



CSE P 501 – Compilers

LR Parsing
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Agenda

- LR Parsing
- Table-driven Parsers
- Parser States
- Shift-Reduce and Reduce-Reduce conflicts



LR(1) Parsing

- We'll look at LR(1) parsers
 - Left to right scan, Rightmost derivation, 1 symbol lookahead
 - Almost all practical programming languages have an LR(1) grammar
 - LALR(1), SLR(1), etc. – subsets of LR(1)
 - LALR(1) can parse most real languages, is more compact, and is used by YACC/Bison/etc.



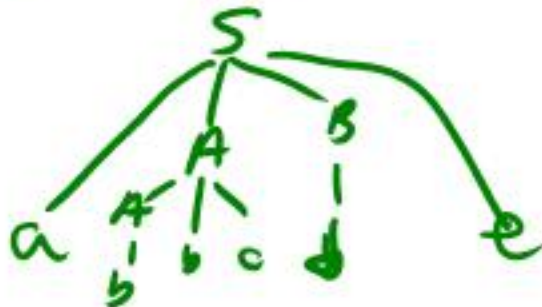
Bottom-Up Parsing

- Idea: Read the input left to right
- Whenever we've matched the right hand side of a production, reduce it to the appropriate non-terminal and add that non-terminal to the parse tree
- The upper edge of this partial parse tree is known as the *frontier*

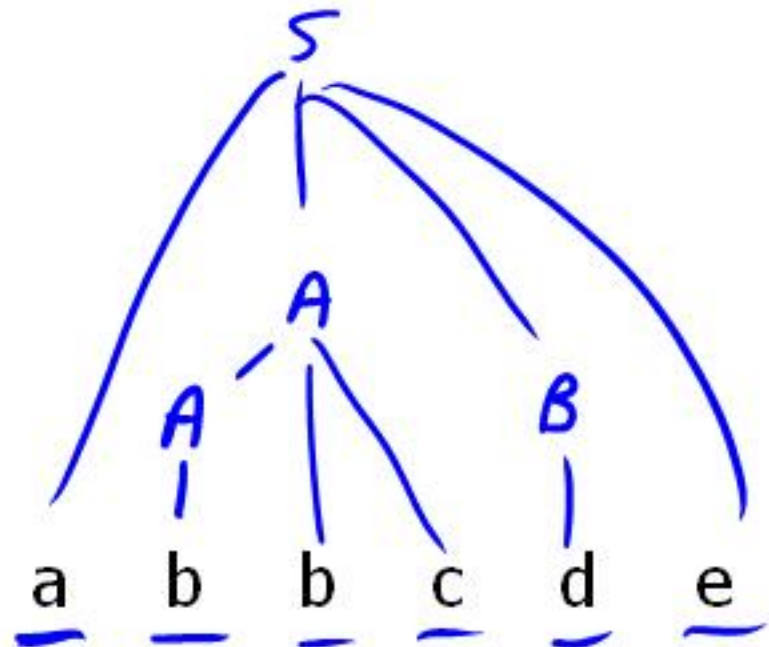
Example

- Grammar

$S ::= aAB e$
 $A ::= \underline{A}bc \mid \underline{b}$
 $B ::= \underline{d}$



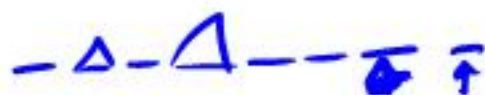
- Bottom-up Parse





Details

- The bottom-up parser reconstructs a reverse rightmost derivation
- Given the rightmost derivation
 - $S \Rightarrow \beta_1 \Rightarrow \beta_2 \Rightarrow \dots \Rightarrow \beta_{n-2} \Rightarrow \beta_{n-1} \Rightarrow \beta_n = W$
the parser will first discover $\beta_{n-1} \Rightarrow \beta_n$, then $\beta_{n-2} \Rightarrow \beta_{n-1}$, etc.
- Parsing terminates when
 - β_1 reduced to S (start symbol, success), or
 - No match can be found (syntax error)



How Do We Parse with This?

