# CSE P 501 – Compilers

LR Parser Construction

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# Agenda

- LR(0) state construction
- FIRST, FOLLOW, and nullable
- Variations: SLR, LR(1), LALR

# LR State Machine

- Idea: Build a DFA that recognizes handles
  - Language generated by a CFG is generally not regular, but
  - Language of handles for a CFG is regular
    - So a DFA can be used to recognize handles
  - Parser reduces when DFA accepts

# Prefixes, Handles, &c (review)

- If $S$ is the start symbol of a grammar $G$,
  - If $S =>^* \alpha$ then $\alpha$ is a *sentential form* of $G$
  - $\gamma$ is a *viable prefix* of $G$ if there is some derivation
    $S =>^*_{rm} \alpha A w =>^*_{rm} \alpha\beta w$
    and $\gamma$ is a prefix of $\alpha\beta$.
  - The occurrence of $\beta$ in $\alpha\beta w$ is a *handle* of $\alpha\beta w$
- An *item* is a marked production (a . at some position in the right hand side)
  - $[A ::= .\ X\ Y]$   $[A ::= X\ .\ Y]$   $[A ::= X\ Y\ .]$

# Building the LR(0) States

- Example grammar

  $S' ::= S \, \$$

  $S ::= ( \; L \; )$

  $S ::= \mathrm{x}$

  $L ::= S$

  $L ::= L \, , \, S$

  - We add a production S′ with the original start symbol followed by end of file ($)
  - Question: What language does this grammar generate?

0. $S' ::= S\,\$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

# Start of LR Parse

- Initially
  - Stack is empty
  - Input is the right hand side of $S'$, i.e., $S\,\$$
  - Initial configuration is $[S' ::= .\ S\,\$]$
  - But, since position is just before $S$, we are also just before anything that can be derived from $S$

0. $S' ::= S\,\$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

# Initial state

$$\boxed{\begin{array}{l} S' ::= .\ S\ \$ \\ S ::= .\ (\ L\ ) \\ S ::= .\ \mathrm{x} \end{array}}$$

start

completion

- A state is just a set of items
  - Start: an initial set of items
  - Completion (or closure): additional productions whose left hand side appears to the right of the dot in some item already in the state

0. $S' ::= S\,\$$
1. $S ::= (\, L\, )$
2. $S ::= x$
3. $L ::= S$
4. $L ::= L\, ,\, S$

# Shift Actions (1)

$$\boxed{\begin{array}{l} S' ::= \,.\, S\,\$ \\ S ::= \,.\, (\,L\,) \\ S ::= \,.\, x \end{array}} \xrightarrow{x} \boxed{S ::= x\,.}$$

- To shift past the x, add a new state with the appropriate item(s)
  - In this case, a single item; the closure adds nothing
  - This state will lead to a reduction since no further shift is possible

0. $S' ::= S\,\$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

# Shift Actions (2)

$$\begin{array}{|l|}\hline S' ::= .\ S\,\$ \\ S ::= .\ (\ L\ ) \\ S ::= .\ \mathrm{x} \\ \hline\end{array} \xrightarrow{\ (\ } \begin{array}{|l|}\hline S ::= (\ .\ L\ ) \\ L ::= .\ L\ ,\ S \\ L ::= .\ S \\ S ::= .\ (\ L\ ) \\ S ::= .\ \mathrm{x} \\ \hline\end{array}$$

- If we shift past the ( , we are at the beginning of $L$
- the closure adds all productions that start with $L$, which requires adding all productions starting with $S$

# Goto Actions

0. $S' ::= S\,\$$
1. $S ::= (\,L\,)$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\,,\,S$

$$\boxed{\begin{array}{l} S' ::= .\,S\,\$ \\ S ::= .\,(\,L\,) \\ S ::= .\,\mathrm{x} \end{array}} \xrightarrow{S} \boxed{S' ::= S\,.\,\$}$$

- Once we reduce *S*, we'll pop the rhs from the stack exposing the first state. Add a *goto* transition on *S* for this.

# Basic Operations

- *Closure* ( *S* )
  - Adds all items implied by items already in *S*
- *Goto* (*I*, *X* )
  - *I* is a set of items
  - *X* is a grammar symbol (terminal or non-terminal)
  - *Goto* moves the dot past the symbol *X* in all appropriate items in set *I*

# Closure Algorithm

- *Closure* ( $S$ ) =

  repeat

  for any item [A ::= $\alpha$ . $X\beta$] in $S$

  for all productions $X ::= \gamma$

  add [$X ::= . \gamma$] to $S$

  until $S$ does not change

  return $S$

# Goto Algorithm

- *Goto* ( $I$, $X$ ) =

  set *new* to the empty set

  for each item [A ::= $\alpha$ . $X$ $\beta$] in $I$

      add [A ::= $\alpha$ $X$ . $\beta$] to *new*

  return *Closure* ( *new* )

