CSE P 501 – Compilers

LR Parser Construction

Hal Perkins

Autumn 2009

Agenda

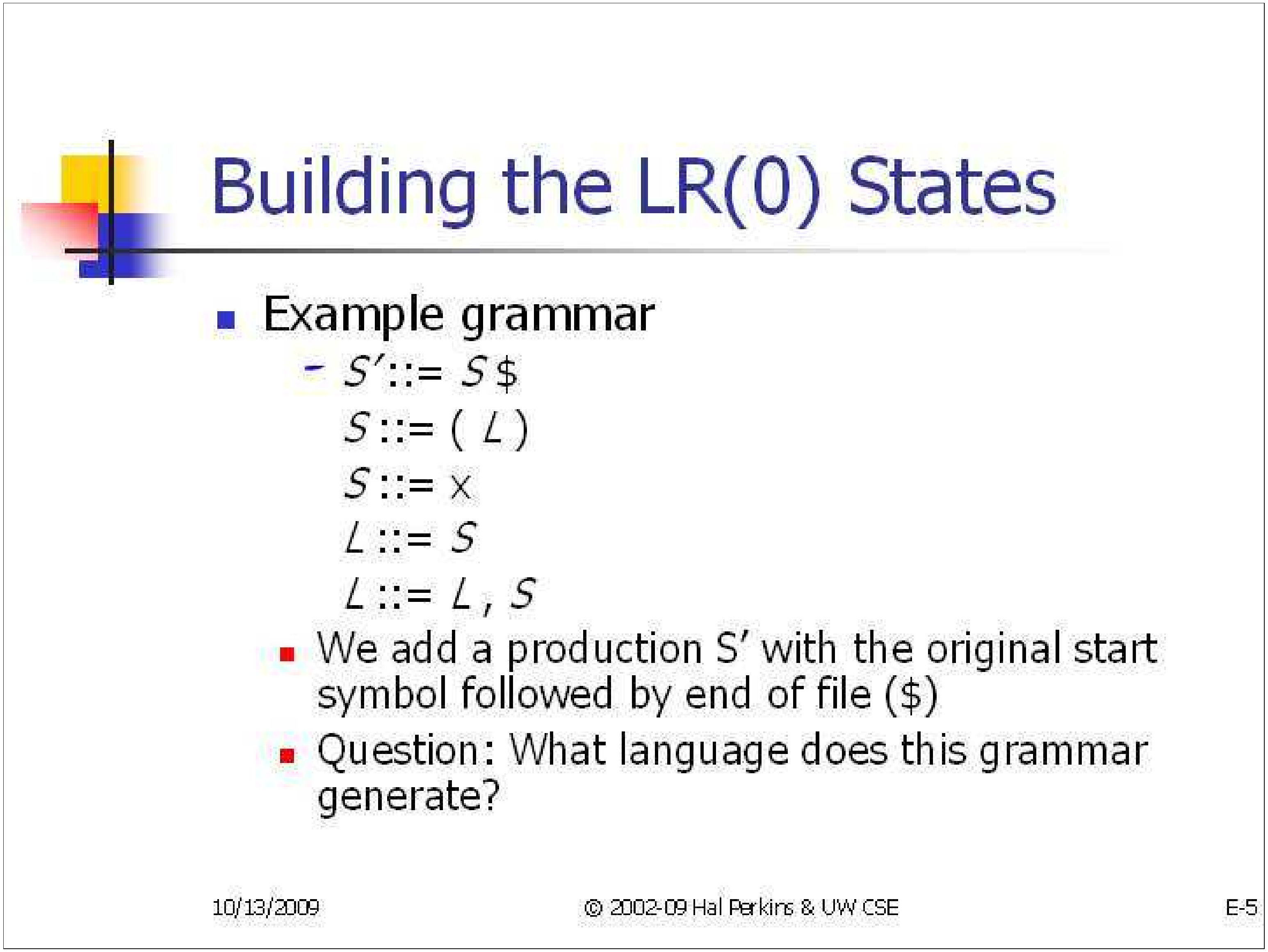
- LR(0) state construction
- FIRST, FOLLOW, and nullable
- Variations: SLR, LR(1), LALR

