CSE P 501 – Compilers

Register Allocation

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Winter 2016

Agenda

- Register allocation constraints
- Local methods
  - Faster compile, slower code, but good enough for lots of things (JITs, …)
- Global allocation – register coloring

k

- Intermediate code typically assumes infinite number of registers
- Real machine has k registers available
- Goals
  - Produce correct code that uses k or fewer registers
  - Minimize added loads and stores
  - Minimize space needed for spilled values
  - Do this efficiently – O(n), O(n log n), maybe $O(n^2)$

Register Allocation

- Task
  - At each point in the code, pick the values to keep in registers
  - Insert code to move values between registers and memory
    - No additional transformations – scheduling should have done its job
      - But we will usually rerun scheduling if we insert spill code
  - Minimize inserted code, both dynamically and statically

Allocation vs Assignment

- Allocation: deciding which values to keep in registers
- Assignment: choosing specific registers for values
- Compiler must do both

Local Register Allocation

- Apply to basic blocks
- Produces decent register usage inside a block
  - But can have inefficiencies at boundaries between blocks
- Two variations: top-down, bottom-up

Top-down Local Allocation

- Principle: keep most heavily used values in registers
  - Priority = # of times register referenced in block
- If more virtual registers than physical,
  - Reserve some registers for values allocated to memory
    - Need enough to address and load two operands and store result
  - Other registers dedicated to "hot" values
    - But are tied up for entire block with particular value, even if only needed for part of the block

Bottom-up Local Allocation (1)

- Keep a list of available registers (initially all registers at beginning of block)
- Scan the code
- Allocate a register when one is needed
- Free register as soon as possible
  - In x:=y op z, free y and z if they are no longer needed before allocating x

Bottom-up Local Allocation (2)

- If no registers are free when one is needed for allocation:
  - Look at values assigned to registers – find the one not needed for longest forward stretch in the code
  - Insert code to spill the value to memory and insert code to reload it when needed later
    - If a copy already exists in memory, no need to spill

Local "bottom-up" Register Allocation, -1

```
1.  ; load v2 from memory
2.  ; load v3 from memory
3.  v1  = v2 + v3
4.  ; load v5, v6 from memory
5.  v4  = v5 - v6
6.  v7  = v2 - 29
7.  ; load v9 from memory
8.  v8  = - v9
9.  v10 = v6 * v4
10. v11 = v10 - v3
```

- Still in LIR. So lots (too many!) virtual registers required (v2, etc).
- Grey instructions (1,2,4,7) load operands from memory into virtual registers.
- We will ignore these going forward. Focus on mapping virtual to physical.

Local "bottom-up" Register Allocation, 0

1. v1 = v2 + v3
2. v4 = v5 - v6
3. v7 = v2 - 29
4. v8 = - v9
5. v10 = v6 * v4
6. v11 = v10 - v3

pReg	vReg
R1	-
R2	-
R3	-
R4	-

vReg	NextRef
v1	1
v2	1
v3	1
v4	2
v5	2
v6	2
v7	3
v8	4
v9	4
v10	5
v11	6

Local "bottom-up" Register Allocation, 1

1. v1 = v2 + v3
2. v4 = v5 - v6
3. v7 = v2 - 29
4. v8 = - v9
5. v10 = v6 * v4
6. v11 = v10 - v3

pReg	vReg
R1	v2
R2	v3
R3	v1
R4	-

R3 = R1 + R2

vReg	NextRef
v1	~~1~~ ∞
v2	~~1~~ 3
v3	~~1~~ 6
v4	2
v5	2
v6	2
v7	3
v8	4
v9	4
v10	5
v11	6

Local "bottom-up" Register Allocation, 2

1. v1 = v2 + v3
2. v4 = v5 - v6
3. v7 = v2 - 29
4. v8 = - v9
5. v10 = v6 * v4
6. v11 = v10 - v3

pReg	vReg
R1	v2
R2	~~v3~~ v4
R3	~~v1~~ v6
R4	v5

vReg	NextRef
v1	∞
v2	3
v3	6
v4	~~2~~ 5
v5	~~2~~ ∞
v6	~~2~~ 5
v7	3
v8	4
v9	4
v10	5
v11	6

```
R3 = R1 + R2
; spill R3
; spill R2? - no - still clean
R2 = R4 - R3
```

Local "bottom-up" Register Allocation, 3

1. v1 = v2 + v3
2. v4 = v5 - v6
3. v7 = v2 - 29
4. v8 = - v9
5. v10 = v6 * v4
6. v11 = v10 - v3

pReg	vReg
R1	v2
R2	v4
R3	v6
R4	~~v5~~ v7

vReg	NextRef
v1	∞
v2	~~3~~ ∞
v3	6
v4	5
v5	∞
v6	5
v7	~~3~~ ∞
v8	4
v9	4
v10	5
v11	6

```
R3 = R1 + R2
; spill R3
; spill R2? - no!
R2 = R4 - R3
; spill R4? - no!
R4 = R1 - 29
```

And so on . . .

