CSE P 501 – Compilers

Loops

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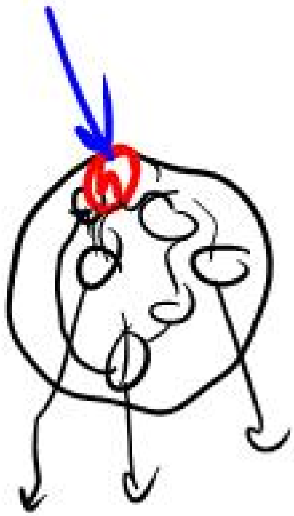
What's a Loop?

- In a control flow graph, a loop is a set of nodes S such that:
  - S includes a *header node* h
  - From any node in S there is a path of directed edges leading to h
  - There is a path from h to any node in S
  - There is no edge from any node outside S to any node in S other than h



Reducible Flow Graphs

- In a reducible flow graph, any two loops are either nested or disjoint
- Roughly, to discover if a flow graph is reducible, repeatedly delete edges and collapse together pairs of nodes (x,y) where x is the only predecessor of y
- If the graph can be reduced to a single node it is reducible
  - Caution: this is the "powerpoint" version of the definition – see a good compiler book for the careful details

Example: Is this Reducible?

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Reducible Flow Graphs in Practice

- Common control-flow constructs yield reducible flow graphs
  - if-then[-else], while, do, for, break(!)
- A C function without goto will always be reducible
- Many dataflow analysis algorithms are very efficient on reducible graphs, but…
- We don't need to assume reducible control-flow graphs to handle loops

Finding Loops in Flow Graphs

- We use *dominators* for this
- Recall
  - Every control flow graph has a unique start node s0
  - Node x dominates node y if every path from s0 to y must go through x
  - A node x dominates itself

Immediate Dominators

- Every node n has a single *immediate dominator* idom(n)
  - idom(n) differs from n
  - idom(n) dominates n
  - idom(n) does not dominate any other dominator of n
- Fact (er, theorem): If a dominates n and b dominates n, then either a dominates b or b dominates a
  - $\therefore$ idom(n) is unique

Back edge	natural loop	Back edge	natural loop
3 → 2	2,3	10 → 5	5,8,9,10
4 → 4	2,4	9 → 8	8,9

Example

Back Edges & Loops

- A flow graph edge from a node n to a node h that dominates n is a *back edge*
- For every back edge there is a corresponding subgraph of the flow graph that is a loop

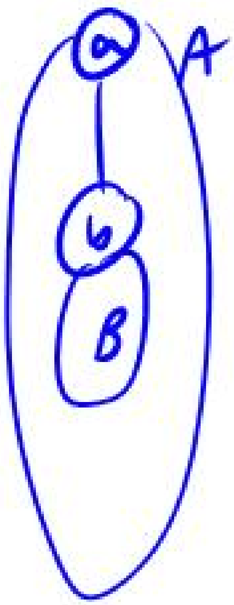
Natural Loops

- If h dominates n and n->h is a back edge, then the *natural loop* of that back edge is the set of nodes x such that
  - h dominates x
  - There is a path from x to n not containing h
- h is the *header* of this loop
- Standard loop optimizations can cope with loops whether they are natural or not

Inner Loops

- Inner loops are more important for optimization because most execution time is expected to be spent there
- If two loops share a header, it is hard to tell which one is "inner"
  - Common way to handle this is to merge natural loops with the same header

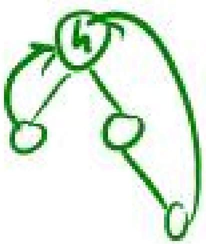
Inner (nested) loops



- Suppose
  - A and B are loops with headers a and b
  - $a \neq b$
  - b is in A
- Then
  - The nodes of B are a proper subset of A
  - B is nested in A, or B is the *inner loop*

