\beta$  is a handle only if it matches the frontier at a point where  $A ::= \beta$  was used in the derivation
  - $\beta$  may appear in many other places in the frontier without being a handle for that particular production



## Handles (cont.)

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- Formally, a *handle* of a right-sentential form  $\gamma$  is a production  $A ::= \beta$  and a position in  $\gamma$  where  $\beta$  may be replaced by  $A$  to produce the previous right-sentential form in the rightmost derivation of  $\gamma$



## Handle Examples

- In the derivation

→  $S \Rightarrow a\underline{AB}e \Rightarrow a\underline{A}de \Rightarrow a\underline{Abc}de \Rightarrow \overset{\downarrow}{\downarrow} \underline{a}bbcde$

- $abbcde$  is a right sentential form whose handle is  $A::=b$  at position 2

- $aAbcde$  is a right sentential form whose handle is  $A::=Abc$  at position 4

- Note: some books take the left of the match as the position



# Implementing Shift-Reduce Parsers

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- Key Data structures
  - A stack holding the frontier of the tree
  - A string with the remaining input



# Shift-Reduce Parser Operations



- *Reduce* – if the top of the stack is the right side of a handle  $A ::= \beta$ , pop the right side  $\beta$  and push the left side  $A$ .
- *Shift* – push the next input symbol onto the stack
- *Accept* – announce success
- *Error* – syntax error discovered



# Shift-Reduce Example

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

| Stack  | Input    | Action |
|--------|----------|--------|
| \$     | abbcde\$ | shift  |
| \$a    | bbcde\$  | shift  |
| \$ab   | bcd e\$  | reduce |
| \$aA   | bcd e\$  | S      |
| \$aAb  | cd e\$   | S      |
| \$aAbc | d e\$    | R      |
| \$aA   | d e\$    | S      |
| \$aAd  | e\$      | R      |
| \$aAB  | e\$      | S      |
| \$aABe | \$       | R      |
| \$S    | \$       | accept |





## How Do We Automate This?

- Def. *Viable prefix* – a prefix of a right-sentential form that can appear on the stack of the shift-reduce parser
  - Equivalent: a prefix of a right-sentential form that does not continue past the rightmost handle of that sentential form
- Idea: Construct a DFA to recognize viable prefixes given the stack and remaining input
  - Perform reductions when we recognize them