Bottom-Up Allocator

- Invented about once per decade
  - Sheldon Best, 1955, for Fortran I
  - Laslo Belady, 1965, for analyzing paging algorithms
  - William Harrison, 1975, ECS compiler work
  - Chris Fraser, 1989, LCC compiler
  - Vincenzo Liberatore, 1997, Rutgers
- Will be reinvented again, no doubt
- Many arguments for optimality of this

Global Register Allocation by Graph Coloring

- How to convert the infinite sequence of temporary data references, t1, t2, … into assignments to finite number of actual registers
- Goal: Use available registers with minimum spilling
- Problem: Minimizing the number of registers is NP-complete … it is equivalent to chromatic number – minimum colors needed to color nodes of a graph so no edge connects same color

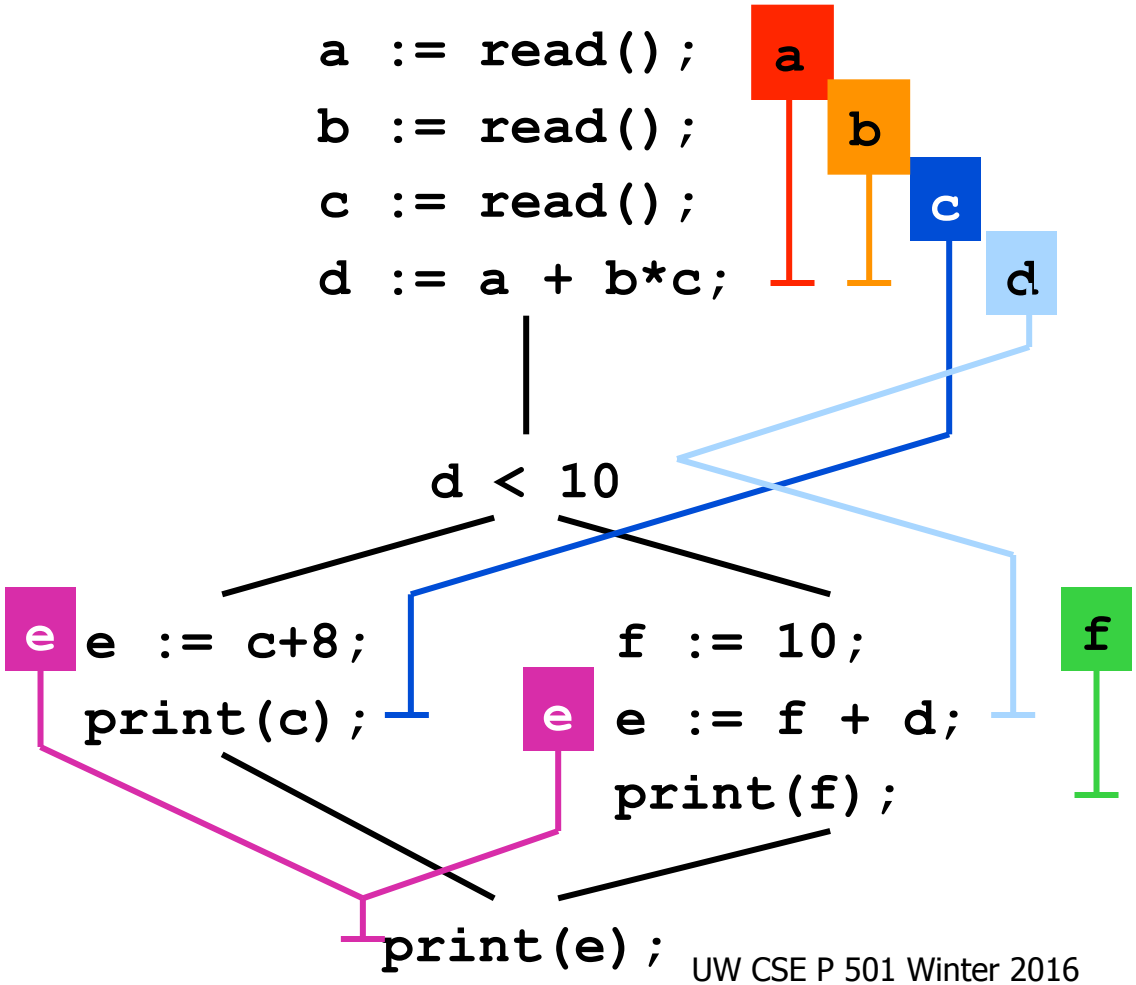
Begin With Data Flow Graph

- procedure-wide register allocation
- only live variables require register storage

dataflow analysis: a variable is live at node N if *the value* it holds is used on some path further down the control-flow graph; otherwise it is dead

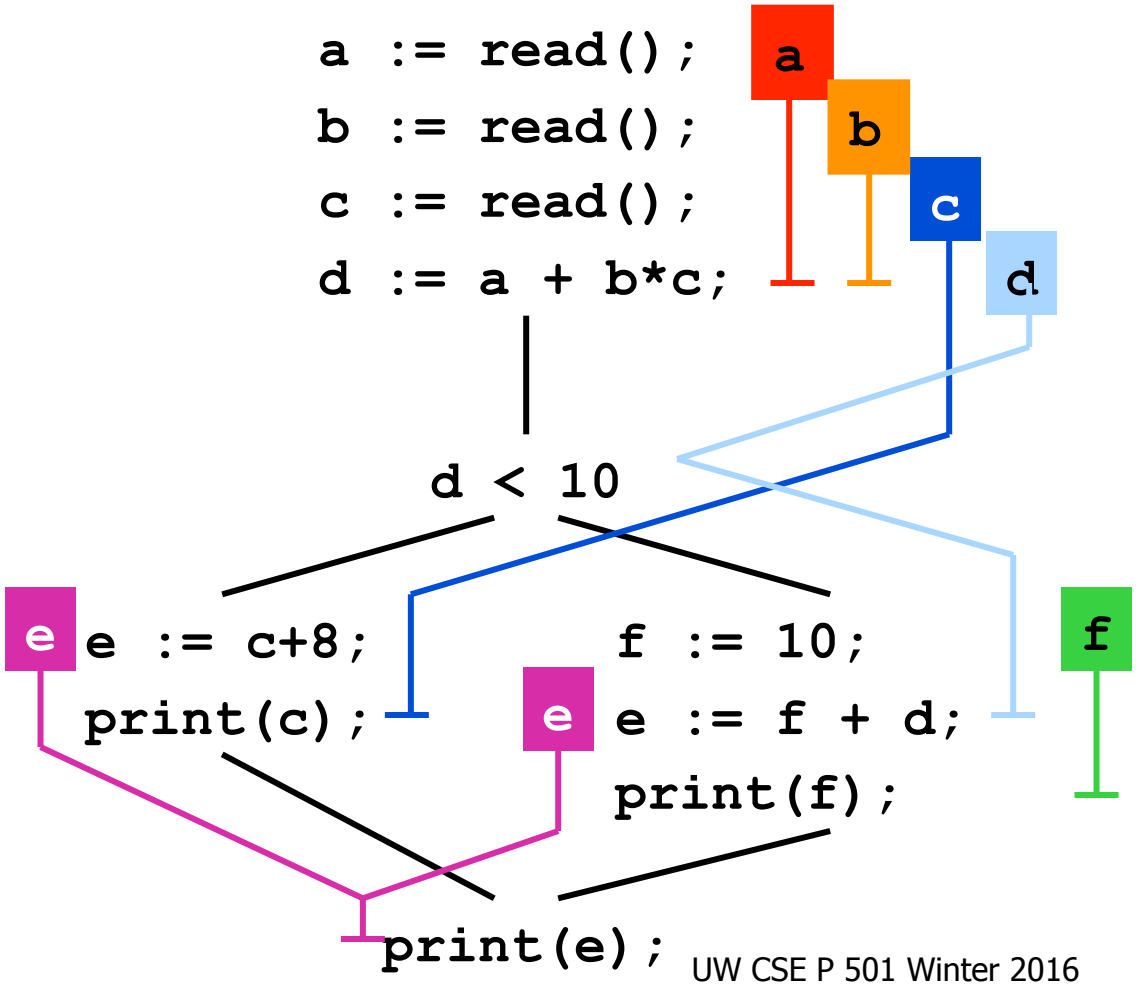
- two variables(values) interfere when their live ranges overlap