  - This may create a new state, or may return an existing one

# LR(0) Construction

- First, augment the grammar with an extra start production $S' ::= S\,\$$
- Let $T$ be the set of states
- Let $E$ be the set of edges
- Initialize $T$ to $Closure(\ [S' ::= .\ S\,\$]\ )$
- Initialize $E$ to empty

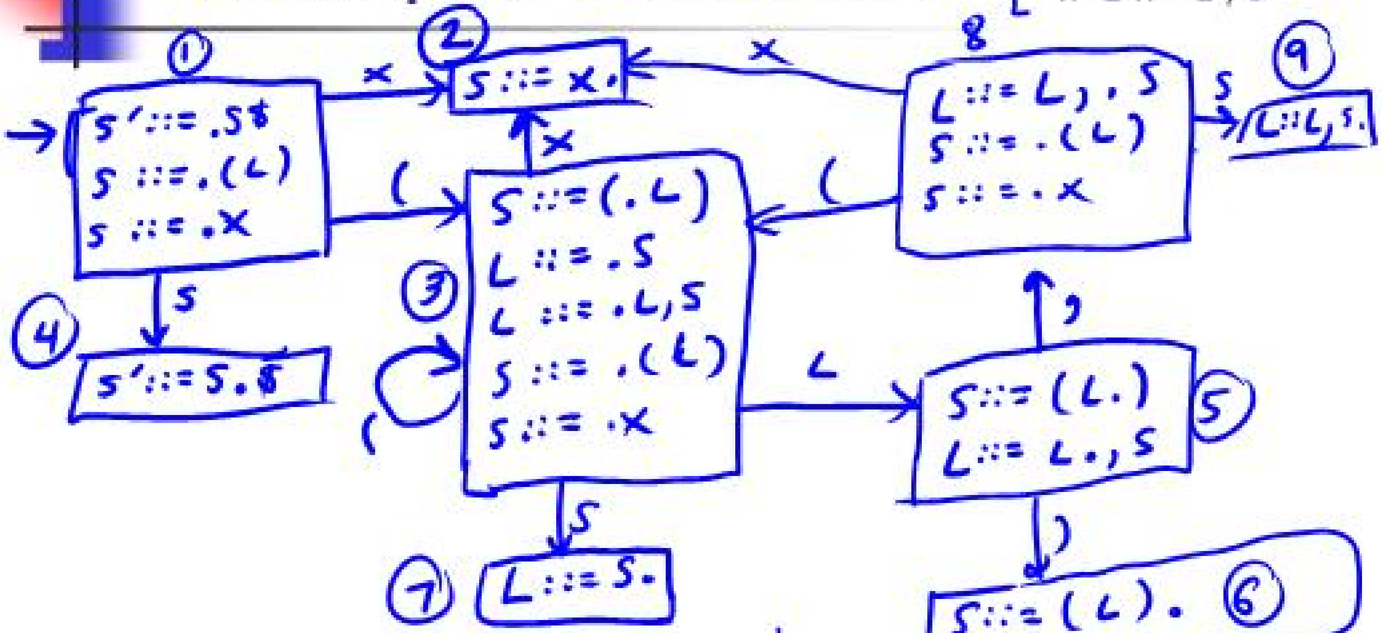
# LR(0) Construction Algorithm

repeat
  for each state $I$ in $T$
    for each item $[A ::= \alpha \,.\, X \beta]$ in $I$
      Let *new* be *Goto* ( $I$, $X$ )
      Add *new* to $T$ if not present
      Add $I \xrightarrow{X}$ *new* to $E$ if not present
until $E$ and $T$ do not change in this iteration

- Footnote: For symbol \$, we don't compute *goto* ($I$, \$); instead, we make this an *accept* action.

# Example: States for

0. S′ ::= S $
1. S ::= ( L )
2. S ::= x
3. L ::= S
4. L ::= L , S




| | ( | ) | x | , | $ | S | L |
|---|---|---|---|---|---|---|---|
| 1 | s3 | | s2 | | | g4 | |
| 2 | r2 | r2 | r2 | r2 | r2 | | |
| 3 | s3 | | s2 | | | g7 | g5 |
| 4 | | | | | acc | | |
| 5 | | s6 | | s8 | | | |
| 6 | r1 | r1 | r1 | r1 | r1 | | |
| 7 | r3 | r3 | r3 | r3 | r3 | | |
| 8 | s3 | | s2 | | | g9 | |
| 9 | r4 | r4 | r4 | r4 | r4 | | |

# Building the Parse Tables (1)

- For each edge $I \xrightarrow{X} J$
  - if X is a terminal, put s$j$ in column X, row $I$ of the action table (shift to state $j$)
  - If X is a non-terminal, put g$j$ in column X, row $I$ of the goto table