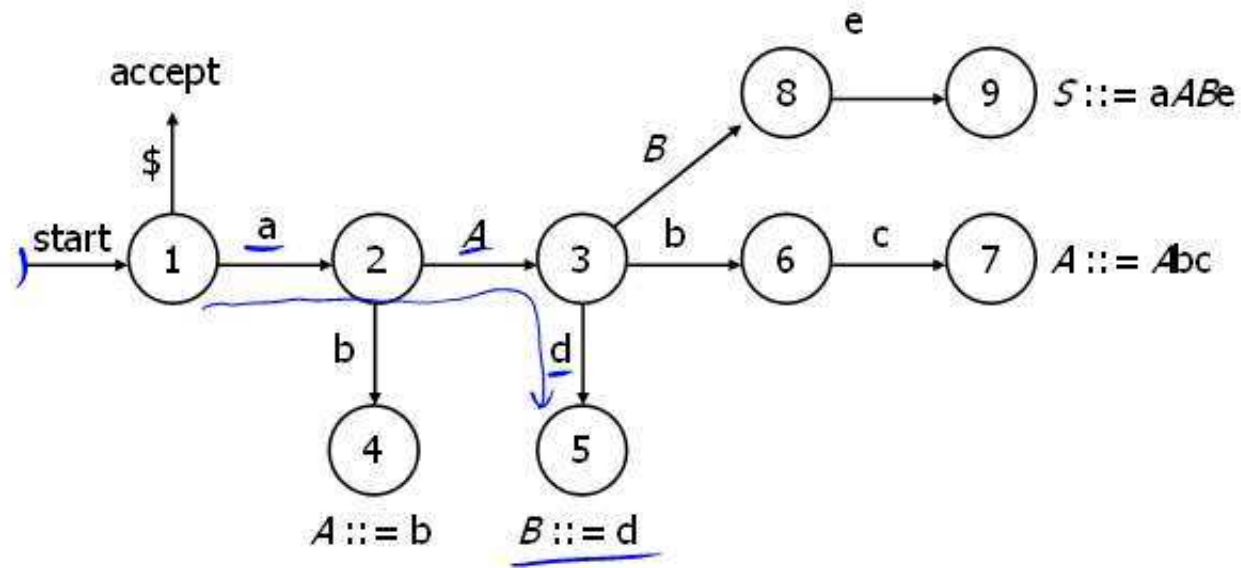


# DFA for prefixes of

$S ::= aABe$

$A ::= Abc \mid b$

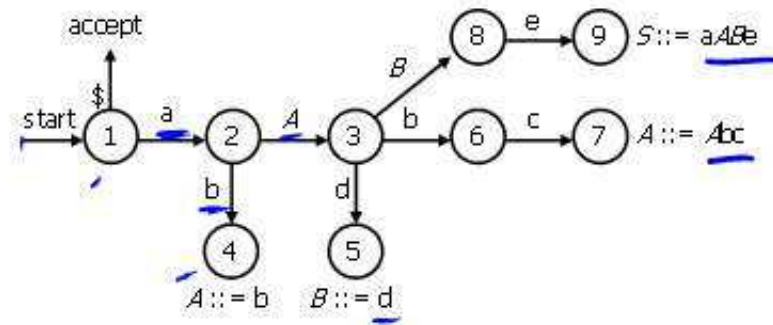
$B ::= d$



# Trace

$S' ::= S\$$   
 $S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

| Stack           | Input   |
|-----------------|---------|
| \$              | abcde\$ |
| \$a             | bcde\$  |
| \$ab            | bcde\$  |
| \$aA            | bcde\$  |
| \$aAb           | cde\$   |
| \$aA <u>b</u> e | de\$    |
| \$aA            | de\$    |
| \$aA <u>d</u>   | e\$     |
| \$aA <u>B</u>   | e\$     |
| \$aA <u>B</u> e | \$      |
| \$S             | \$      |





## Observations

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- Way too much backtracking
  - We want the parser to run in time proportional to the length of the input
- Where the heck did this DFA come from anyway?
  - From the underlying grammar
  - We'll defer construction details for now



## Avoiding DFA Rescanning

- Observation: after a reduction, the contents of the stack are the same as before except for the new non-terminal on top
  - $\therefore$  Scanning the stack will take us through the same transitions as before until the last one
  - $\therefore$  If we record state numbers on the stack, we can go directly to the appropriate state when we pop the right hand side of a production from the stack



# Stack

- Change the stack to contain pairs of states and symbols from the grammar
$$\$s_0 X_1 (S_1) X_2 S_2 \dots X_n S_n$$
  - State  $s_0$  represents the accept state
    - (Not always added – depends on particular presentation)
- Observation: in an actual parser, only the state numbers need to be pushed, since they implicitly contain the symbol information, but for explanations, it's clearer to use both.



## Encoding the DFA in a Table

- A shift-reduce parser's DFA can be encoded in two tables
  - One row for each state
  - *action* table encodes what to do given the current state and the next input symbol
  - *goto* table encodes the transitions to take after a reduction





## Actions (1)

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- Given the current state and input symbol, the main possible actions are
  - $s/i$  – shift the input symbol and state  $i$  onto the stack (i.e., shift and move to state  $i$ )
  - $rj$  – reduce using grammar production  $j$ 
    - The production number tells us how many  $\langle \text{symbol}, \text{state} \rangle$  pairs to pop off the stack





## Actions (2)

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- Other possible *action* table entries
  - *accept*
  - blank – no transition – syntax error
    - A LR parser will detect an error as soon as possible on a left-to-right scan
    - A real compiler needs to produce an error message, recover, and continue parsing when this happens



## Goto

- When a reduction is performed, <symbol, state> pairs are popped from the stack revealing a state *uncovered s* on the top of the stack
- `goto[uncovered s, A]` is the new state to push on the stack when reducing production  $A ::= \beta$  (after popping  $\beta$  and finding state *uncovered\_s* on top)