Loop-Nest Tree

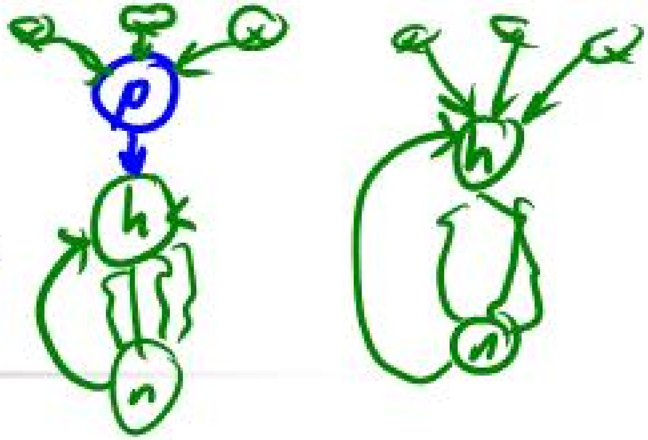
- Given a flow graph G
  1. Compute the dominators of G
  2. Construct the dominator tree
  3. Find the natural loops (thus all loop-header nodes)
  4. For each loop header h, merge all natural loops of h into a single loop: loop[h]
  5. Construct a tree of loop headers s.t. h1 is above h2 if h2 is in loop[h1]



Loop-Nest Tree details

- Leaves of this tree are the innermost loops
- Need to put all non-loop nodes somewhere
  - Convention: lump these into the root of the loop-nest tree

Loop Preheader

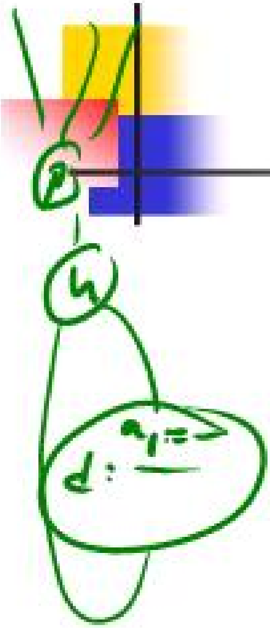


- Often we need a place to park code right before the beginning of a loop
- Easy if there is a single node preceding the loop header h
  - But this isn't the case in general
- So insert a *preheader* node p
  - Include an edge p->h
  - Change all edges x->h to be x->p ← x outside loop

Loop-Invariant Computations

- Idea: If `x := a1 op a2` always does the same thing each time around the loop, we'd like to *hoist* it and do it once outside the loop
- But can't always tell if a1 and a2 will have the same value
  - Need a conservative (safe) approximation

Loop-Invariant Computations



- d: x := a1 op a2 is *loop-invariant* if for each a_i
  - a_i is a constant, or
  - All the definitions of a_i that reach d are outside the loop, or
  - Only one definition of a_i reaches d, and that definition is loop invariant
- **Use this to build an iterative algorithm**
  - Base cases: constants and operands defined outside the loop
  - Then: repeatedly find definitions with loop-invariant operands

Hoisting

- Assume that d: x := a1 op a2 is loop invariant. We can hoist it to the loop preheader if
  - d dominates all loop exits where x is live-out, and
  - There is only one definition of x in the loop, and
  - x is not live-out of the loop preheader
- Need to modify this if a1 op a2 could have side effects or raise an exception

Hoisting: Possible?

- Example 1

```
  L0:t := 0
  L1: i := i + 1
d:  t := a op b
    M[i] := t
    if i < n goto L1
  L2:x := t
```

ok

do-while

- Example 2

```
  L0:t := 0
  L1:if i ≥ n goto L2
     i := i + 1
d:   t := a op b
     M[i] := t
     goto L1
  L2:x := t
```

while

Hoisting: Possible?

- Example 3

```
L0:t := 0
L1: i := i + 1
d:  t := a op b
    M[i] := t
    t := 0
    M[j] := t
    if i < n goto L1
L2:x := t
```

- Example 4

```
L0:t := 0
L1:M[j] := t
   i := i + 1
d: t := a op b
   M[i] := t
   if i < n goto L1
L2:x := t
```

Induction Variables

for(i=0; i<N; i++)
TOP: {
a[i] = 0
j = 4*i
m[A+j] = 0
i++
if(i<N) goto TOP

- Suppose inside a loop
  - Variable i is incremented or decremented
  - Variable j is set to i*c+d where c and d are loop-invariant
- Then we can calculate j's value without using i
  - Whenever i is incremented by a, increment j by c*a

Example

- Original

```
      s := 0
      i := 0
L1:   if i ≥ n goto L2
      j := i*4
      k := j+a
      x := M[k]
      s := s+x
      i := i+1
      goto L1
L2:
```

- To optimize, do…
  - Induction-variable analysis to discover i and j are related induction variables
  - Strength reduction to replace *4 with an addition
  - Induction-variable elimination to replace i ≥ n
  - Assorted copy propagation

Result

- Original

```
      s := 0
      i := 0
L1:   if i ≥ n goto L2
      j := i*4
      k := j+a
      x := M[k]
      s := s+x
      i := i+1
      goto L1
L2:
```

