

# CSE P 501 – Compilers

LR Parsing  
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# Agenda

- LR Parsing
- Table-driven Parsers
- Parser States
- Shift-Reduce and Reduce-Reduce conflicts

## Bottom-Up Parsing



- Idea: Read the input left to right
- Whenever we've matched the right hand side of a production, reduce it to the appropriate non-terminal and add that non-terminal to the parse tree
- The upper edge of this partial parse tree is known as the *frontier*

# Example

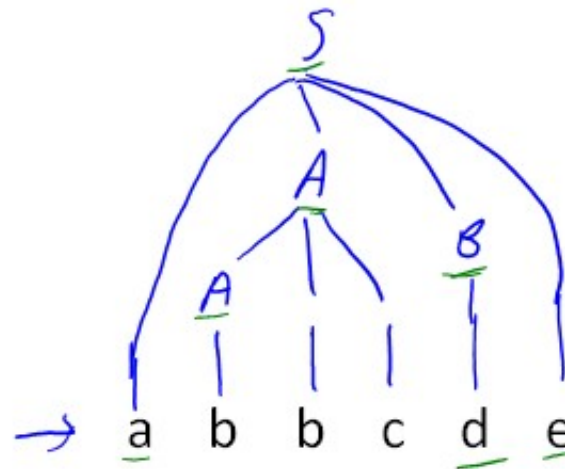
- Grammar

$S ::= aAB e$

$A ::= A\underline{bc} \mid \underline{b}$

$B ::= \underline{d}$

- Bottom-up Parse



# LR(1) Parsing

- We'll look at LR(1) parsers
  - Left to right scan, Rightmost derivation, 1 symbol lookahead
  - Almost all practical programming languages have a LR(1) grammar
  - LALR(1), SLR(1), etc. – subsets of LR(1)
    - LALR(1) can parse most real languages, tables are more compact, and is used by YACC/Bison/CUP/etc.

## LR Parsing in Greek

- The bottom-up parser reconstructs a reverse rightmost derivation
- Given the rightmost derivation

$$\underline{S} \Rightarrow \beta_1 \Rightarrow \beta_2 \Rightarrow \dots \Rightarrow \beta_{n-2} \Rightarrow \beta_{n-1} \Rightarrow \underline{\beta_n = w}$$

the parser will first discover  $\beta_{n-1} \Rightarrow \beta_n$ , then  $\beta_{n-2} \Rightarrow \beta_{n-1}$ , etc.

- Parsing terminates when
  - $\beta_1$  reduced to  $S$  (start symbol, success), or
  - No match can be found (syntax error)

## How Do We Parse with This?



- Key: given what we've already seen and the next input symbol (the lookahead), decide what to do.
- Choices:
  - Perform a reduction
  - Look ahead further
- Can reduce  $A \Rightarrow \beta$  if both of these hold:
  - $A \Rightarrow \beta$  is a valid production, *and*
  - $A \Rightarrow \beta$  is a step in *this* rightmost derivation
- This is known as a *shift-reduce* parser

## Sentential Forms

- If  $S \Rightarrow^* \underline{\alpha}$ , the string  $\alpha$  is called a *sentential form* of the grammar
- In the derivation  
 $\underline{S} \Rightarrow \beta_1 \Rightarrow \beta_2 \Rightarrow \dots \Rightarrow \beta_{n-2} \Rightarrow \beta_{n-1} \Rightarrow \beta_n = \underline{w}$   
each of the  $\beta_i$  are sentential forms
- A sentential form in a rightmost derivation is called a right-sentential form (similarly for leftmost and left-sentential)



## Handles

- Informally, a production whose right hand side matches a substring of the tree frontier *that is part of the rightmost derivation of the current input string* (i.e., the “correct” production)
  - Even if  $A ::= \beta$  is a production, it is a handle only if  $\beta$  matches the frontier at a point where  $A ::= \beta$  was used in *this specific* derivation
  - $\beta$  may appear in many other places in the frontier without designating a handle
- Bottom-up parsing is all about finding handles