LR State Machine

- Idea: Build a DFA that recognizes handles
  - Language generated by a CFG is generally not regular, but
  - Language of handles for a CFG is regular
    - So a DFA can be used to recognize handles
  - Parser reduces when DFA accepts

Prefixes, Handles, &c (review)

- If S is the start symbol of a grammar G,
  - If $S =>^* \alpha$ then α is a *sentential form* of G
  - γ is a *viable prefix* of G if there is some derivation $S =>^*_{rm} \alpha A w =>^*_{rm} \alpha\beta w$ and γ is a prefix of $\alpha\beta$.
  - The occurrence of β in $\alpha\beta w$ is a *handle* of $\alpha\beta w$
- An *item* is a marked production (a . at some position in the right hand side)
  - $[A ::= .\ X\ Y]$ $[A ::= X\ .\ Y]$ $[A ::= X\ Y\ .]$



Building the LR(0) States

- Example grammar
  - $S' ::= S \$$
    $S ::= (L)$
    $S ::= x$
    $L ::= S$
    $L ::= L , S$
  - We add a production S′ with the original start symbol followed by end of file ($)
  - Question: What language does this grammar generate?

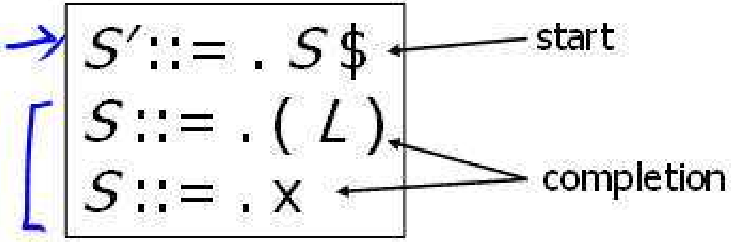
0. $S' ::= S \$$
1. $S ::= (L)$
2. $S ::= x$
3. $L ::= S$
4. $L ::= L , S$

Start of LR Parse

- Initially
  - Stack is empty
  - Input is the right hand side of *S′*, i.e., $\underline{S \$}$
  - Initial configuration is $\underline{[S' ::= \,.\, S \$]}$
  - But, since position is just before *S*, we are also just before anything that can be derived from *S*

0. $S' ::= S\ \$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

Initial state



$S' ::= .\ S\ \$$

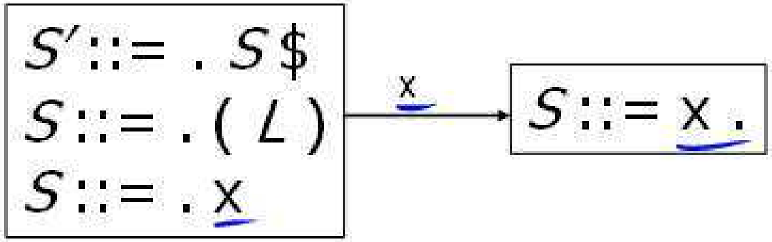
$S ::= .\ (\ L\ )$

$S ::= .\ \mathrm{x}$

- A state is just a set of items
  - Start: an initial set of items
  - Completion (or closure): additional productions whose left hand side appears to the right of the dot in some item already in the state

0. *S′* ::= *S* $
1. *S* ::= (*L*)
2. *S* ::= x
3. *L* ::= *S*
4. *L* ::= *L* , *S*

Shift Actions (1)