- Transformed

```
      s := 0
      k′ = a
      b = n*4
      c = a+b
L1:   if k′ ≥ c goto L2
      x := M[k′]
      s := s+x
      k′ := k′+4
      goto L1
L2:
```

Details are somewhat messy – see your favorite compiler book

Basic and Derived Induction Variables

- Variable i is a *basic induction variable* in loop L with header h if the only definitions of i in L have the form i:=i±c where c is loop invariant
- Variable k is a *derived induction variable* in L if:
  - There is only one definition of k in L of the form k:=j*c or k:=j+d where j is an induction variable and c, d are loop-invariant, *and*
  - if j is a derived variable in the family of i, then:
    - The only definition of j that reaches k is the one in the loop, *and*
    - there is no definition of i on any path between the definition of j and the definition of k

Optimizating Induction Variables

- Strength reduction: if a derived induction variable is defined with j:=i*c, try to replace it with an addition inside the loop
- Elimination: after strength reduction some induction variables are not used or are only compared to loop-invariant variables; delete them
- Rewrite comparisons: If a variable is used only in comparisons against loop-invariant variables and in its own definition, modify the comparison to use a related induction variable

Loop Unrolling

- If the body of a loop is small, most of the time is spent in the "increment and test" code
- Idea: reduce overhead by *unrolling* – put two or more copies of the loop body inside the loop

Loop Unrolling

- Basic idea: Given loop L with header node h and back edges s_i->h
  1. Copy the nodes to make loop L′ with header h′ and back edges s_i′->h′
  2. Change all backedges in L from s_i->h to s_i->h′
  3. Change all back edges in L′ from s_i′->h′ to s_i′->h

Unrolling Algorithm Results

- Before

```
L1:x := M[i]
     s := s + x
     i := i + 4
   if i<n goto L1 else L2
L2:
```

- After

```
L1: x := M[i]
     s := s + x
     i := i + 4
     if i<n goto L1' else L2
L1':x := M[i]
     s := s + x
     i := i + 4
   if i<n goto L1 else L2
L2:
```

Hmmmm….

- Not so great – just code bloat
- But: use induction variables and various loop transformations to clean up

After Some Optimizations

- Before

```
L1: x := M[i]
    s := s + x
    i := i + 4
    if i<n goto L1′ else L2
L1′:x := M[i]
    s := s + x
    i := i + 4
  if i<n goto L1 else L2
L2:
```

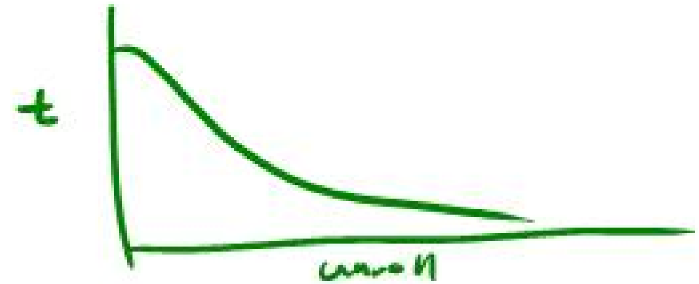
- After

```
L1: x := M[i]
    s := s + x
    x := M[i+4]
    s := s + x
    i := i + 8
    if i<n goto L1 else L2
L2:
```

Still Broken…

- But in a different, better(?) way
- Good code, but only correct if original number of loop iterations was even
- Fix: add an epilogue to handle the "odd" leftover iteration

Fixed



- Before

```
L1: x := M[i]
      s := s + x
      x := M[i+4]
      s := s + x
      i := i + 8
    if i<n goto L1 else L2
L2:
```

- After

```
    if i<n-8 goto L1 else L2
L1: x := M[i]
    s := s + x
    x := M[i+4]
    s := s + x
    i := i + 8
    if i<n-8 goto L1 else L2
L2: x := M[i]
    s := s+x
    i := i+4
    if i < n goto L2 else L3
L3:
```

Postscript

- This example only unrolls the loop by a factor of 2
- More typically, unroll by a factor of K
  - Then need an epilogue that is a loop like the original that iterates up to K-1 times

Memory Heirarchies

- One of the great triumphs of computer design
- Effect is a large, fast memory
- Reality is a series of progressively larger, slower, cheaper stores, with frequently accessed data automatically staged to faster storage (cache, main storage, disk)
- Programmer/compiler typically treats it as one large store. Bug or feature?