## Handle Examples

- In the derivation

$\rightarrow S \Rightarrow aABe \Rightarrow aAde \Rightarrow aAbcde \Rightarrow \underline{abbcde}$

–  $a\underline{b}bcde$  is a right sentential form whose handle is  $\underline{A::=b}$  at position 2

–  $a\underline{Abc}de$  is a right sentential form whose handle is  $\underline{A::=Abc}$  at position 4

- Note: some books take the left end of the match as the position

## Handles – The Dragon Book Defn.

- Formally, a *handle* of a right-sentential form  $\gamma$  is a production  $A ::= \beta$  and a position in  $\gamma$  where  $\beta$  may be replaced by  $A$  to produce the previous right-sentential form in the rightmost derivation of  $\gamma$

## Implementing Shift-Reduce Parsers

- Key Data structures
  - A stack holding the frontier of the tree
  - A string with the remaining input (tokens)
- We also need something to encode the rules that tell us what action to take next, given the state of the stack and the lookahead symbol
  - Typically a table that encodes a finite automata

## Shift-Reduce Parser Operations

- *Reduce* – if the top of the stack is the right side of a handle  $A ::= \beta$ , pop the right side  $\beta$  and push the left side  $A$
- *Shift* – push the next input symbol onto the stack
- *Accept* – announce success
- *Error* – syntax error discovered

# Shift-Reduce Example

$S ::= aABe$   
 $A ::= \underline{Abc} \mid b$   
 $B ::= d$

Stack	Input	Action
\$	abbcde\$	shift
\$a	bbcde\$	shift
\$ab	bcd e\$	reduce
\$aA	bcd e\$	shift
\$aAb	cde\$	shift
\$aAbc	de\$	reduce
\$aA	de\$	shift
\$aAd	e\$	reduce
\$aAB	e\$	shift
\$aABd	\$	reduce
\$S	\$	acc

## How Do We Automate This?

- Cannot use clairvoyance in a real parser (alas...)
- Defn. *Viable prefix* – a prefix of a right-sentential form that can appear on the stack of the shift-reduce parser
  - Equivalent: a prefix of a right-sentential form that does not continue past the rightmost handle of that sentential form
  - In Greek:  $\gamma$  is a *viable prefix* of  $G$  if there is some derivation  $S \Rightarrow_{rm}^* \alpha A w \Rightarrow_{rm}^* \alpha \beta w$  and  $\gamma$  is a prefix of  $\alpha \beta$ .
  - The occurrence of  $\beta$  in  $\alpha \beta w$  is a *handle* of  $\alpha \beta w$



## How Do We Automate This?

- Fact: the set of viable prefixes of a CFG is a regular language(!)
- Idea: Construct a DFA to recognize viable prefixes given the stack and remaining input
  - Perform reductions when we recognize them