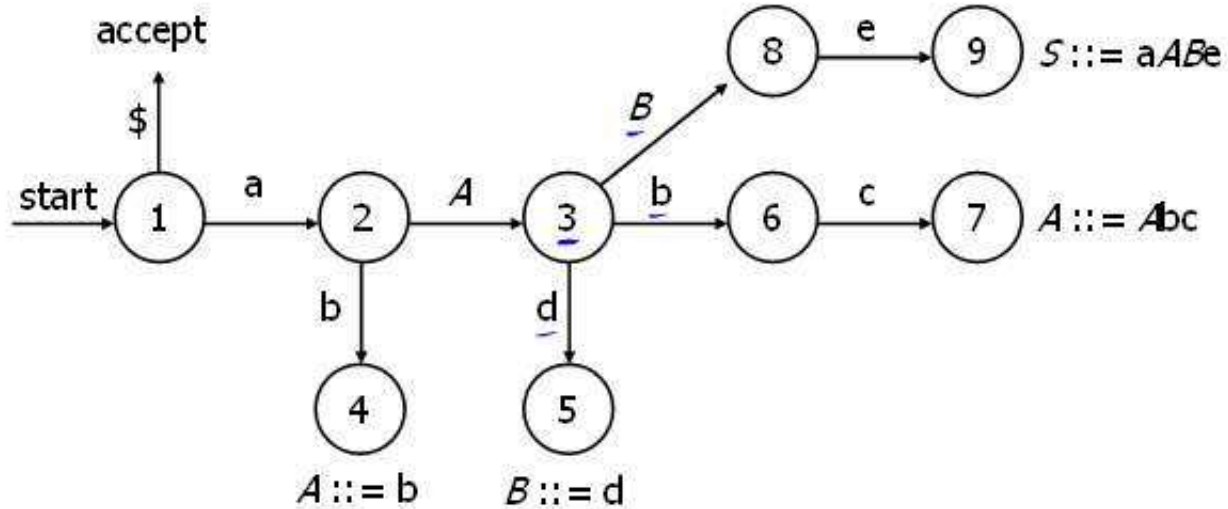


# Reminder: DFA for

$S ::= aABe$

$A ::= Abc \mid b$

$B ::= d$





# LR Parse Table for

1.  $S ::= aABe$
2.  $A ::= Abc$
3.  $A ::= b$
4.  $B ::= d$

| State | action    |    |    |    |    |     | goto |    |    |
|-------|-----------|----|----|----|----|-----|------|----|----|
|       | a         | b  | c  | d  | e  | \$  | A    | B  | S  |
| 1     | <u>s2</u> |    |    |    |    | acc |      |    | g1 |
| 2     |           | s4 |    |    |    |     | g3   |    |    |
| 3     |           | s6 |    | s5 |    |     |      | g8 |    |
| 4     | <u>r3</u> | r3 | r3 | r3 | r3 | r3  |      |    |    |
| 5     | r4        | r4 | r4 | r4 | r4 | r4  |      |    |    |
| 6     |           |    | s7 |    |    |     |      |    |    |
| 7     | r2        | r2 | r2 | r2 | r2 | r2  |      |    |    |
| 8     |           |    |    |    | s9 |     |      |    |    |
| 9     | r1        | r1 | r1 | r1 | r1 | r1  |      |    |    |



# LR Parsing Algorithm (1)

```
word = scanner.getToken();
while (true) {
    s = top of stack;
    if (action[s, word] = s/) {
        push word; push / (state);
        word = scanner.getToken();
    } else if (action[s, word] = rj) {
        pop 2 * length of right side of
            production  $j$  ( $2 * |\beta|$ );
        uncovered_s = top of stack;
        push left side  $A$  of production  $j$ ;
        push state goto[uncovered_s,  $A$ ];
    }
    } else if (action[s, word] = accept ) {
        return;
    } else {
        // no entry in action table
        report syntax error;
        halt or attempt recovery;
    }
}
```

# Example

1.  $S ::= aABe$
- 2.  $A ::= \overline{A}bc$
- 3.  $A ::= b$
4.  $\overline{B} ::= \overline{d}$

Stack

\$1  
 \$1a2  
 \$1a2b4  
 \$1a2A3  
 \$1a2A3b6  
 \$1a2A3b6c7  
 \$1a2A3  
 \$1a2A3d5  
 \$1a2A3B8  
 \$1a2A3B8e9  
 \$1S