S′ ::= . *S* $
S ::= . (*L*)
S ::= . x

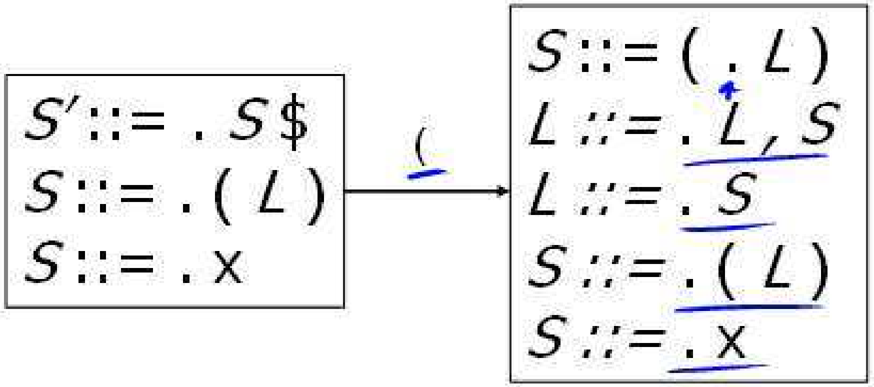
x

S ::= x .

- To shift past the x, add a new state with the appropriate item(s)
  - In this case, a single item; the closure adds nothing
  - This state will lead to a reduction since no further shift is possible

0. $S' ::= S\ \$$
1. $S ::= (\ L\ )$
2. $S ::= x$
3. $L ::= S$
4. $L ::= L\ ,\ S$

Shift Actions (2)



$S' ::= .\ S\ \$$
$S ::= .\ (\ L\ )$
$S ::= .\ x$

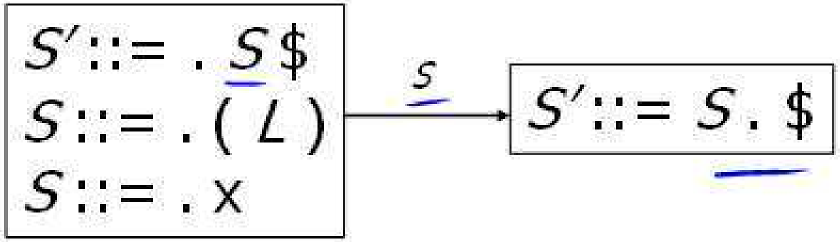
$\xrightarrow{(}$

$S ::= (\ .\ L\ )$
$L ::= .\ L\ ,\ S$
$L ::= .\ S$
$S ::= .\ (\ L\ )$
$S ::= .\ x$

- If we shift past the (, we are at the beginning of L
- the closure adds all productions that start with L, which requires adding all productions starting with S

0. $S' ::= S\ \$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

Goto Actions


$S' ::= .\ \underline{S}\ \$$
$S ::= .\ (\ L\ )$
$S ::= .\ \mathrm{x}$

$\xrightarrow{\underline{S}}$

$S' ::= S\ .\ \$$

- Once we reduce *S*, we'll pop the rhs from the stack exposing the first state. Add a *goto* transition on *S* for this.

Basic Operations

- *Closure* (*S*)
  - Adds all items implied by items already in *S*
- *Goto* (*I*, *X*)
  - *I* is a set of items
  - *X* is a grammar symbol (terminal or non-terminal)
  - *Goto* moves the dot past the symbol *X* in all appropriate items in set *I*

Closure Algorithm

- *Closure* (*S*) =

```
repeat
  for any item [A ::= $\alpha$ . $X\,\beta$] in $S$
    for all productions $X$ ::= $\gamma$
      add [$X$ ::= . $\gamma$] to $S$
until $S$ does not change
return $S$
```

Goto Algorithm

- *Goto* (*I*, *X*) =

```
set new to the empty set
for each item $[A ::= \alpha\ \underline{.\ X}\ \beta]$ in $I$
        add $[A ::= \alpha\ \underline{X\ .}\ \beta]$ to new
return Closure (new)
```

  - This may create a new state, or may return an existing one

LR(0) Construction

- First, augment the grammar with an extra start production $S' ::= S\ \$$
- Let T be the set of states
- Let E be the set of edges
- Initialize T to *Closure* ($[S' ::= .\ S\ \$]$)
- Initialize E to empty

LR(0) Construction Algorithm

```
repeat
  for each state $I$ in $T$
    for each item $[A ::= \alpha \, . \, X \, \beta]$ in $I$
      Let $new$ be $Goto(I, X)$
      Add $new$ to $T$ if not present
      Add $I \xrightarrow{X} new$ to $E$ if not present
until $E$ and $T$ do not change in this iteration
```

- Footnote: For symbol \$, we don't compute *goto* (I, \$); instead, we make this an *accept* action.