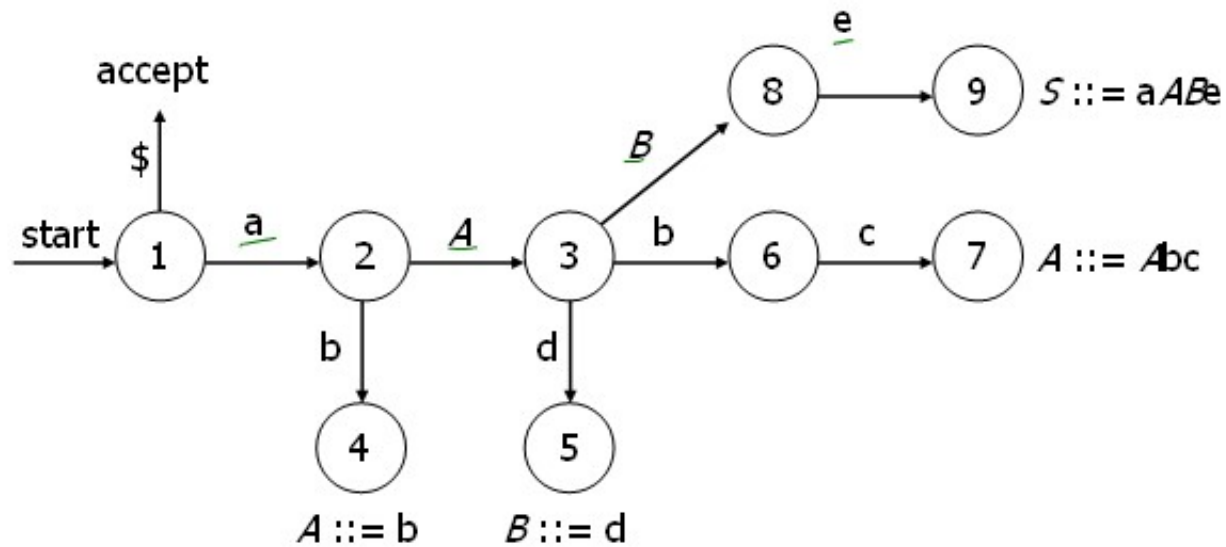


# DFA for prefixes of

$S ::= aABe$

$A ::= Abc \mid b$

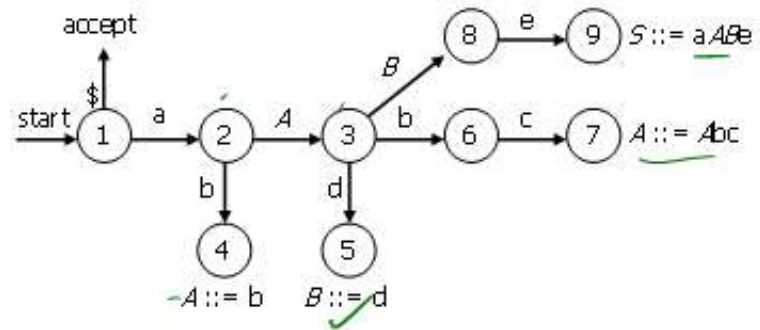
$B ::= d$



# Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input
\$	abcde\$ <i>shift</i>
\$a	bcd e\$ <i>shift</i>
\$ <u>ab</u>	bcd e\$ <i>reduce</i>
\$aA	bcd e\$ <i>shift</i>
\$aAb	cde\$ <i>shift</i>
\$a <u>Abc</u>	cde\$ <i>reduce</i>
\$aA	d e\$ <i>shift</i>
✓ \$aA <u>d</u>	e\$ <i>reduce</i>
\$aAB	e\$ <i>shift</i>
\$a <u>ABe</u>	\$ <i>reduce</i>
\$S	\$ <i>acc</i>



## Observations

- Way too much backtracking
  - We want the parser to run in time proportional to the length of the input
- Where the heck did this DFA come from anyway?
  - From the underlying grammar
  - Defer construction details for now

## Avoiding DFA Rescanning

- Observation: no need to restart DFA after a shift. Stay in the same state and process next token.
- Observation: after a reduction, the contents of the stack are the same as before except for the new non-terminal on top
  - $\therefore$  Scanning the stack will take us through the same transitions as before until the last one
  - $\therefore$  If we record state numbers on the stack, we can go directly to the appropriate state when we pop the right hand side of a production from the stack

## Stack

- Change the stack to contain pairs of states and symbols from the grammar

$\$s_0 X_1 s_1 X_2 s_2 \dots X_n s_n$

- State  $s_0$  represents the accept (start) state  
(Not always explicitly on stack – depends on particular presentation)
  - When we push a symbol on the stack, push the symbol plus the FA state
  - When we reduce, popping the handle will reveal the state of the FA just prior to reading the handle
- Observation: in an actual parser, only the state numbers are needed since they implicitly contain the symbol information. But for explanations / examples it can help to show both.