- Key: given what we've already seen and the next input symbol, decide what to do.
- Choices:
 - Perform a reduction
 - Look ahead further
- Can reduce $A \Rightarrow \beta$ if both of these hold:
 - $A \Rightarrow \beta$ is a valid production
 - $A \Rightarrow \beta$ is a step in *this* rightmost derivation
- This is known as a shift-reduce parser



Sentential Forms

- If $S \Rightarrow^* \alpha$, the string α is called a *sentential form* of the grammar
- In the derivation
 $S \Rightarrow \underline{\beta_1} \Rightarrow \underline{\beta_2} \Rightarrow \dots \Rightarrow \underline{\beta_{n-2}} \Rightarrow \underline{\beta_{n-1}} \Rightarrow \underline{\beta_n} = \underline{W}$
each of the β_i are sentential forms
- A sentential form in a rightmost derivation is called a right-sentential form (similarly for leftmost and left-sentential)



Handles

- Informally, a substring of the tree frontier that matches the right side of a production
 - Even if $A ::= \beta$ is a production, β is a handle only if it matches the frontier at a point where $A ::= \beta$ was used in the derivation
 - β may appear in many other places in the frontier without being a handle for that particular production



Handles (cont.)

- Formally, a *handle* of a right-sentential form γ is a production $A ::= \beta$ and a position in γ where β may be replaced by A to produce the previous right-sentential form in the rightmost derivation of γ

Handle Examples

- In the derivation

➤ $S \Rightarrow aABe \Rightarrow aAde \Rightarrow \underline{aAbcde} \Rightarrow \underline{abbcde}$

- $abbcde$ is a right sentential form whose handle is $A ::= b$ at position 2
- $aAbcde$ is a right sentential form whose handle is $A ::= Abc$ at position 4
 - Note: some books take the left of the match as the position

Implementing Shift-Reduce Parsers

- Key Data structures

- A stack holding the frontier of the tree
- A string with the remaining input





Shift-Reduce Parser Operations

- *Reduce* – if the top of the stack is the right side of a handle $A ::= \beta$, pop the right side β and push the left side A .
- *Shift* – push the next input symbol onto the stack
- *Accept* – announce success
- *Error* – syntax error discovered

Shift-Reduce Example

$S ::= aABe$
 $A ::= Abc \mid b$
 $\bar{B} ::= d$

| Stack | Input | Action |
|--------|----------|--------------|
| \$ | abbcde\$ | <u>shift</u> |
| \$a | bbcdes | shift |
| \$ab | bcdes | reduce |
| \$aA | bcdes | shift |
| \$aAb | cdes | shift |
| \$aAbc | des | reduce |
| \$aA | des | shift |
| \$aAd | es | reduce |
| \$aAB | es | shift |
| \$aABe | s | reduce |
| \$S | \$ | accept |





How Do We Automate This?