Input

abbcdes\$  
 bbcdes  
 bcdes  
 bcdes  
 cdes  
 des  
 des  
 es  
 es  
 \$  
 \$

| S | action |    |    |    |    |    | goto |    |    |
|---|--------|----|----|----|----|----|------|----|----|
|   | a      | b  | c  | d  | e  | \$ | A    | B  | S  |
| 1 | s2     |    |    |    |    | ac |      |    | g1 |
| 2 |        | s4 |    |    |    |    | g3   |    |    |
| 3 |        | s6 |    | s5 |    |    |      | g8 |    |
| 4 | r3     | r3 | r3 | r3 | r3 | r3 |      |    |    |
| 5 | r4     | r4 | r4 | r4 | r4 | r4 |      |    |    |
| 6 |        |    | s7 |    |    |    |      |    |    |
| 7 | r2     | r2 | r2 | r2 | r2 | r2 |      |    |    |
| 8 |        |    |    |    | s9 |    |      |    |    |
| 9 | r1     | r1 | r1 | r1 | r1 | r1 |      |    |    |



## LR States

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- Idea is that each state encodes
  - The set of all possible productions that we could be looking at, given the current state of the parse, and
  - *Where* we are in the right hand side of each of those productions





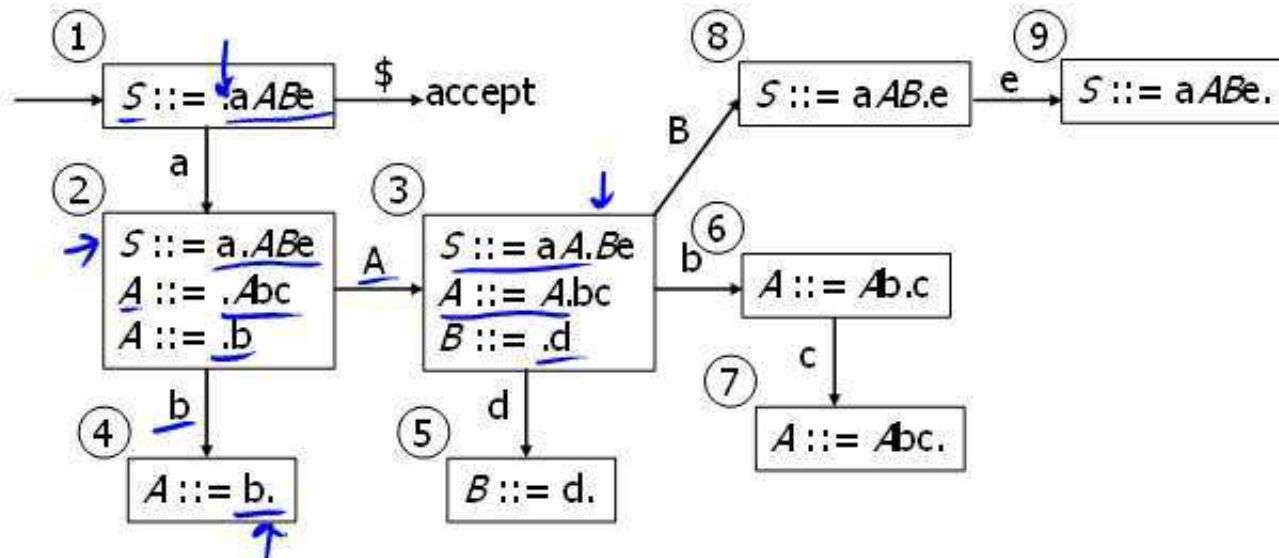
## Items

- An *item* is a production with a dot in the right hand side
- Example: Items for production  $A ::= XY$ 
  - $A ::= .XY$
  - $\rightarrow A ::= \underline{X}.\underline{Y}$
  - $A ::= XY.$
- Idea: The dot represents a position in the production



# DFA for

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$





## Problems with Grammars

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- Grammars can cause to problems when constructing a LR parser
  - Shift-reduce conflicts
  - Reduce-reduce conflicts