## Encoding the DFA in a Table



- A shift-reduce parser's DFA can be encoded in two tables
  - One row for each state
  - *action* table encodes what to do given the current state and the next input symbol
  - *goto* table encodes the transitions to take after a reduction

## Actions (1)

- Given the current state and input symbol, the main possible actions are
  - $si$  – shift the input symbol and **state  $i$**  onto the stack (i.e., shift and move to state  $i$ )
  - $rj$  – reduce using grammar **production  $j$** 
    - The production number tells us how many <symbol, state> pairs to pop off the stack (= number of symbols on rhs of production)

## Actions (2)

- Other possible *action* table entries
  - *accept*
  - **blank** – no transition – syntax error
    - A LR parser will detect an error as soon as possible on a left-to-right scan
    - A real compiler needs to produce an error message, recover, and continue parsing when this happens



## Goto

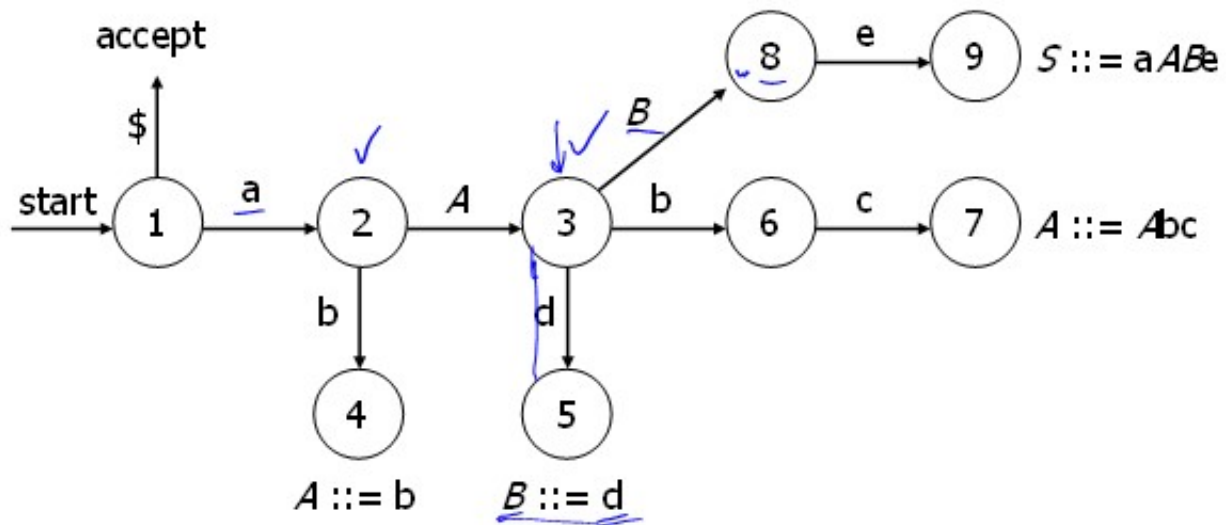
- When a reduction is performed using  $A ::= \underline{\beta}$ , we pop  $|\beta|$   $\langle$ symbol, state $\rangle$  pairs from the stack revealing a state  $uncovered\_s$  on the top of the stack
- $goto[uncovered\_s, A]$  is the new state to push on the stack when reducing production  $A ::= \beta$  (after popping handle  $\beta$  and pushing  $A$ )

# Reminder: DFA for

$S ::= aABe$

$A ::= Abc \mid b$

$B ::= d$



# LR Parse Table for

1.  $S ::= aABe$
2.  $A ::= Abc$
3.  $A ::= b$
4.  $B ::= d$

State	action						goto		
	a	b	c	d	e	\$	A	B	S
0						acc			
1	s2								g0
2		s4					<u>g3</u>		
3		s6		s5				g8	
4	r3	r3	r3	r3	r3	r3			
5	r4	r4	r4	r4	r4	r4			
6			s7						
7	r2	r2	r2	r2	r2	r2			
8					s9				
9	r1	r1	r1	r1	r1	r1			

# LR Parsing Algorithm

```
tok = scanner.getToken();
while (true) {
    s = top of stack;
    if (action[s, tok] = si ) {
        push tok; push i (state);
        tok = scanner.getToken();
    } else if (action[s, tok] = rj ) {
        pop 2 * length of right side of
        production j (2 * |β|);
        uncovered_s = top of stack;
        push left side A of production j ;
        push state goto[uncovered_s, A];
    }
    } else if (action[s, tok] = accept ) {
        return;
    } else {
        // no entry in action table
        report syntax error;
        halt or attempt recovery;
    }
}
```