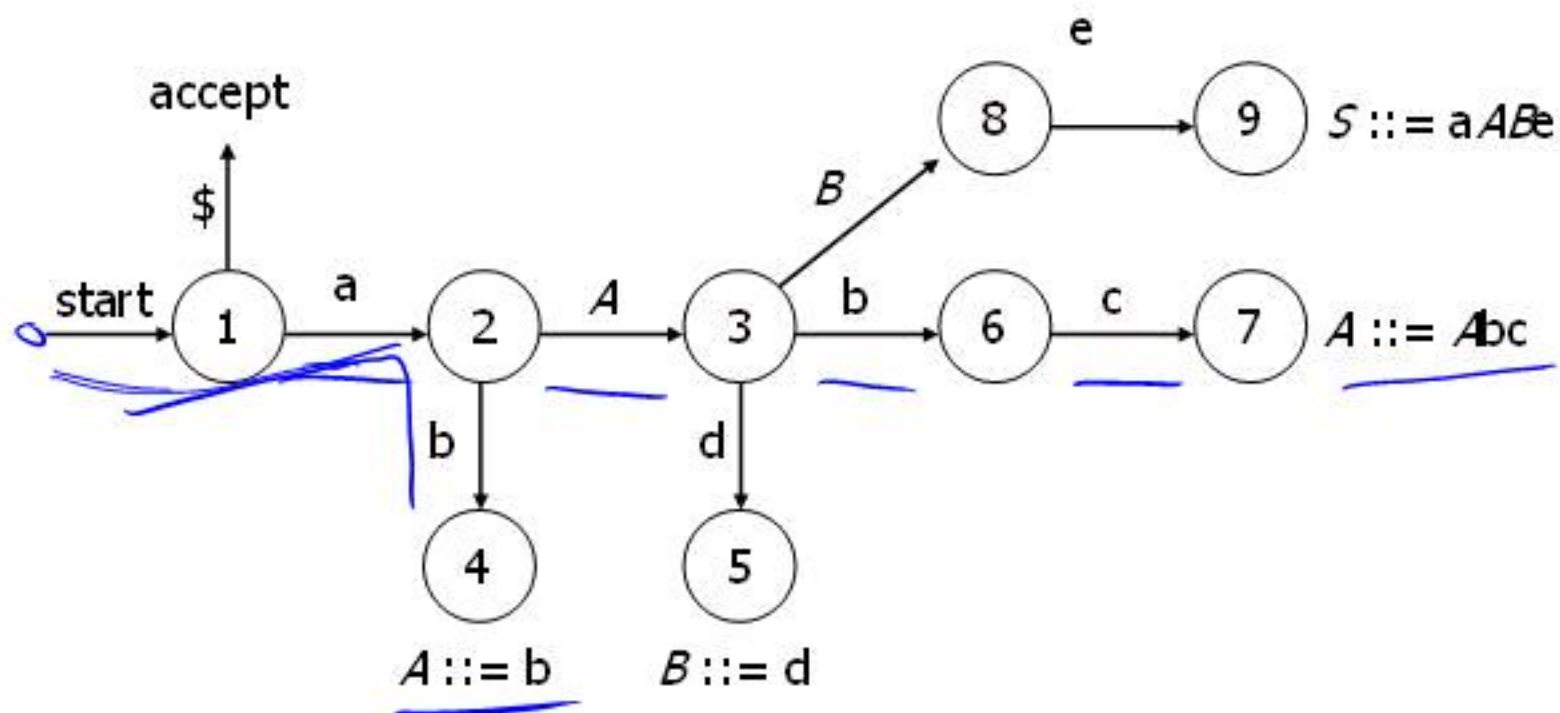
- Def. *Viable prefix* – a prefix of a right-sentential form that can appear on the stack of the shift-reduce parser
 - Equivalent: a prefix of a right-sentential form that does not continue past the rightmost handle of that sentential form
- Idea: Construct a DFA to recognize viable prefixes given the stack and remaining input
 - Perform reductions when we recognize them

DFA for prefixes of

$S ::= aABe$

$A ::= Abc \mid b$

$B ::= d$



Trace

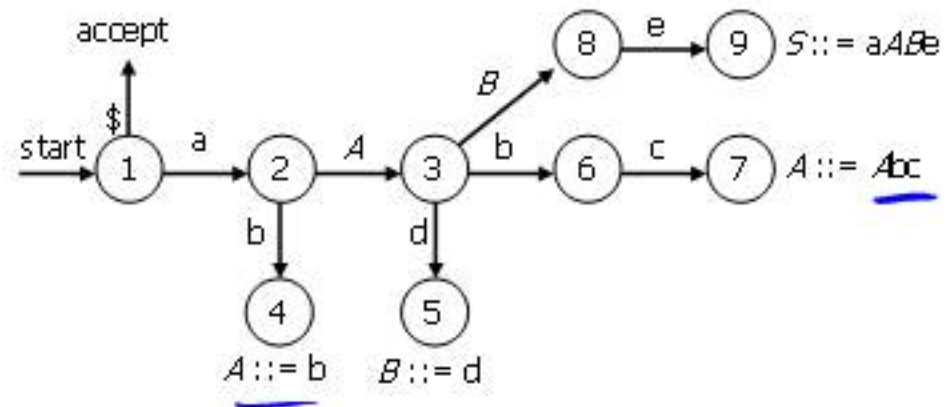
$S ::= aABe$
 $A ::= Abc \mid b$
 $B ::= d$

Stack

$\$$
 $_$
 $\$ a$
 $\$ a b$
 $\$ a A$
 $\$ a Ab$
 $\$ a Abc$
 $\$ a A$
 $\$ a Ad$
 $\$ a AB$
 $\$ a ABe$
 $\$ \underline{S}$