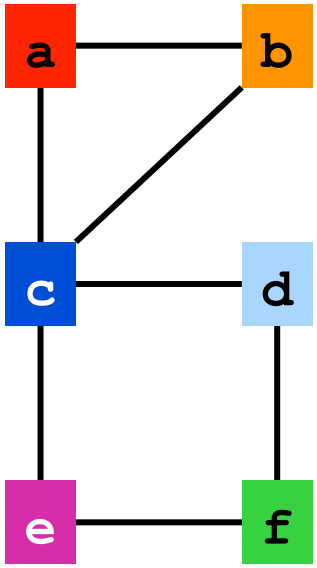
Live Variable Analysis




```
a := read();
b := read();
c := read();
d := a + b*c;
if (d < 10 ) then
    e := c+8;
    print(c);
else
    f := 10;
    e := f + d;
    print(f);
fi
print(e);
```

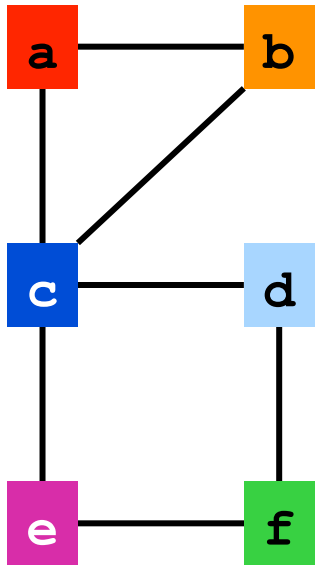
Register Interference Graph





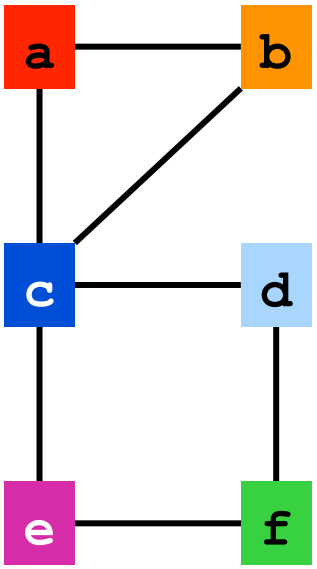
Graph Coloring

- NP complete problem

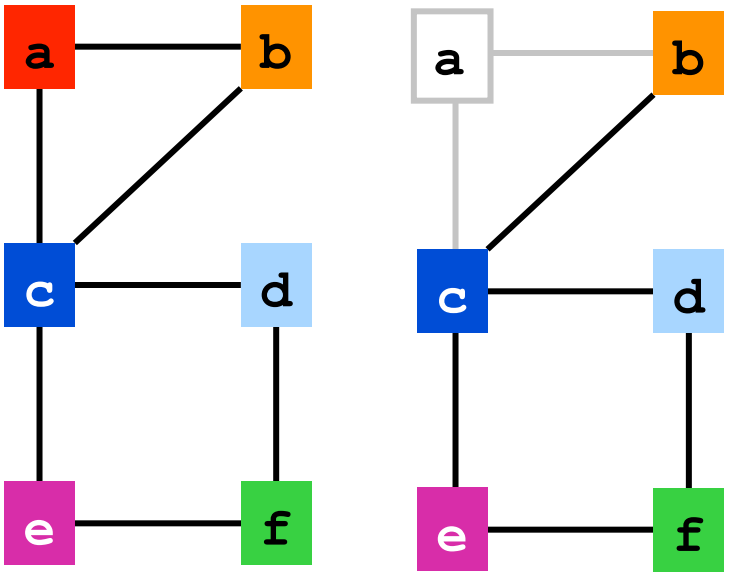


- Heuristic: color easy nodes last
  - find node N with lowest degree
  - remove N from the graph
  - color the simplified graph
  - set color of N to the first color that is not used by any of N 's neighbors
- Basics due to Chaitin (1982), refined by Briggs (1992)

