## CSEP 521 - Spring 2005 Assignment 2

## Due 3/14/05

- 1. Imagine there is a set of resources and demands where each demand can be satisfied by one of two specific resources. Each resource can satisfy multiple demands. The optimization problem is to find the minimum number of resources that can satisfy all the demands. Such a problem can be modeled by an undirected graph, where the resources are the vertices and the edges are the demands. More specifically, this resource allocation problem is called the *vertex cover optimization problem* defined as follows.
  - Input: G = (V, E), an undirected graph.
  - Output: The smallest number m such that there exist  $V' \subseteq V$  where
    - (a) The cardinality of V' is m.
    - (b) For every edge  $\{x, y\} \in E$  one or both of x and y is a member of V'.
  - (a) Design a reasonable decision version of the vertex cover optimization problem.
  - (b) Design a reasonable reporting version of your decision problem.
  - (c) Show how to reduce reporting to decision. Your reduction should run in polynomial time with the decision problem as an oracle.
  - (d) Show how to reduce optimality to decision. Your reduction should run in polynomial time with the decision problem as an oracle
- 2. Problem 35.5-2 on page 1017 of CLRS. If you have 0-1 valued variables  $x_1, y_1, \ldots, x_n, y_n$ , think of linear constraints on these variables that force at most one of  $x_i$  and  $y_i$  to have the value 1, and force all the clauses in a 3-CNF formula to be satisfied were  $y_i$  represents  $\neg x_i$  in the formula.