Input

$abbcde\$$
 $bbcde\$$
 $bcde\$$
 $bcde\$$
 $cde\$$
 $de\$$
 $de\$$
 $e\$$
 $e\$$
 $\$$
 $\$$





Observations

- Way too much backtracking
 - We want the parser to run in time proportional to the length of the input
- Where the heck did this DFA come from anyway?
 - From the underlying grammar
 - We'll defer construction details for now



Avoiding DFA Rescanning

- Observation: after a reduction, the contents of the stack are the same as before except for the new non-terminal on top
 - \therefore Scanning the stack will take us through the same transitions as before until the last one
 - \therefore If we record state numbers on the stack, we can go directly to the appropriate state when we pop the right hand side of a production from the stack



Stack

- Change the stack to contain pairs of states and symbols from the grammar
$$\$s_0 \ X_1 \ S_1 \ X_2 \ S_2 \ \dots \ X_n \ S_n$$
 - State s_0 represents the accept state
 - (Not always added – depends on particular presentation)
- Observation: in an actual parser, only the state numbers need to be pushed, since they implicitly contain the symbol information, but for explanations, it's clearer to use both.



Encoding the DFA in a Table

- A shift-reduce parser's DFA can be encoded in two tables
 - One row for each state
 - *action* table encodes what to do given the current state and the next input symbol
 - *goto* table encodes the transitions to take after a reduction



Actions (1)

- Given the current state and input symbol, the main possible actions are
 - si – shift the input symbol and **state i** onto the stack (i.e., shift and move to state i)
 - rj – reduce using grammar **production j**
 - The production number tells us how many \langle symbol, state \rangle pairs to pop off the stack



Actions (2)

- Other possible *action* table entries
 - *accept*
 - **blank** – no transition – syntax error
 - A LR parser will detect an error as soon as possible on a left-to-right scan
 - A real compiler needs to produce an error message, recover, and continue parsing when this happens



Goto

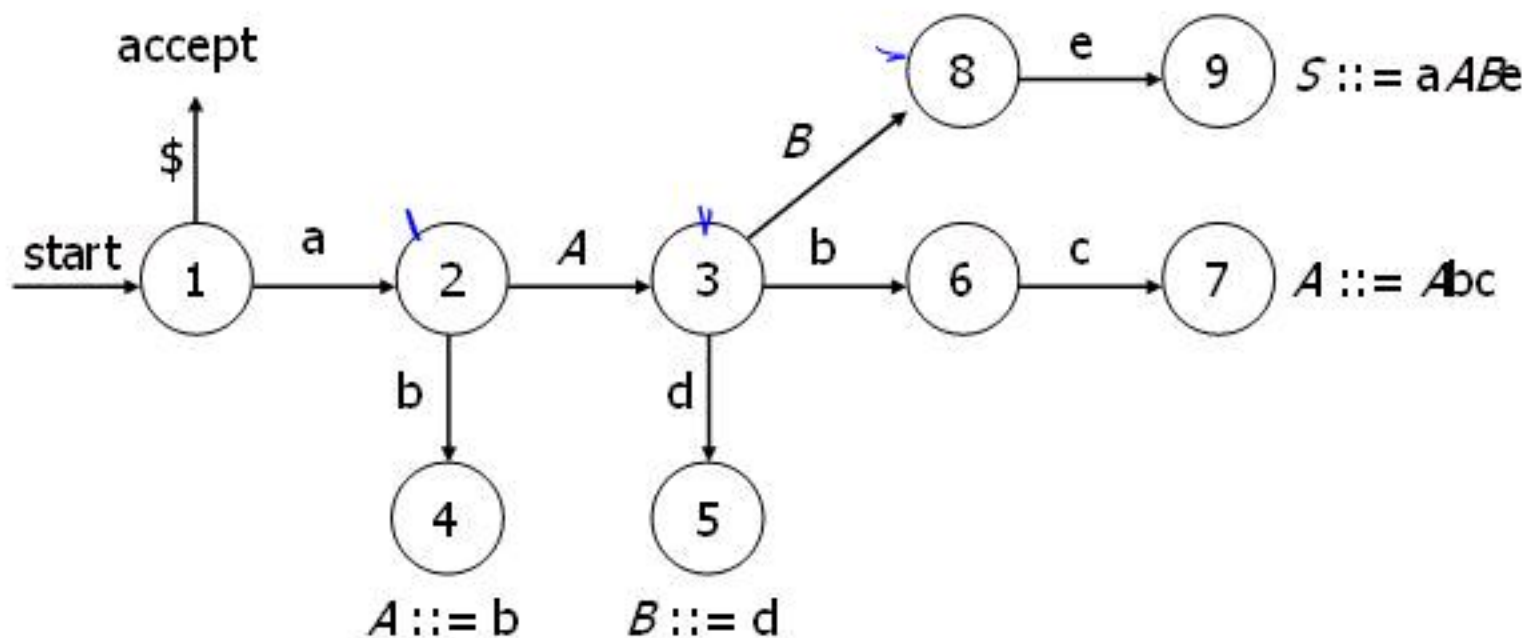
- When a reduction is performed, $\langle \text{symbol}, \text{state} \rangle$ pairs are popped from the stack revealing a state *uncovered_s* on the top of the stack
- $\text{goto}[\text{uncovered}_s, A]$ is the new state to push on the stack when reducing production $A ::= \beta$ (after popping β and finding state *uncovered_s* on top)

