CSEP 521
Applied Algorithms

Richard Anderson

Winter 2013

Lecture 1

CSEP 521 Course Introduction

- CSEP 521, Applied Algorithms
  - Monday's, 6:30-9:20 pm
  - CSE 305 and Microsoft Building 99
- Instructor
  - Richard Anderson, anderson@cs.washington.edu
  - Office hours:
    - CSE 582
    - Monday, 4:00-5:00 pm or by appointment
- Teaching Assistant
  - Tanvir Aumi, tanvir@cs.washington.edu
  - Office hours:
    - TBD

Announcements

- It's on the web.
- Homework due at start of class on Mondays
  - HW 1, Due January 14, 2013
  - It's on the web

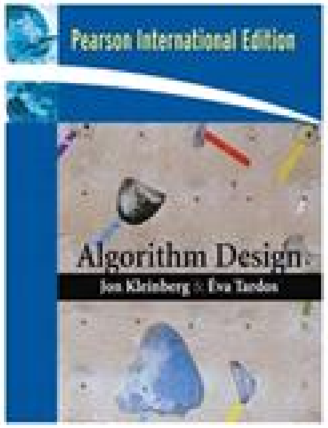
http://www.cs.washington.edu/education/courses/csep521/13wi/

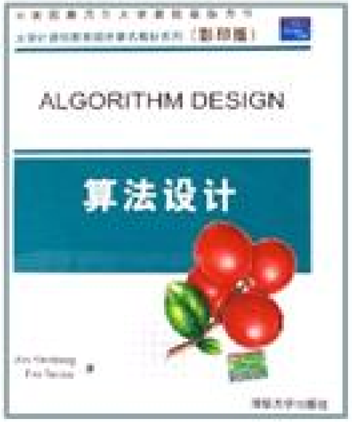
Text book

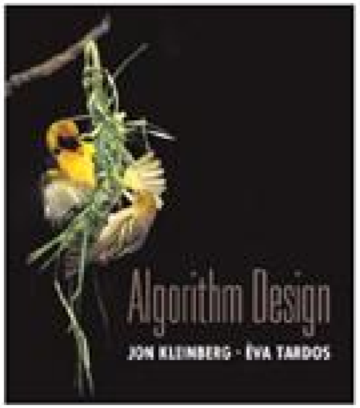
- Algorithm Design
- Jon Kleinberg, Eva Tardos

- Read Chapters 1 & 2

- Expected coverage:
  – Chapter 1 through 7







Recorded lectures

- This is a distance course, so lectures are recorded and will be available on line for later viewing
- However, low attendance in the distance PMP course is a concern
  - Various draconian measures are under discussion
- We will make lectures available
  - Please attend class, and participate
  - Participation may be a component of the class grade

Lecture schedule

- Monday holidays:
  - Monday, January 21, MLK
  - Monday, February 18, President's day
- Make up lectures will be scheduled, which will be recorded for offline viewing
  - Hopefully, some students will attend, so there is a studio audience
  - First makeup lecture:
    - Thursday, January 17, 5:00-6:30 pm
- Additional makeup lectures to accommodate RJA's travel schedule

Course Mechanics

- Homework
  - Due Mondays
  - Textbook problems and programming exercises
    - Choice of language
    - Expectation that Algorithmic Code is original
  - Target: 1 week turnaround on grading
  - Late Policy: Two assignments may be turned in up to one week late
- Exams (In class, ~~tentative~~)
  - Midterm, Monday, Feb 11 (60 minutes)
  - Final, Monday, March 18, 6:30-8:20 pm
- Approximate grade weighting
  - HW: 50, MT: 15, Final: 35

All of Computer Science is the Study of Algorithms

How to study algorithms

- Zoology
- Mine is faster than yours is
- Algorithmic ideas
  - Where algorithms apply
  - What makes an algorithm work
  - Algorithmic thinking