# Example

1.  $S ::= aABe$
2.  $A ::= Abc$
3.  $A ::= b$
4.  $B ::= d$

Stack

\$0  
 \$0a2  
 \$0a2b4  
 \$0a2A3  
 \$0a2A3b6  
 \$0a2A3b6c7  
 \$0a2A3  
 \$0a2A3d5  
 \$0a2A3B8  
 \$0a2A3B8e9  
 \$050

Input

abbcde\$ r2  
 bbcdes  
 bcde\$  
 bcdef  
 cde\$  
 de\$  
 de\$  
 e\$  
 e\$  
 \$  
 \$

S	action						↓ goto		
	a	b	c	d	e	\$	A	B	S
0	<u>s2</u>					<u>ac</u>			
1	s2								g0
2		s4					<u>g3</u>		
3		s6		s5				<u>g8</u>	
4	r3	r3	r3	r3	r3	r3			
5	r4	r4	r4	r4	r4	r4			
6			s7						
7	r2	r2	r2	r2	r2	r2			
8					s9				
9	r1	r1	r1	r1	r1	r1			

## LR States

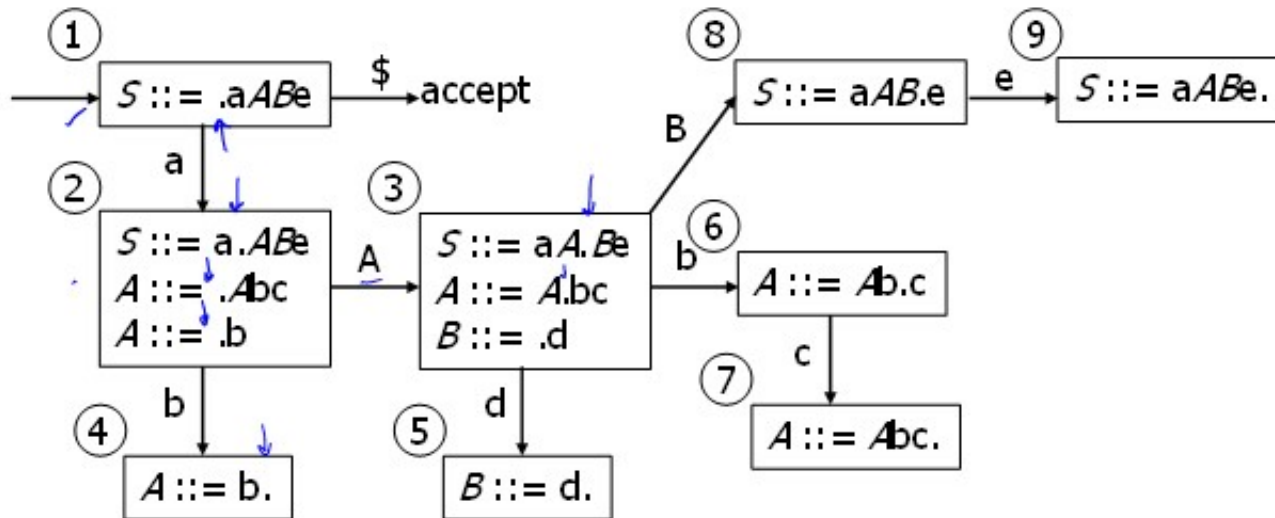
- Idea is that each state encodes
  - The set of all possible productions that we could be looking at, given the current state of the parse, and
  - *Where* we are in the right hand side of each of those productions

## Items

- An *item* is a production with a dot in the right hand side
- Example: Items for production  $A ::= X Y$ 
  - $A ::= . X Y$
  - $A ::= X . Y$
  - $A ::= X Y .$
- Idea: The dot represents a position in the production