Reminder: DFA for

$S ::= aABe$

$A ::= Abc \mid b$

$B ::= d$



LR Parse Table for

- ✓ 1. $S ::= aABe$
- ✓ 2. $A ::= Abc$
- ✓ 3. $A ::= b$
- ✓ 4. $B ::= d$

| State | <i>action</i> | | | | | | <i>goto</i> | | |
|-------|---------------|----|----|----|----|-----|-------------|----|----|
| | a | b | c | d | e | \$ | A | B | S |
| 1 | s2 | | | | | acc | | | g1 |
| 2 | | s4 | | | | | g3 | | |
| 3 | | s6 | | s5 | | | | g8 | |
| 4 | r3 | r3 | r3 | r3 | r3 | r3 | | | |
| 5 | r4 | r4 | r4 | r4 | r4 | r4 | | | |
| 6 | | | s7 | | | | | | |
| 7 | r2 | r2 | r2 | r2 | r2 | r2 | | | |
| 8 | | | | | s9 | | | | |
| 9 | r1 | r1 | r1 | r1 | r1 | r1 | | | |

LR Parsing Algorithm (1)

```
word = scanner.getToken();
while (true) {
    s = top of stack;
    - if (action[s, word] = s/) {
        push word; push / (state);
        word = scanner.getToken();
    } else if (action[s, word] = rj) {
        pop 2 * length of right side of
        production j (2*|β|);
        uncovered_s = top of stack;
        push left side A of production j;
        push state goto[uncovered_s, A];
    }
    } else if (action[s, word] = accept ) {
        return;
    } else {
        // no entry in action table
        report syntax error;
        halt or attempt recovery;
    }
}
```

Example

1. $S ::= aABe$
2. $A ::= \underline{Abc}$
3. $A ::= b$
4. $B ::= d$



Stack

$\$1$
 $\$1a2$
 $\$1a2b4$
 $\$1a2A3$
 $\$1a2A3b6$
 $\$1a2A3b6c7$
 $\$1a2A3$
 $\$1a2A3d5$
 $\$1a2A3B8$
 $\$1a2A3B8e9$
 $\$15$

Input

abbcde\$
 b b c d e \$
 b c d e \$
 b c d e \$
 c d e \$
 d e \$
 d e \$
 e \$
 e \$
 \$
 \$

| S | action | | | | | | goto | | |
|---|-----------|----|----|----|----|-----------|-----------|----|----|
| | a | b | c | d | e | \$ | A | B | S |
| 1 | <u>s2</u> | | | | | <u>ac</u> | | | g1 |
| 2 | | s4 | | | | | <u>g3</u> | | |
| 3 | | s6 | | s5 | | | | g8 | |
| 4 | r3 | r3 | r3 | r3 | r3 | r3 | | | |
| 5 | r4 | r4 | r4 | r4 | r4 | r4 | | | |
| 6 | | | s7 | | | | | | |
| 7 | r2 | r2 | r2 | r2 | r2 | r2 | | | |
| 8 | | | | | s9 | | | | |
| 9 | r1 | r1 | r1 | r1 | r1 | r1 | | | |



