## CSE P 501 - Compilers

## Parsing \& Context-Free Grammars Hal Perkins <br> Autumn 2011

## Agenda for Today

- Parsing overview
- Context free grammars
- Ambiguous grammars
- Reading: Cooper/Torczon ch. 3, or Dragon Book ch. 4, or Appel ch. 3


## Parsing

- The syntax of most programming languages can be specified by a context-free grammar (CGF)
- Parsing: Given a grammar $G$ and a sentence $w$ in $L(G)$, traverse the derivation (parse tree) for $w$ in some standard order and do something useful at each node
- The tree might not be produced explicitly, but the control flow of a parser corresponds to a traversal


## Old Example

program


## "Standard Order"

- For practical reasons we want the parser to be deterministic (no backtracking), and we want to examine the source program from left to right.
- (i.e., parse the program in linear time in the order it appears in the source file)


## Common Orderings

- Top-down
- Start with the root
- Traverse the parse tree depth-first, left-to-right (leftmost derivation)
- LL(k)
- Bottom-up
- Start at leaves and build up to the root
- Effectively a rightmost derivation in reverse(!)
- LR(k) and subsets (LALR(k), SLR(k), etc.)


## "Something Useful"

- At each point (node) in the traversal, perform some semantic action
- Construct nodes of full parse tree (rare)
- Construct abstract syntax tree (common)
- Construct linear, lower-level representation (more common in later parts of a modern compiler)
- Generate target code on the fly (1-pass compiler; not common in production compilers - can't generate very good code in one pass - but great if you need a quick ' $n$ dirty working compiler)


## Context-Free Grammars

- Formally, a grammar $G$ is a tuple $\langle N, \Sigma, P, S\rangle$ where
- $N$ a finite set of non-terminal symbols
- $\Sigma$ a finite set of terminal symbols
- $P$ a finite set of productions
- A subset of $N \times(N \cup \Sigma)^{*}$
- $S$ the start symbol, a distinguished element of $N$
- If not specified otherwise, this is usually assumed to be the non-terminal on the left of the first production


## Standard Notations

- a, b, c elements of $\Sigma$
- w, x, y, z elements of $\Sigma^{*}$
- A, B, C elements of $N$
- X, Y, Z elements of $N \cup \Sigma$
- $\alpha, \beta, \gamma$ elements of $(N \cup \Sigma)^{*}$
- $\mathrm{A} \rightarrow \alpha$ or $\mathrm{A}::=\alpha$ if $\langle\mathrm{A}, \alpha\rangle$ in $P$


## Derivation Relations (1)

- $\alpha \mathrm{A} \gamma=>\alpha \beta \gamma$ iff $\mathrm{A}::=\beta$ in $P$
- derives
- A = ${ }^{*} \alpha$ if there is a chain of productions starting with A that generates $\alpha$
- transitive closure


## Derivation Relations (2)

- w $\mathrm{A} \gamma=>_{\text {Im }} \mathrm{w} \beta \gamma$ iff $\mathrm{A}::=\beta$ in $P$
- derives leftmost
- $\alpha$ A w $=>_{r m} \alpha \beta$ w iff $A::=\beta$ in $P$
- derives rightmost
- We will only be interested in leftmost and rightmost derivations - not random orderings


## Languages

- For A in $N, L(\mathrm{~A})=\left\{\mathrm{w} \mid \mathrm{A}=>^{*} \mathrm{w}\right\}$
- If $S$ is the start symbol of grammar $G$, define $L(G)=L(S)$
- Nonterminal on the left of the first rule is taken to be the start symbol if one is not specified explicitly


## Reduced Grammars

- Grammar $G$ is reduced iff for every production $\mathrm{A}::=\alpha$ in $G$ there is some derivation

$$
S=>^{*} x A z=>x \alpha z=>^{*} x y z
$$

- i.e., no production is useless
- Convention: we will use only reduced grammars


## Ambiguity

- Grammar $G$ is unambiguous iff every $w$ in $L(G)$ has a unique leftmost (or rightmost) derivation
- Fact: unique leftmost or unique rightmost implies the other
- A grammar lacking this property is ambiguous
- Note that other grammars that generate the same language may be unambiguous
- We need unambiguous grammars for parsing


## Example: Ambiguous Grammar for Arithmetic Expressions

$$
\begin{aligned}
\text { expr }:: & =\text { expr }+ \text { expr } \mid \text { expr }- \text { expr } \\
& \mid \text { expr }{ }^{*} \text { expr } \mid \text { expr }::=0|1| 2|3| 4|5| 6|7| 8 \mid 9
\end{aligned}
$$

- Exercise: show that this is ambiguous
- How? Show two different leftmost or rightmost derivations for the same string
- Equivalently: show two different parse trees for the same string


