

Homework 2, Due Monday, January 21, 2013

*For this assignment, electronic turnin only.* Note that Monday, January 21 is a holiday - but we will stick with the regular deadline.

**Problem 1 (10 points):**

Page 108, Exercise 7.

**Problem 2 (10 points):**

Page 109, Exercise 8.

**Problem 3 (10 points):**

Page 110, Exercise 9.

**Problem 4 (10 points):**

~~Page 189, Exercise 3.~~ This problem will be included on Homework 3.

**Problem 5 (10 points):**

Consider a directed graph on  $n$  vertices, where each vertex has exactly one out going edge. This graph consists of a collection of cycles as well as additional vertices that have paths to the cycles, which we will call the branches. We define the *weight* of the cycle to be the total number of vertices that are either on the cycle or on branches that are connected to the cycle.

Describe a linear time algorithm that identifies all of the cycles and computes the length and weight of each cycle.

**Programming Problem 6 (20 points) :**

(10 points) Implement your algorithm for finding the cycles in an out-degree one graph. Your algorithm should be designed to work on very large graphs, e.g., with  $n = 100,000,000$ .

Write an input generator which creates completely random out-degree one graphs where each vertex points to another vertex chosen uniformly at random.

(10 points) As the size of the problem increases - how does the number of cycles, and the length, and weight of the cycles change, when the input is a random graph with out-degree one? Submit a write up showing how these quantities relate to the number of vertices. Can you come up with a conjecture on how any of these quantities vary as a function of  $n$ ?

You are free to write in any programming language you like. The quality of your algorithm may be graded (but you can use the one in the book), but the actual quality of the code will not be graded. The expectation is that you will write the algorithmic code yourself - but you can use other code or libraries for supporting operations.