LR States

- Idea is that each state encodes
 - The set of all possible productions that we could be looking at, given the current state of the parse, and
 - *Where* we are in the right hand side of each of those productions



Items

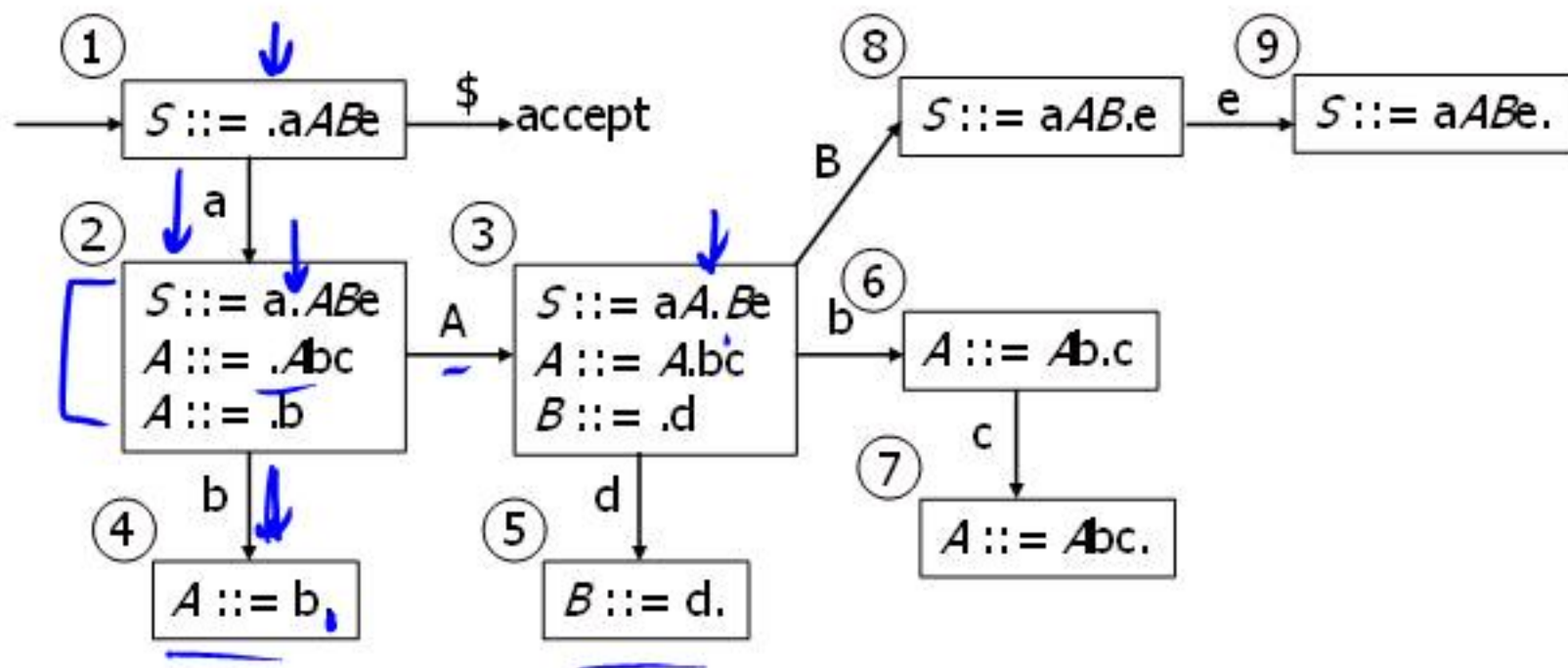
- An *item* is a production with a dot in the right hand side
- Example: Items for production $A ::= XY$
 - $A ::= .XY$
 - $A ::= X.Y$
 - $A ::= XY.$
- Idea: The dot represents a position in the production

DFA for

$S ::= aABe$

$A ::= Abc \mid b$

$B ::= d$





Problems with Grammars

- Grammars can cause to problems when constructing a LR parser
 - Shift-reduce conflicts
 - Reduce-reduce conflicts