# Where Do We Stand?

- We have built the LR(0) state machine and parser tables
  - No lookahead yet
  - Different variations of LR parsers add lookahead information, but basic idea of states, closures, and edges remains the same

# A Grammar that is not LR(0)

- Build the state machine and parse tables for a simple expression grammar

$S ::= E\,\$$

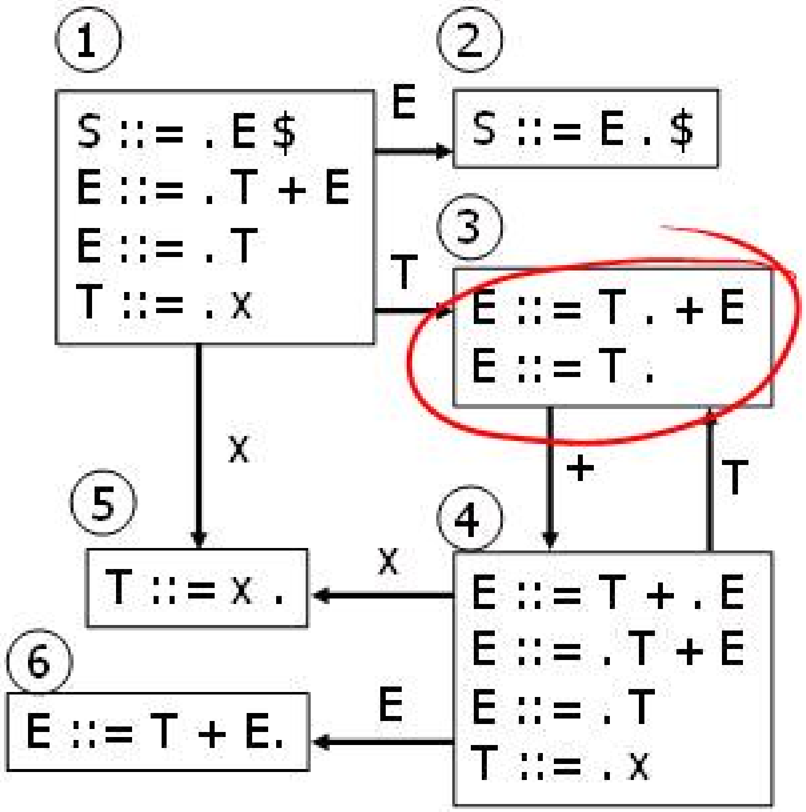
$E ::= T + E$

$E ::= T$

$T ::= \mathrm{x}$

# LR(0) Parser for

0. $S ::= E\ \$$
1. $E ::= T + E$
2. $E ::= T$
3. $T ::= x$




| | x | + | $ | E | T |
|---|---|---|---|---|---|
| 1 | s5 | | | g2 | G3 |
| 2 | | | acc | | |
| 3 | r2 | s4,r2 | r2 | | |
| 4 | s5 | | | g6 | G3 |
| 5 | r3 | r3 | r3 | | |
| 6 | r1 | r1 | r1 | | |

- State 3 is has two possible actions on +:

  shift 4 or reduce 2

- ∴ Grammar is not LR(0)

# SLR Parsers

- Idea: Use information about what can follow a non-terminal to decide if we should perform a reduction
- Easiest form is SLR – Simple LR
- We need to be able to compute FOLLOW(*A* ) – the set of symbols that can follow *A* in any possible derivation
  - i.e., t is in FOLLOW(A) if any derivation contains At
  - To compute this, we need to compute FIRST($\gamma$) for strings $\gamma$ that can follow *A*

# Calculating FIRST($\gamma$)

- Sounds easy… If $\gamma = X\ Y\ Z$, then FIRST($\gamma$) is FIRST($X$), right?
  - But what if we have the rule $X ::= \varepsilon$?
  - In that case, FIRST($\gamma$) includes anything that can follow an $X$ – i.e. FOLLOW($X$)

# FIRST, FOLLOW, and nullable

- nullable($X$) is true if $X$ can derive the empty string
- Given a string $\gamma$ of terminals and non-terminals, FIRST($\gamma$) is the set of terminals that can begin strings derived from $\gamma$.
- FOLLOW($X$) is the set of terminals that can immediately follow $X$ in some derivation
- All three of these are computed together