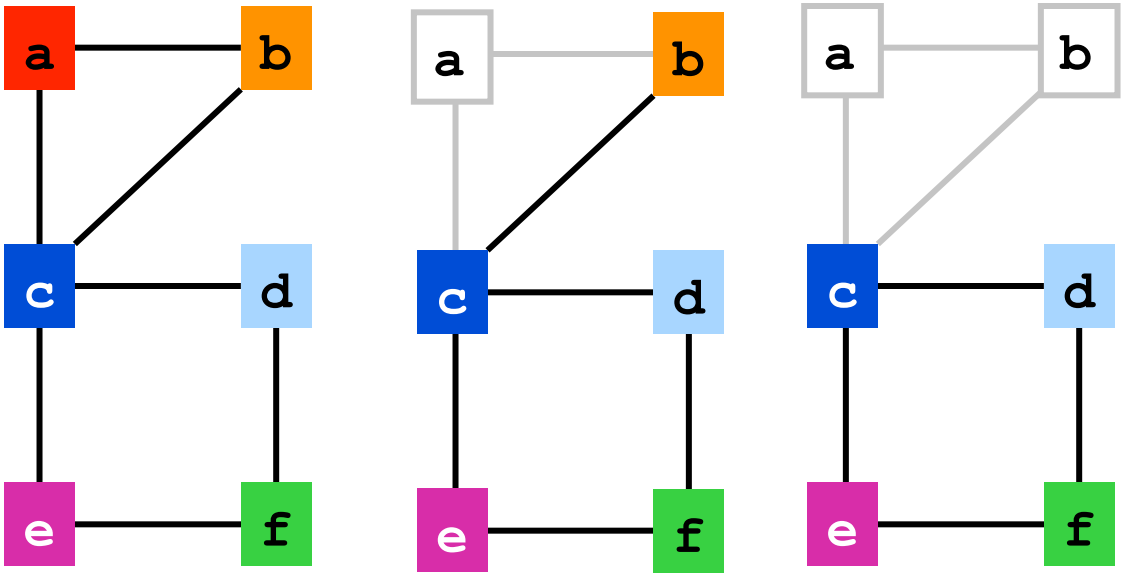
Apply Heuristic



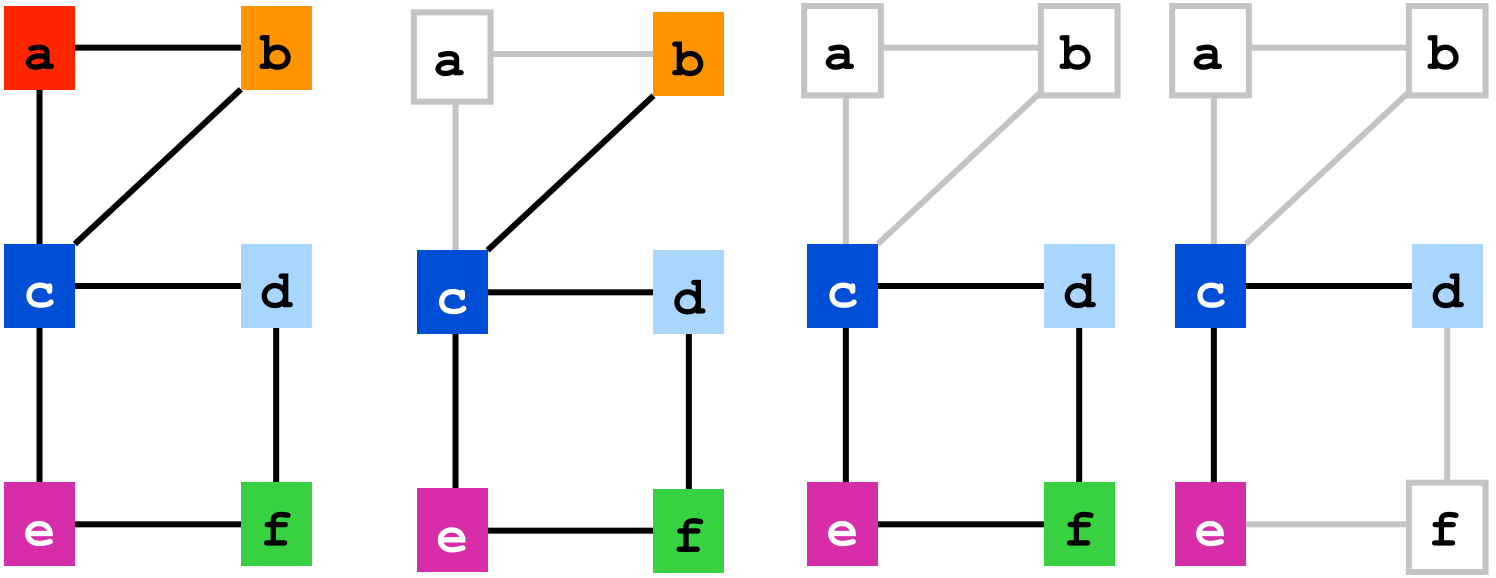
Apply Heuristic



Apply Heuristic



Apply Heuristic



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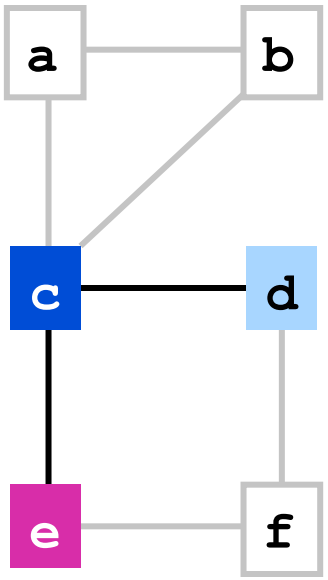