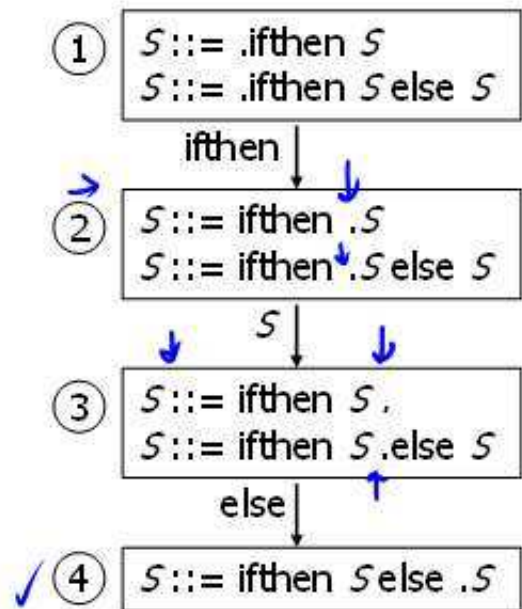


## Shift-Reduce Conflicts

- Situation: both a shift and a reduce are possible at a given point in the parse (equivalently: in a particular state of the DFA)
- Classic example: if-else statement  
$$S ::= \underline{\text{ifthen } S} \mid \underline{\text{ifthen } S \text{ else } S}$$



## Parser States for $\left\{ \begin{array}{l} 1. S ::= \text{ifthen } S \\ 2. S ::= \text{ifthen } S \text{ else } S \end{array} \right.$



### ■ State 3 has a shift-reduce conflict

- Can shift past else into state 4 (s4)
- Can reduce (r1)  
 $S ::= \text{ifthen } S$

(Note: other  $S ::= \text{.ifthen}$  items not included in states 2-4 to save space)



## Solving Shift-Reduce Conflicts

- Fix the grammar
  - Done in Java reference grammar, others
- Use a parse tool with a “longest match” rule – i.e., if there is a conflict, choose to shift instead of reduce
  - Does exactly what we want for if-else case
  - Guideline: a few shift-reduce conflicts are fine, but be sure they do what you want



## Reduce-Reduce Conflicts

- Situation: two different reductions are possible in a given state
- Contrived example

$S ::= A$

$S ::= B$

$A ::= x$

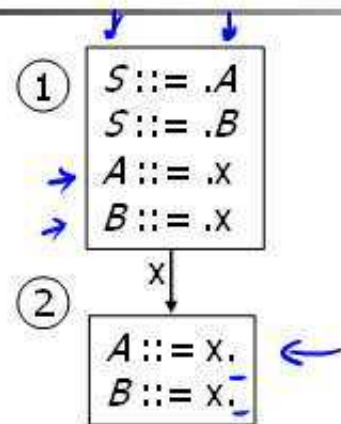
$B ::= x$





## Parser States for

1.  $S ::= A$
2.  $S ::= B$
3.  $A ::= x$
4.  $B ::= x$



- State 2 has a reduce-reduce conflict (r3, r4)



## Handling Reduce-Reduce Conflicts

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- These normally indicate a serious problem with the grammar.
- Fixes
  - Use a different kind of parser generator that takes lookahead information into account when constructing the states (LR(1) instead of SLR(1) for example)
    - Most practical tools use this information
  - Fix the grammar



## Another Reduce-Reduce Conflict

- Suppose the grammar separates arithmetic and boolean expressions

$$expr ::= aexp \mid bexp$$
$$aexp ::= aexp * aident \mid \underline{aident}$$
$$bexp ::= bexp \&\& bident \mid \underline{bident}$$
$$\underline{aident} ::= \underline{id}$$
$$\underline{bident} ::= \underline{id}$$

- This will create a reduce-reduce conflict



## Covering Grammars

- A solution is to merge *aident* and *bident* into a single non-terminal (or use *id* in place of *aident* and *bident* everywhere they appear)
- This is a *covering grammar*
  - Includes some programs that are not generated by the original grammar
  - Use the type checker or other static semantic analysis to weed out illegal programs later



## Coming Attractions

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- Constructing LR tables
  - We'll present a simple version (SLR(0)) in lecture, then talk about extending it to LR(1)
- LL parsers and recursive descent
- Continue reading ch. 4