# Computing FIRST, FOLLOW, and nullable (1)

- Initialization

    set FIRST and FOLLOW to be empty sets

    set nullable(X) false for all non-terminals X

    set FIRST[a] to a for all terminal symbols a

# Computing FIRST, FOLLOW, and nullable (2)

repeat

  for each production $X := Y_1\ Y_2 \ldots Y_k$

    if $Y_1 \ldots Y_k$ are all nullable (or if $k = 0$)

      set nullable[$X$] = true

    for each $i$ from 1 to k and each $j$ from $i$+1 to $k$

      if $Y_1 \ldots Y_{i-1}$ are all nullable (or if $i = 1$)

        add FIRST[$Y_i$] to FIRST[$X$]

      if $Y_{i+1} \ldots Y_k$ are all nullable (or if $i = k$)

        add FOLLOW[$X$] to FOLLOW[$Y_i$]

      if $Y_{i+1} \ldots Y_{j-1}$ are all nullable (or if i+1=j)

        add FIRST[$Y_j$] to FOLLOW[$Y_i$]

Until FIRST, FOLLOW, and nullable do not change





$$X ::= \underbrace{Y_1\ Y_2 \cdots Y_{i-1}}\ Y_i\ Y_{i+1} \cdots Y_{j-1}\ Y_j\ Y_{j+1} \cdots Y_k$$

Pass 1

# Example

- Grammar

  $Z ::= d$

  $Z ::= X\ Y\ Z$

  $Y ::= \varepsilon$

  $Y ::= c$

  $X ::= Y$

  $X ::= a$

| | nullable | FIRST | FOLLOW |
|---|---|---|---|
| $X$ | true ~~nullable~~ ~~false~~ | c, a | c, d, a |
| $Y$ | true ~~false~~ | c | d, a, c |
| $Z$ | false | d, a, c | |

# LR(0) Reduce Actions

- In an LR(0) parser, if a state contains a reduction, it is unconditional regardless of the next input symbol
- Algorithm:

  Initialize $R$ to empty
  for each state $I$ in $T$
    for each item $[A ::= \alpha\ .]$ in $I$
      add $(I, A ::= \alpha)$ to $R$

# SLR Construction

- This is identical to LR(0) – states, etc., except for the calculation of reduce actions
- Algorithm:

  Initialize $R$ to empty

  for each state $I$ in $T$

  for each item $[A ::= \alpha\ .]$ in $I$

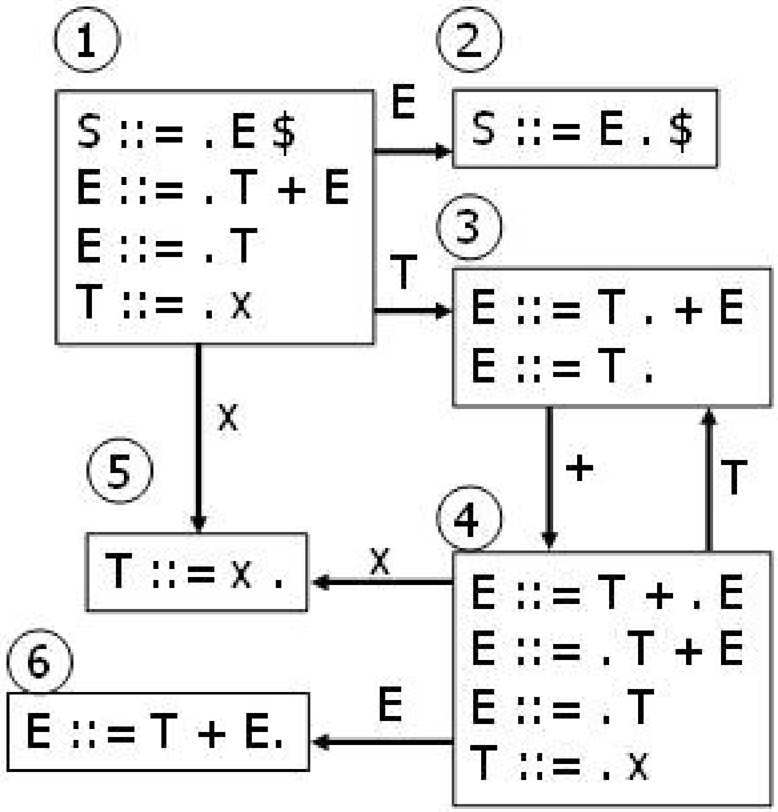
  for each terminal a in FOLLOW($A$)

  add ($I$, a, $A ::= \alpha$) to $R$

  - i.e., reduce $\alpha$ to $A$ in state $I$ only on lookahead a

# SLR Parser for

0. S ::= E $
1. E ::= T + E
2. E ::= T
3. T ::= x




| | x | + | $ | E | T |
|---|---|---|---|---|---|
| 1 | s5 | | | g2 | g3 |
| 2 | | | acc | | |
| 3 | r2 | s4, r2 | r2 | | |
| 4 | s5 | | | g6 | g3 |
| 5 | r3 | r3 | r3 | | |
| 6 | r1 | r1 | r1 | | |