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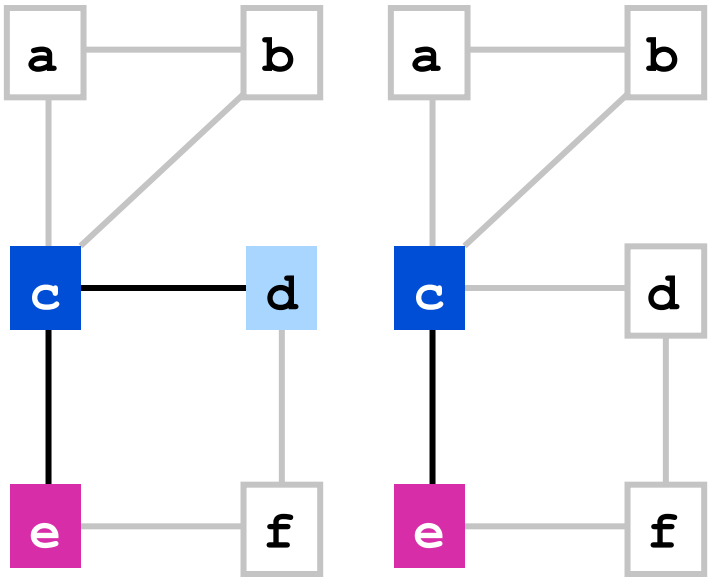

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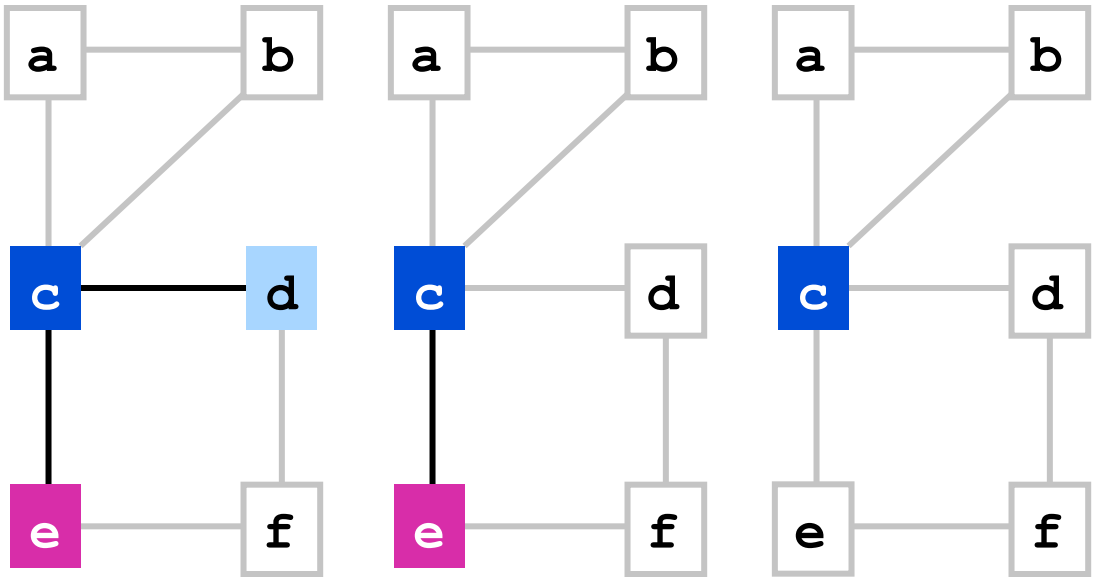

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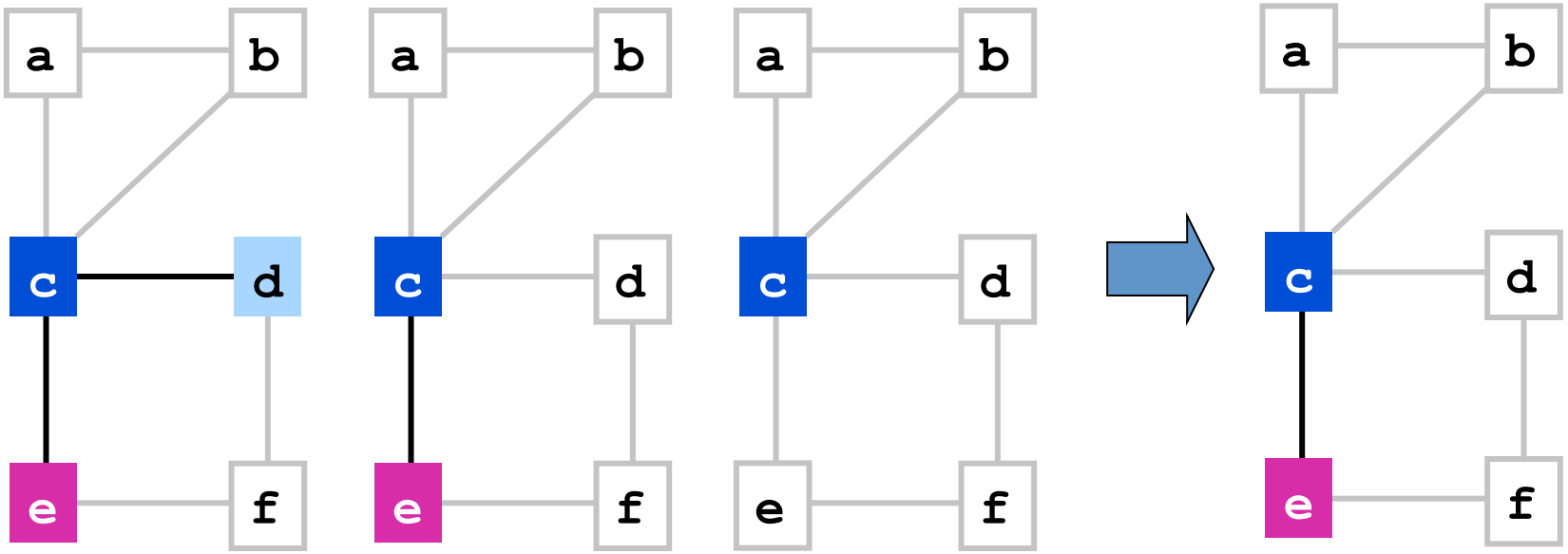