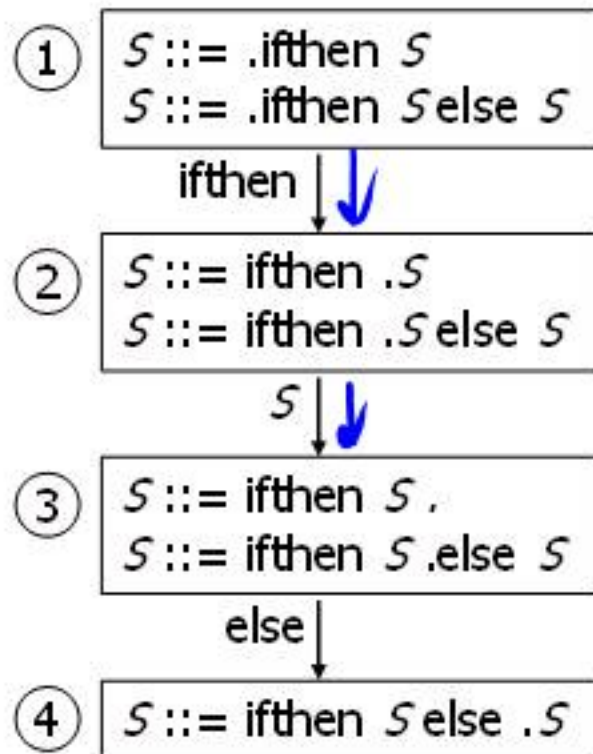
Shift-Reduce Conflicts

- Situation: both a shift and a reduce are possible at a given point in the parse (equivalently: in a particular state of the DFA)
- Classic example: if-else statement
$$S ::= \text{ifthen } S \mid \text{ifthen } S \text{ else } S$$



Parser States for

1. $S ::= \text{ifthen } S$
2. $S ::= \text{ifthen } S \text{ else } S$



■ State 3 has a shift-reduce conflict

- Can shift past else into state 4 (s4)
- Can reduce (r1)
 $S ::= \text{ifthen } S$

(Note: other $S ::= \text{.ifthen}$ items not included in states 2-4 to save space)



Solving Shift-Reduce Conflicts

- Fix the grammar
 - Done in Java reference grammar, others
- Use a parse tool with a “longest match” rule – i.e., if there is a conflict, choose to shift instead of reduce
 - Does exactly what we want for if-else case
 - Guideline: a few shift-reduce conflicts are fine, but be sure they do what you want



Reduce-Reduce Conflicts

- Situation: two different reductions are possible in a given state
- Contrived example

$S ::= A$

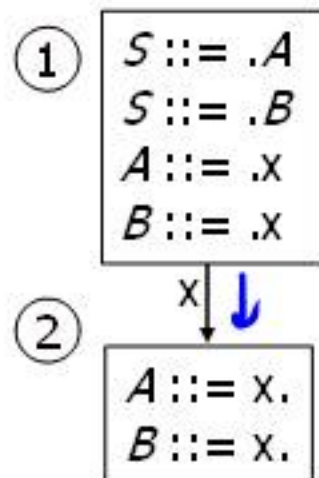
$S ::= B$

$A ::= x$

$B ::= x$

Parser States for

1. $S ::= A$
2. $S ::= B$
3. $A ::= x$
4. $B ::= x$



- State 2 has a reduce-reduce conflict (r3, r4)



Handling Reduce-Reduce Conflicts

- These normally indicate a serious problem with the grammar.
- Fixes
 - Use a different kind of parser generator that takes lookahead information into account when constructing the states (LR(1) instead of SLR(1) for example)
 - Most practical tools use this information
 - Fix the grammar



Another Reduce-Reduce Conflict

- Suppose the grammar separates arithmetic and boolean expressions

$expr ::= aexp \mid bexp$

- $aexp ::= aexp * aident \mid aident$

- $bexp ::= bexp \&\& bident \mid bident$

[$aident ::= id$
 $bident ::= id$

- This will create a reduce-reduce conflict



Covering Grammars

- A solution is to merge *aident* and *bident* into a single non-terminal (or use *id* in place of *aident* and *bident* everywhere they appear)
- This is a *covering grammar*
 - Includes some programs that are not generated by the original grammar
 - Use the type checker or other static semantic analysis to weed out illegal programs later



Coming Attractions

- Constructing LR tables
 - We'll present a simple version (SLR(0)) in lecture, then talk about extending it to LR(1)
- LL parsers and recursive descent
- Continue reading ch. 4