LR(0) Reduce Actions

- Algorithm:

```
Initialize $R$ to empty
for each state $I$ in $T$
   for each item $[A ::= \alpha\ .]$ in $I$
      add $(I, A ::= \alpha)$ to $R$
```

Building the Parse Tables (1)

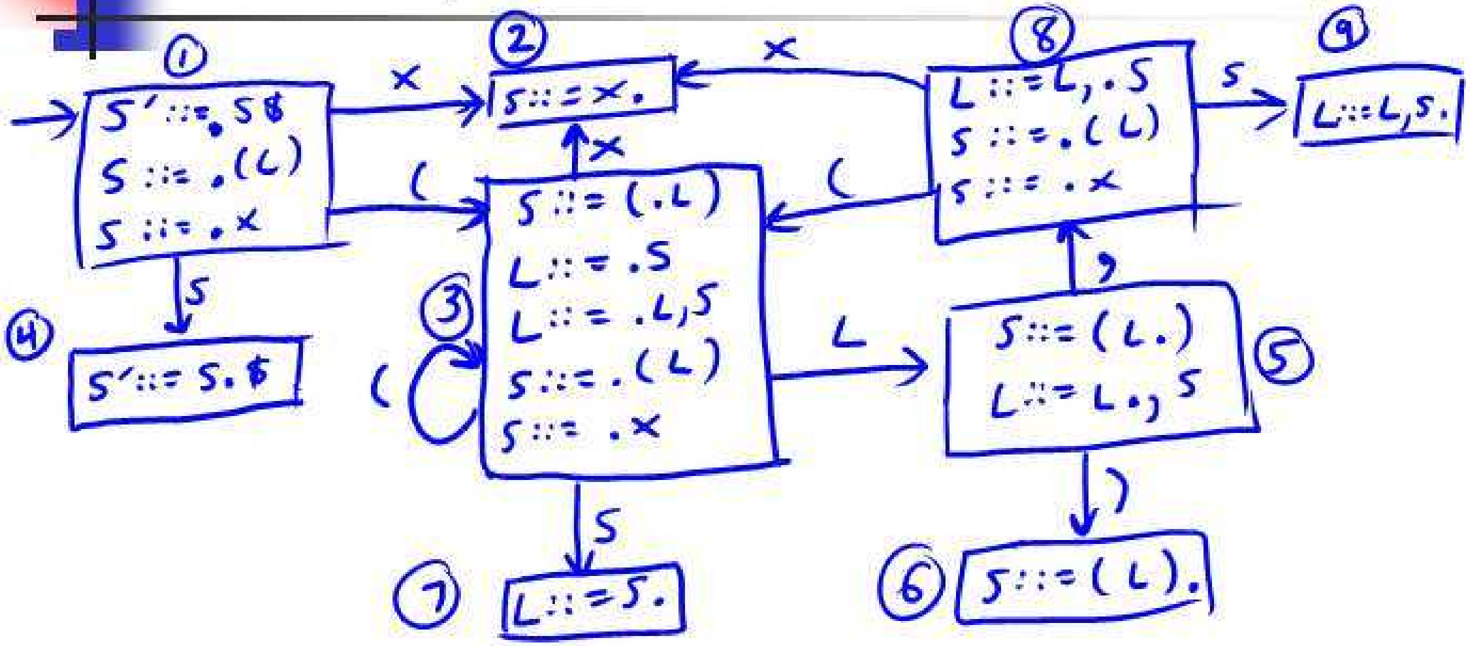
- For each edge $I \xrightarrow{X} J$
  - if X is a terminal, put s*j* in column X, row *I* of the action table (shift to state *j*)
  - If X is a non-terminal, put g*j* in column X, row *I* of the goto table