# DFA for

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$





## Problems with Grammars

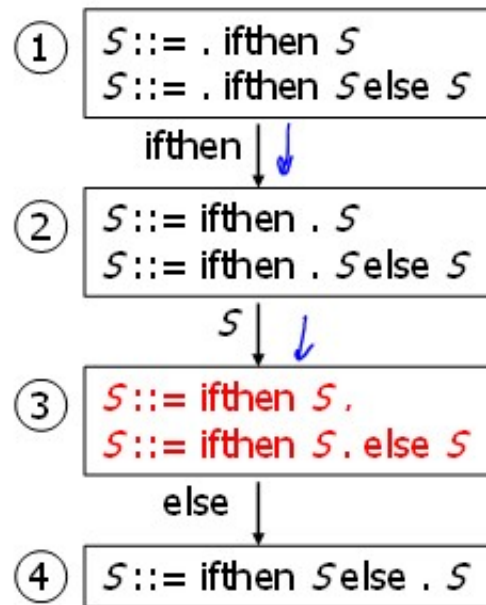
- Grammars can cause problems when constructing a LR parser
  - Shift-reduce conflicts
  - Reduce-reduce conflicts

## Shift-Reduce Conflicts

- Situation: both a shift and a reduce are possible at a given point in the parse (equivalently: in a particular state of the DFA)
- Classic example: if-else statement  
 $S ::= \text{ifthen } S \mid \text{ifthen } S \text{ else } S$

# Parser States for

1.  $S ::= \text{ifthen } S$
2.  $S ::= \text{ifthen } S \text{ else } S$



- State 3 has a shift-reduce conflict
  - Can shift past else into state 4 (s4)
  - Can reduce (r1)  
 $S ::= \text{ifthen } S$

(Note: other  $S ::= . \text{ifthen}$  items not included in states 2-4 to save space)

## Solving Shift-Reduce Conflicts

- Fix the grammar
  - Done in Java reference grammar, others
- Use a parse tool with a “longest match” rule – i.e., if there is a conflict, choose to shift instead of reduce
  - Does exactly what we want for if-else case
  - Guideline: a few shift-reduce conflicts are fine, but be sure they do what you want (and that this behavior is guaranteed by the tool specification)

## Reduce-Reduce Conflicts

- Situation: two different reductions are possible in a given state
- Contrived example

$S ::= A$

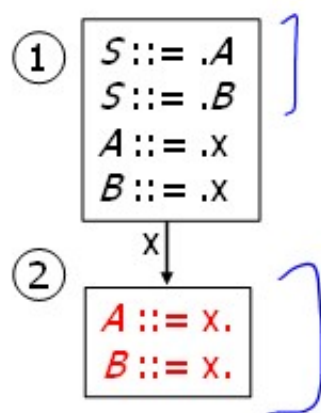
$S ::= B$

$A ::= x$

$B ::= x$

## Parser States for

1.  $S ::= A$
2.  $S ::= B$
3.  $A ::= x$
4.  $B ::= x$



- State 2 has a reduce-reduce conflict (r3, r4)

## Handling Reduce-Reduce Conflicts

- These normally indicate a serious problem with the grammar.
- Fixes
  - Use a different kind of parser generator that takes lookahead information into account when constructing the states
    - Most practical tools use this information
  - Fix the grammar

## Another Reduce-Reduce Conflict

- Suppose the grammar tries to separate arithmetic and boolean expressions

$expr ::= aexp \mid bexp$

$aexp ::= aexp * \underline{aident} \mid \underline{aident}$

$bexp ::= bexp \&\& \underline{bident} \mid \underline{bident}$

$aident ::= \underline{id}$

$bident ::= \underline{id}$

- This will create a reduce-reduce conflict after recognizing  $id$



## Covering Grammars

- A solution is to merge *aident* and *bident* into a single non-terminal like *ident* (or just use *id* in place of *aident* and *bident* everywhere they appear)
- This is a *covering grammar*
  - Will generate some programs (sentences) that are not generated by the original grammar
  - Use the type checker or other static semantic analysis to weed out illegal programs later

## Coming Attractions

- Constructing LR tables
  - We'll present a simple version (SLR(0)) in lecture, then talk about adding lookahead and then a little bit about how this relates to LALR(1) used in most parser generators
- LL parsers and recursive descent
- Continue reading ch. 3