## expr ::= expr + expr $\mid$ expr - expr <br> | expr* expr | expr/ expr| int int $::=0|1| 2|3| 4|5| 6|7| 8 \mid 9$ <br> Example (cont)

- Give a leftmost derivation of $2+3 * 4$ and show the parse tree


## expr ::= expr + expr | expr - expr <br> | expr* expr | expr/ expr | int int ::=0|1|2|3|4|5|6|7|8|9 <br> Example (cont)

- Give a different leftmost derivation of $2+3 * 4$ and show the parse tree


## expr::= expr + expr | expr- expr <br> | expr* expr | expr/ expr | int int $::=0|1| 2|3| 4|5| 6|7| 8 \mid 9$ <br> Another example

- Give two different derivations of 5+6+7


## What's going on here?

- The grammar has no notion of precedence or associatively
- Solution
- Create a non-terminal for each level of precedence
- Isolate the corresponding part of the grammar
- Force the parser to recognize higher precedence subexpressions first


## Classic Expression Grammar

expr $::=$ expr + term | expr - term $\mid$ term
term ::= term * factor $\mid$ term | factor $\mid$ factor factor ::= int | ( expr)
int $::=0|1| 2|3| 4|5| 6 \mid 7$

$$
\text { expr }::=\text { expr }+ \text { term } \mid \text { expr }- \text { term } \mid \text { term }
$$

$$
\text { term }::=\text { term } * \text { factor } \mid \text { term } / \text { factor } \mid \text { factor }
$$

$$
\text { factor }::=\text { int } \mid \text { ( expr })
$$

$$
\text { int }::=0|1| 2|3| 4|5| 6 \mid 7
$$

## Check: Derive $2+3$ * 4

$$
\begin{aligned}
& \text { expr }::=\text { expr }+ \text { term } \mid \text { expr }- \text { term } \mid \text { term } \\
& \text { term }::=\text { term } * \text { factor } \mid \text { term } / \text { factor } \mid \text { factor } \\
& \text { factor }::=\text { int } \mid(\text { expr }) \\
& \text { int }::=0|1| 2|3| 4|5| 6 \mid 7
\end{aligned}
$$

## Check: Derive $5+6+7$

- Note interaction between left- vs right-recursive rules and resulting associativity
expr $::=$ expr + term $\mid$ expr- term $\mid$ term
term $::=$ term $*$ factor $\mid$ term $/$ factor $\mid$ factor factor $::=$ int $\mid$ ( expr)
int $::=0|1| 2|3| 4|5| 6 \mid 7$


## Check: Derive $5+(6+7)$

## Another Classic Example

- Grammar for conditional statements stmt ::= if ( cond ) stmt | if (cond) stmt else stmt
- Exercise: show that this is ambiguous
- How?

```
stmt ::= if ( cond ) stmt
    | if ( cond) stmt else stmt
```


## One Derivation

if ( cond ) if ( cond ) stmt else stmt

```
stmt ::= if ( cond ) stmt
    | if ( cond) stmt else stmt
```


## Another Derivation

if ( cond ) if ( cond ) stmt else stmt

## Solving "if" Ambiguity

- Fix the grammar to separate if statements with else clause and if statements with no else
- Done in Java reference grammar
- Adds lots of non-terminals
- Use some ad-hoc rule in parser
- "else matches closest unpaired if"
- Change the language
- You better have permission to do this


## Resolving Ambiguity with Grammar (1)

Stmt ::= MatchedStmt | UnmatchedStmt MatchedStmt ::=... |
if ( Expr ) MatchedStmt else MatchedStmt
UnmatchedStmt ::= if (Expr ) Stmt |
if ( Expr ) MatchedStmt else UnmatchedStmt

- formal, no additional rules beyond syntax
- sometimes obscures original grammar


## Resolving Ambiguity with Grammar (2)

- If you can (re-)design the language, avoid the problem entirely

```
Stmt ::= ... |
if Expr then Stmt end |
if Expr then Stmt else Stmt end
```

- formal, clear, elegant
- allows sequence of Stmts in then and else branches, no \{ , \} needed
- extra end required for every if
(But maybe this is a good idea anyway?)


## Parser Tools and Operators

- Most parser tools can cope with ambiguous grammars
- Makes life simpler if used with discipline
- Usually can specify operator precedence \& associativity
- Allows simpler, ambiguous grammar with fewer nonterminals as basis for generated parser, without creating problems


## Parser Tools and Ambiguous Grammars

- Possible rules for resolving other problems
- Earlier productions in the grammar preferred to later ones
- Longest match used if there is a choice
- Parser tools normally allow for this
- But be sure that what the tool does is really what you want


## Coming Attractions

- Next topic: LR parsing
- Continue reading ch. 3 or 4 or 3 (depending on your book)