Building the Parse Tables (2)

- For each state I containing an item $[S' ::= S\ .\ \$]$, put *accept* in column \$ of row I
- Finally, for any state containing $[A ::= \gamma\ .]$ put action rn in every column of row I in the table, where n is the production number

Example: States for

0. $S' ::= S\ \$$
1. $S ::= (\ L\ )$
2. $S ::= x$
3. $L ::= S$
4. $L ::= L\ ,\ S$



Example: Tables for

0. $S' ::= S\,\$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					acc		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8			s2			g9	
9	r4	r4	r4	r4	r4		

Where Do We Stand?

- We have built the LR(0) state machine and parser tables
  - No lookahead yet
  - Different variations of LR parsers add lookahead information, but basic idea of states, closures, and edges remains the same

A Grammar that is not LR(0)

- Build the state machine and parse tables for a simple expression grammar

  $S ::= E\,\$$

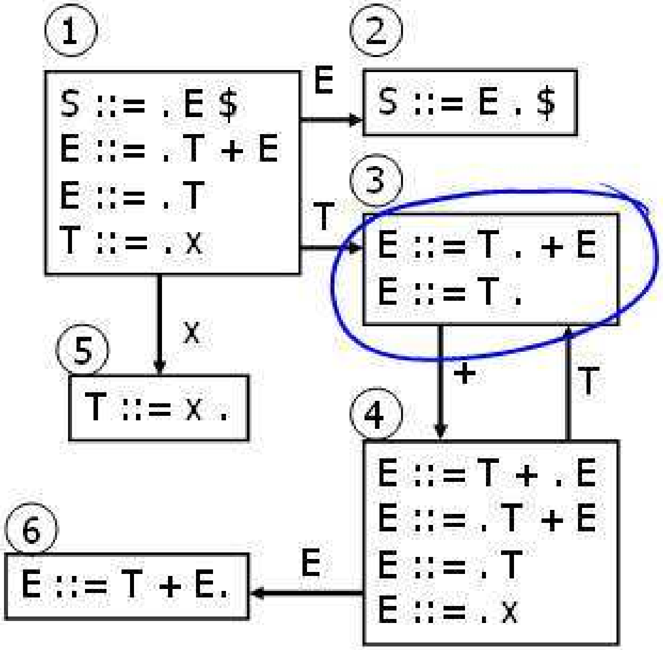
  $E ::= T + E$

  $E ::= T$

  $T ::= \mathrm{x}$

LR(0) Parser for

0. *S* ::= *E* $
1. *E* ::= *T* + *E*
2. *E* ::= *T*
3. *T* ::= x



	x	+	$	E	T
1	s5			g2	G3
2			acc		
3	r2	s4,r2	r2		
4	s5			g6	G3
5	r3	r3	r3		
6	r1	r1	r1		

- State 3 is has two possible actions on +
  - shift 4, or reduce 2
- ∴ Grammar is not LR(0)

SLR Parsers

- Idea: Use information about what can follow a non-terminal to decide if we should perform a reduction
- Easiest form is SLR – Simple LR
- So we need to be able to compute FOLLOW(*A*) – the set of symbols that can follow *A* in any possible derivation
  - But to do this, we need to compute FIRST(γ) for strings γ that can follow *A*

$A\gamma$

Calculating FIRST(γ)

- Sounds easy… If $\gamma = X\,Y\,Z$, then FIRST(γ) is FIRST(X), right?
  - But what if we have the rule $X ::= \varepsilon$?
  - In that case, FIRST(γ) includes anything that can follow an X – i.e. FOLLOW(X)