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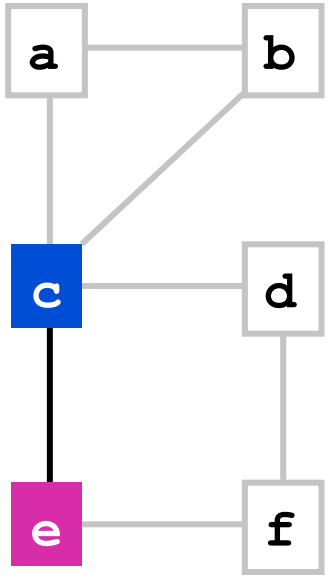

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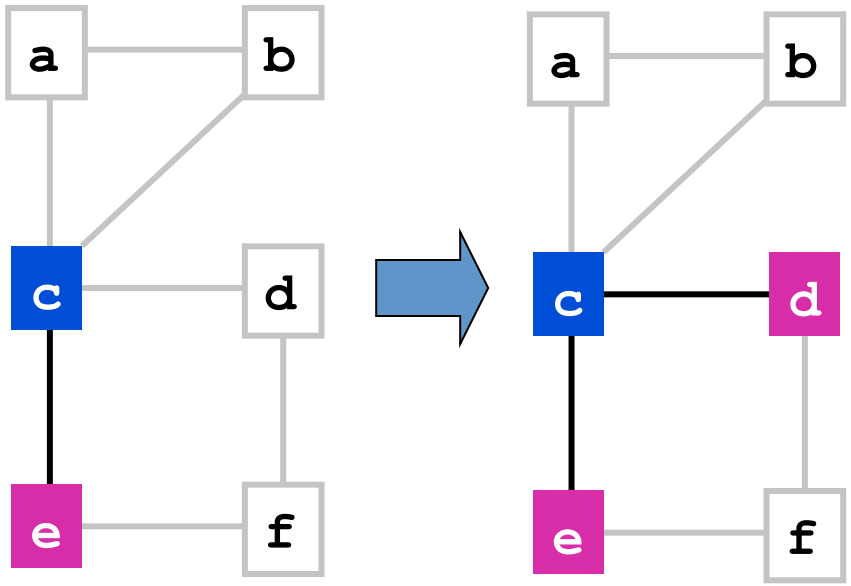

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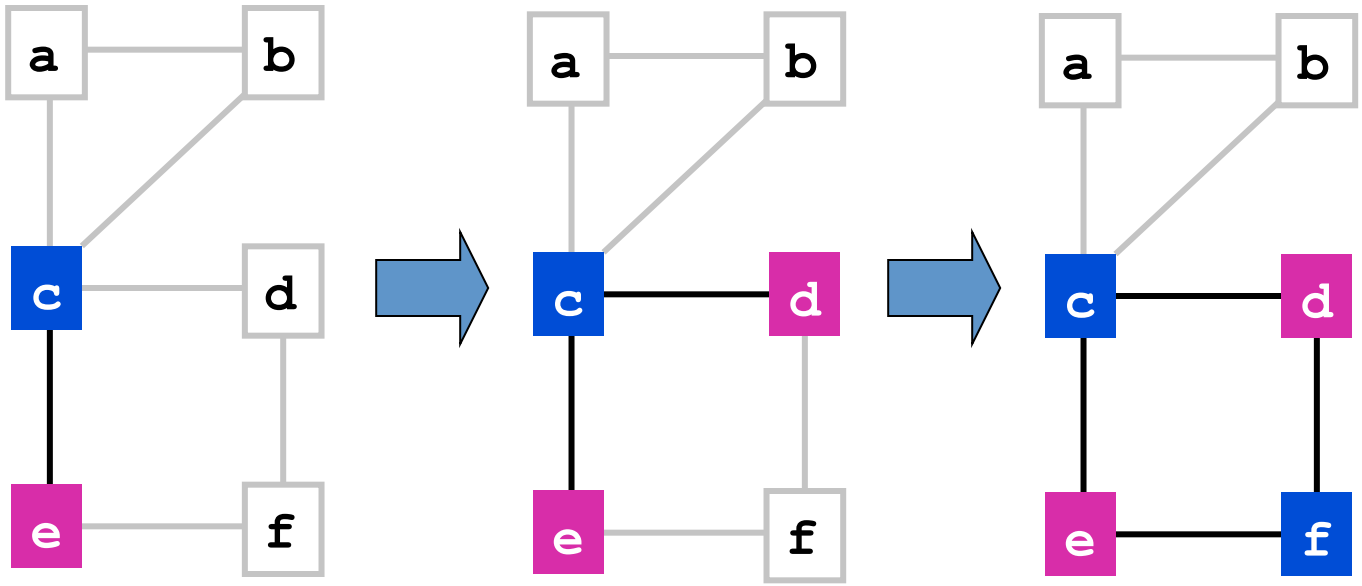

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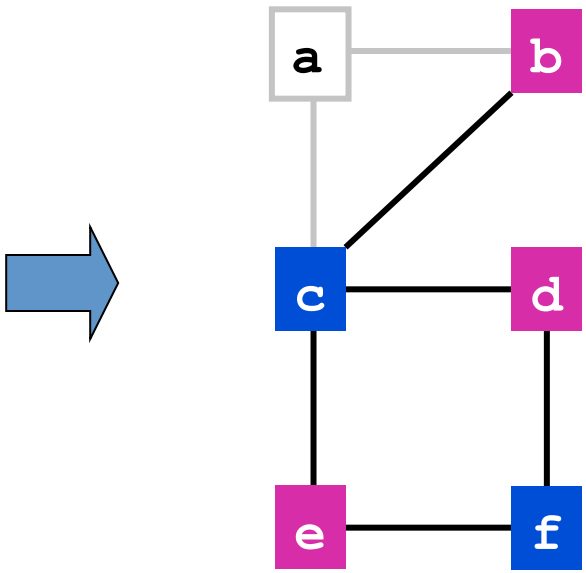