FIRST, FOLLOW, and nullable

- nullable(X) is true if X can derive the empty string
- Given a string γ of terminals and non-terminals, FIRST(γ) is the set of terminals that can begin strings derived from γ.
- FOLLOW(X) is the set of terminals that can immediately follow X in some derivation
- All three of these are computed together

Computing FIRST, FOLLOW, and nullable (1)

- Initialization

  set FIRST and FOLLOW to be empty sets
  set nullable to false for all non-terminals
  set FIRST[a] to a for all terminal symbols a

$X ::= Y_1 Y_2 \cdots Y_{i-1} Y_i Y_{i+1} \cdots Y_{j-1} Y_j \cdots Y_k$

Computing FIRST, FOLLOW, and nullable (2)

```
repeat
    for each production $X := Y_1\ Y_2 \ldots Y_k$
        if $Y_1 \ldots Y_k$ are all nullable (or if $k = 0$)
            set nullable[$X$] = true
        for each $i$ from 1 to k and each $j$ from $i$+1 to $k$
            if $Y_1 \ldots Y_{i-1}$ are all nullable (or if $i = 1$)
                add FIRST[$Y_i$] to FIRST[$X$]
            if $Y_{i+1} \ldots Y_k$ are all nullable (or if $i = k$)
                add FOLLOW[$X$] to FOLLOW[$Y_i$]
            if $Y_{i+1} \ldots Y_{j-1}$ are all nullable (or if i+1=j)
                add FIRST[$Y_j$] to FOLLOW[$Y_i$]
Until FIRST, FOLLOW, and nullable do not change
```

Example

- **Grammar**
  - $Z ::= d$
  - $Z ::= X\ Y\ Z$
  - $Y ::= \varepsilon$
  - $Y ::= c$
  - $X ::= Y$
  - $X ::= a$

	nullable	FIRST	FOLLOW
X	true ~~false~~	{a, c }	{c, d, a}
Y	true ~~false~~	{c }	{d, a, c}
Z	false	{d, a, c }	{ }

SLR Construction

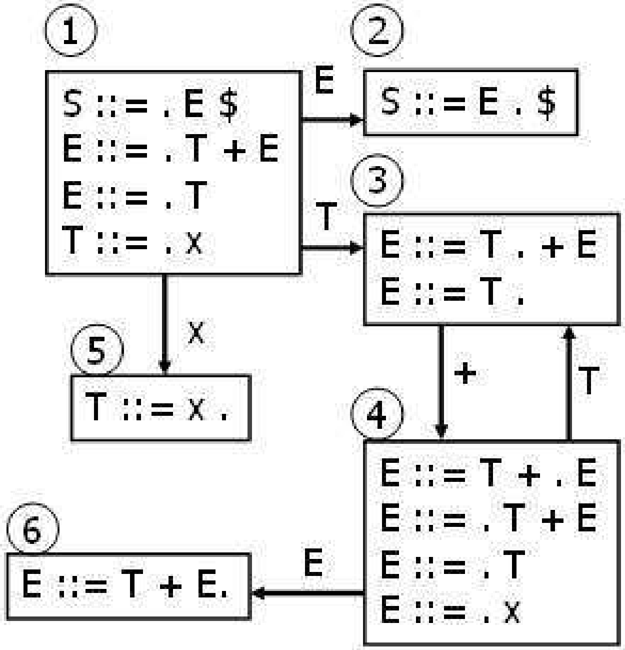
- This is identical to LR(0) – states, etc., except for the calculation of reduce actions
- Algorithm:

```
Initialize $R$ to empty
for each state $I$ in $T$
  for each item $[A ::= \alpha\ .]$ in $I$
    for each terminal a in FOLLOW($A$)
      add $(I, a, A ::= \alpha)$ to $R$
```

  - i.e., reduce α to A in state I only on lookahead a

SLR Parser for

0. S ::= E $
1. E ::= T + E
2. E ::= T
3. T ::= x



	x	+	$	E	T
1	s5			g2	g3
2			acc		
3	r2	s4,r2	r2		
4	s5			g6	g3
5	r3	r3	r3		
6	r1	r1	r1		