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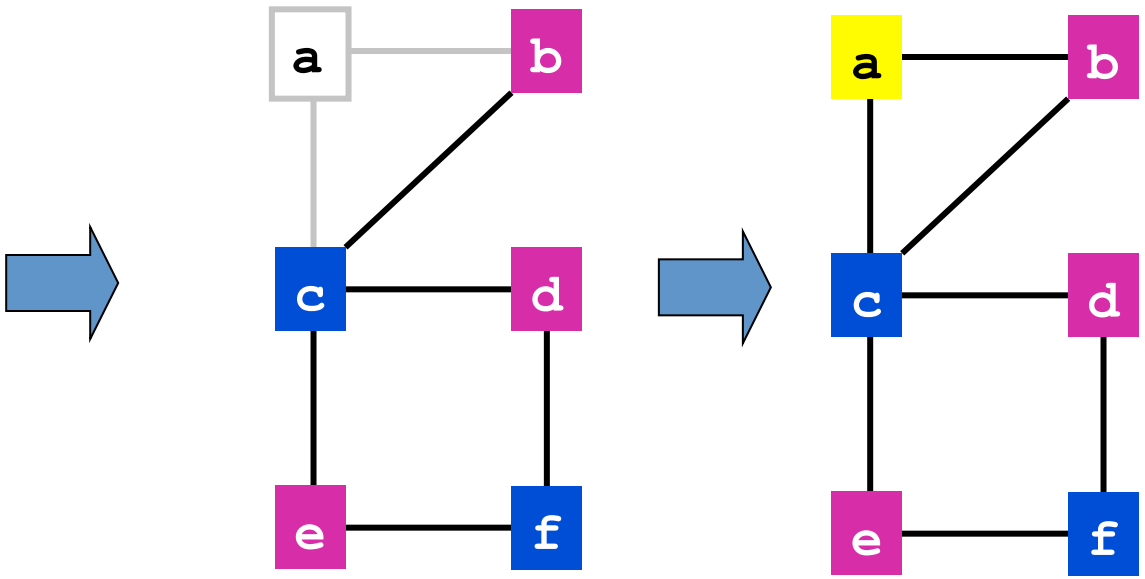
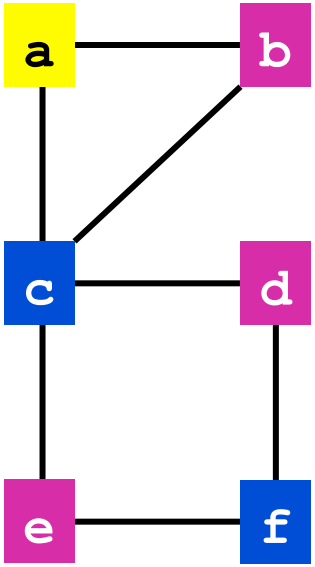