On To LR(1)

- Many practical grammars are SLR
- LR(1) is more powerful yet
- Similar construction, but notion of an item is more complex, incorporating lookahead information

LR(1) Items

- An LR(1) item $[A ::= \alpha \;.\; \beta, a]$ is
  - A grammar production ($A ::= \alpha\beta$)
  - A right hand side position (the dot)
  - A lookahead symbol (a)
- Idea: This item indicates that α is the top of the stack and the next input is derivable from βa.
- Full construction: see the book

LR(1) Tradeoffs

- LR(1)
  - Pro: extremely precise; largest set of grammars
  - Con: potentially very large parse tables with many states

LALR(1)

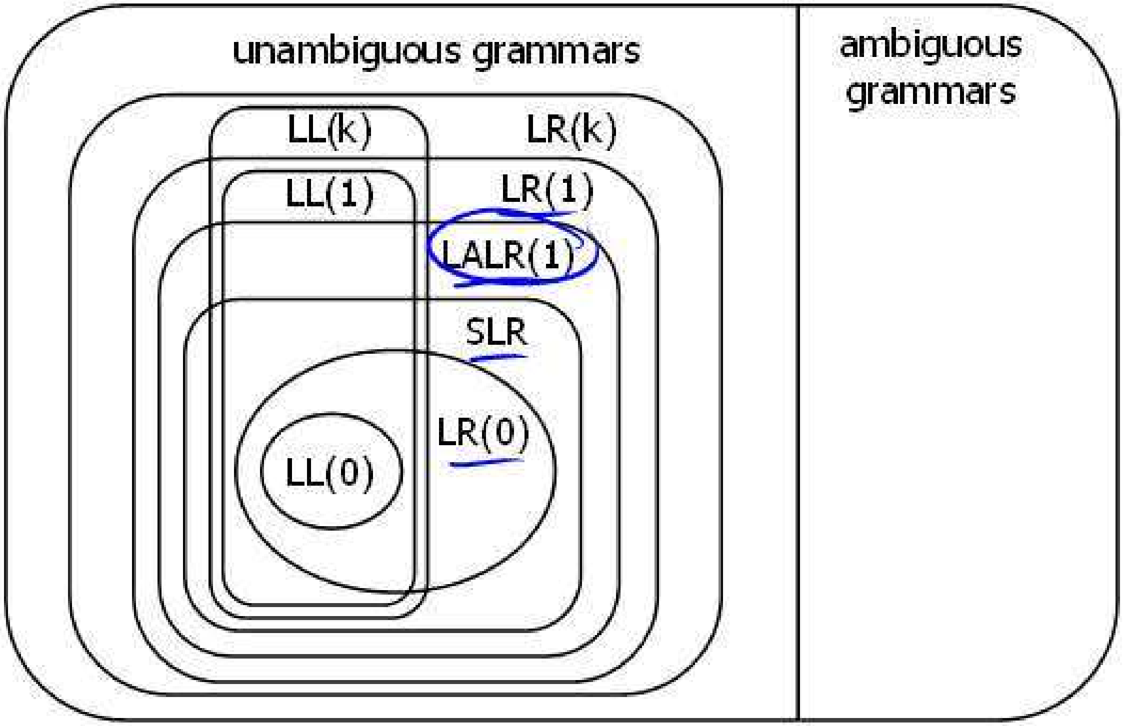
- Variation of LR(1), but merge any two states that differ only in lookahead
  - Example: these two would be merged

    [*A* ::= x . , a]
    [*A* ::= x . , b]

LALR(1) vs LR(1)

- LALR(1) tables can have many fewer states than LR(1)
- LALR(1) may have reduce conflicts where LR(1) would not (but in practice this doesn't happen often)

Language Heirarchies



Coming Attractions

- LL(k) Parsing – Top-Down
- Recursive Descent Parsers
  - What you can do if you need a parser in a hurry