Final Assignment

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b := read();
c := read();
d := a + b*c;
if (d < 10 ) then
    e := c+8;
    print(c);
else
    f := 10;
    e := f + d;
    print(f);
fi
print(e);
```

Some Graph Coloring Issues

- May run out of registers
  - Solution: insert spill code and reallocate
- Special-purpose and dedicated registers
  - Examples: function return register, function argument registers, registers required for particular instructions
  - Solution: "pre-color" some nodes to force allocation to a particular register

Global Register Allocation (for real)

- Graph coloring is the standard technique, but
- Nodes are *live ranges* not variables
- Use control and dataflow (actually SSA) graphs to derive interference graph
  - Edge between (t1,t2) when live ranges t1 and t2 cannot be assigned to the same register
    - Most commonly, t1 and t2 are both live at the same time
    - Can also use to express constraints about registers, etc.
- Then color the nodes in the graph
  - Two nodes connected by an edge may not have same color (i.e., cannot allocate to same register)
  - If more than k colors are needed, insert spill code

Live Ranges (1)

- A live range is the set of definitions and uses that are related because they flow together
  - Every definition can reach every use
  - Every use that a definition can reach is in the same live range

Live Ranges (2)

- The idea relies on the notion of *liveness*, but not the same as either the set of variables or set of values
  - Every value is part of some live range, even anonymous temporaries
  - Same name may be part of several different live ranges

Live Ranges: Example

1. loadi … → rfp
2. loadai rfp, 0 → rw
3. loadi 2 → r2
4. loadai rfp,xoffset → rx
5. loadai rfp,yoffset → ry
6. loadai rfp,zoffset → rz
7. mult rw, r2 → rw
8. mult rw, rx → rw
9. mult rw, ry → rw
10. mult rw, rz → rw
11. storeai rw → rfp, 0

Register	Interval
rfp	[1,11]
rw	[2,7]
rw	[7,8]
rw	[8,9]
rw	[9,10]
rw	[10,11]
r2	[3,7]
rx	[4,8]
ry	[5,9]
rz	[6,10]

Coloring by Simplification

- Linear-time approximation that generally gives good results
  1. Build: Construct the interference graph
  2. Simplify: Color the graph by repeatedly simplification
  3. Spill: If simplify cannot reduce the graph completely, mark some node for spilling
  4. Select: Assign colors to nodes in the graph

1. Build

- Construct the interference graph
- Find live ranges – SSA!
  - Build SSA form of IR
  - Each SSA name is initially a singleton set
  - A Φ-function means form the union of the sets that includes those names (union-find algorithm)
  - Resulting sets represent live ranges
  - Either rewrite code to use live range names or keep a mapping between SSA names and live-range names

1. Build

- Use dataflow information to build interference graph
  - Nodes = live ranges
  - Add an edge in the graph for each pair of live ranges that overlap
    - But watch copy operations. MOV ri → rj does not create interference between ri, rj since they can be the same register if the ranges do not otherwise interfere

2. Simplify

- Heuristic: Assume we have K registers
- Find a node *m* with fewer than K neighbors
- Remove *m* from the graph. If the resulting graph can be colored, then so can the original graph (the neighbors of m have at most K-1 colors among them)
- Repeat by removing and pushing on a stack all nodes with degree less than K
  - Each simplification decreases other node degrees – may make more simplifications possible

3. Spill

- If simplify stops because all nodes have degree ≥ k, mark some node for spilling
  - This node is in memory during execution
  - ∴ Spilled node no longer interferes with remaining nodes, reducing their degree.
  - Continue by removing spilled node and push on the stack (optimistic – hope that spilled node does not interfere with remaining nodes – Briggs allocator)

3. Spill

- Spill decisions should be based on costs of spilling different values
- Issues
  - Address computation needed for spill
  - Cost of memory operation
  - Estimated execution frequency

    (e.g., inner loops first)

4. Select

- Assign nodes to colors in the graph:
  - Start with empty graph
  - Rebuild original graph by repeatedly adding node from top of the stack
    - (When we do this, there must be a color for it if it didn't represent a potential spill – pick a different color from any adjacent node)
  - When a potential spill node is popped it may not be colorable (neighbors may have k colors already). This is an actual spill.

5. Start Over

- If Select phase cannot color some node (must be a potential spill node), add loads before each use and stores after each definition
  - Creates new temporaries with tiny live ranges
- Repeat from beginning
  - Iterate until Simplify succeeds
  - In practice a couple of iterations are enough

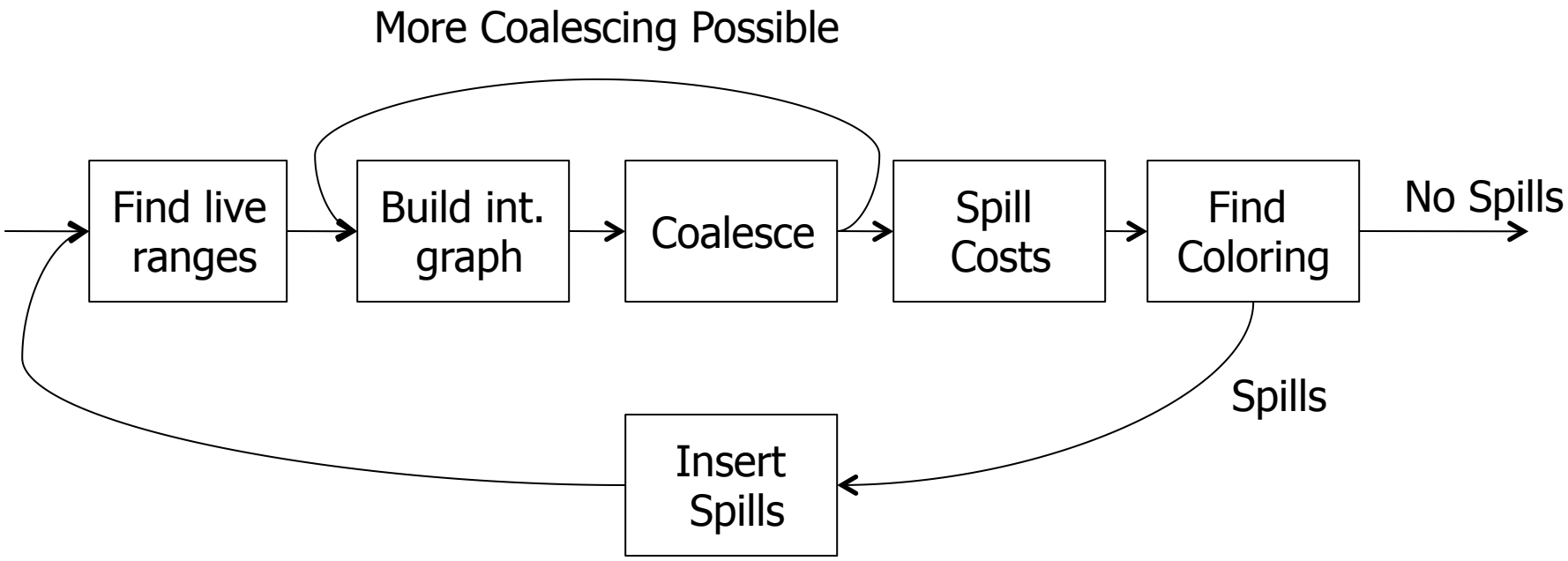
Coalescing Live Ranges

- Idea: if two live ranges are connected by a copy operation (MOV ri → rj) but do not otherwise interfere, then the live ranges can be coalesced (combined)
  - Rewrite all references to rj to use ri
  - Remove the copy instruction
- Then need to fix up interference graph

Advantages?

- Makes the code smaller, faster (no copy operation)
- Shrinks set of live ranges
- Reduces the degree of any live range that interfered with both live ranges ri, rj
- But: coalescing two live ranges can prevent coalescing of others, so ordering matters
  - Best: Coalesce most frequently executed ranges first (e.g., inner loops)
- Can have a substantial payoff – do it!

Overall Structure



Complications

- Need to deal with irregularities in the register set
  - Some operations require dedicated registers (idiv in x86, split address/data registers in M68k and others), register overlap (AH, AL, AX, EAX, RAX) in x86 and x86-64
  - Register conventions like function results, use of registers across calls, etc.
- Model by precoloring nodes, adding constraints in the graph, etc.

Graph Representation

- The interference graph representation drives the time and space requirements for the allocator (and maybe the compiler)
- Not unknown to have O(5K) nodes and O(1M) edges
- Dual representation works best
  - Triangular bit matrix for efficient access to interference information
  - Vector of adjacency vectors for efficient access to node neighbors

And That's It

- Modulo all the picky details, that is…