$O(n^2)$

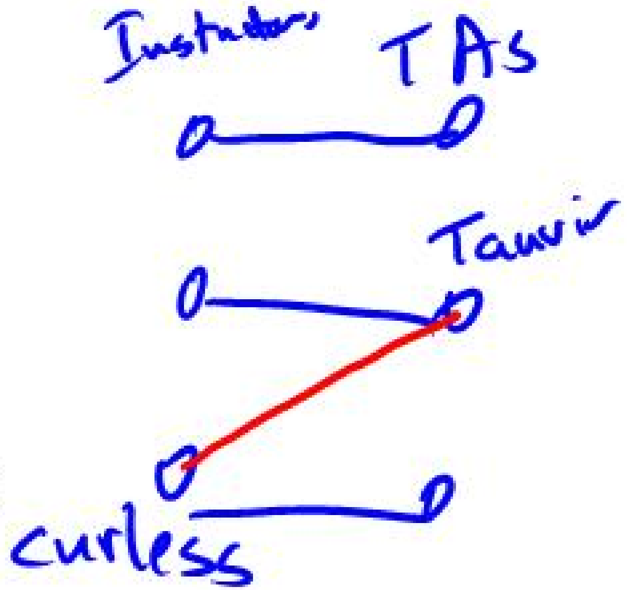
$O\left(\frac{n^2}{\log n}\right)$

$O(n \log n)$

$O(n \log \log n)$

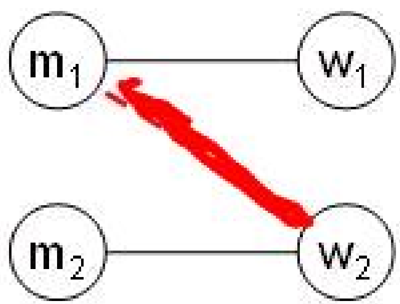
Introductory Problem: Stable Matching

- Setting:
  - Assign TAs to Instructors
  - Avoid having TAs and Instructors wanting changes
    - E.g., Prof A. would rather have student X than her current TA, and student X would rather work for Prof A. than his current instructor.



Formal notions

- Perfect matching
- Ranked preference lists
- Stability



m_1 : w_1 w_2

m_2 : w_2 w_1

w_1 : m_1 m_2

w_2 : m_2 m_1

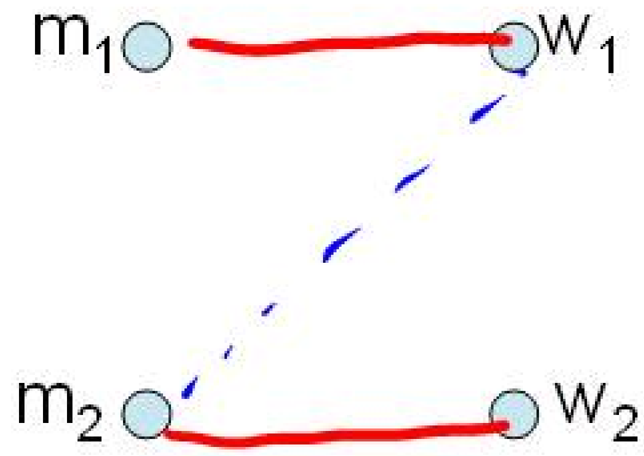
Example (1 of 3)

m_1: w_1 w_2

m_2: w_2 w_1

w_1: m_1 m_2

w_2: m_2 m_1



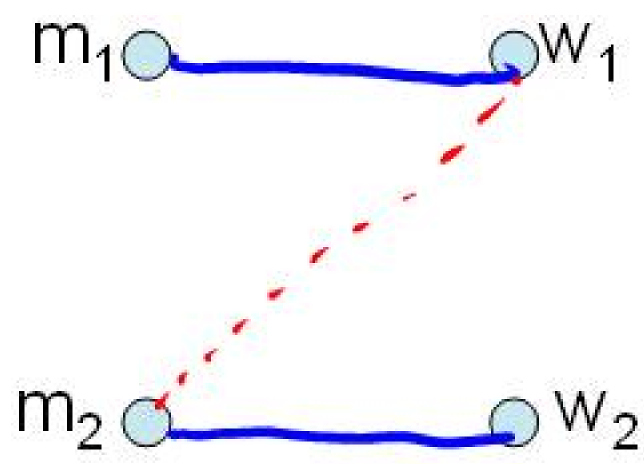
Example (2 of 3)

m_1: w_1 w_2

m_2: w_1 w_2

w_1: m_1 m_2

w_2: m_1 m_2



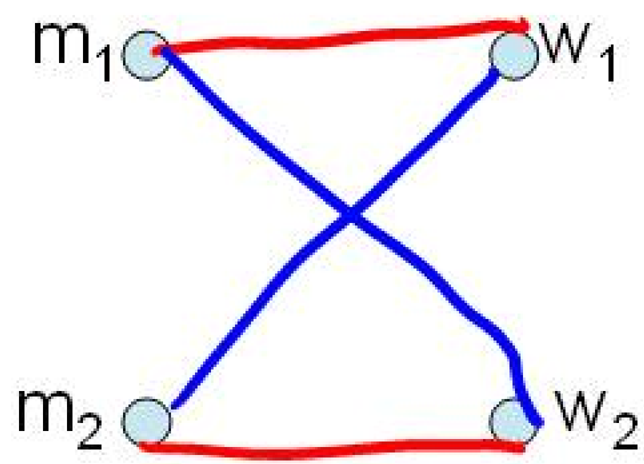
Example (3 of 3)

m_1: w_1 w_2

m_2: w_2 w_1

w_1: m_2 m_1

w_2: m_1 m_2



Formal Problem

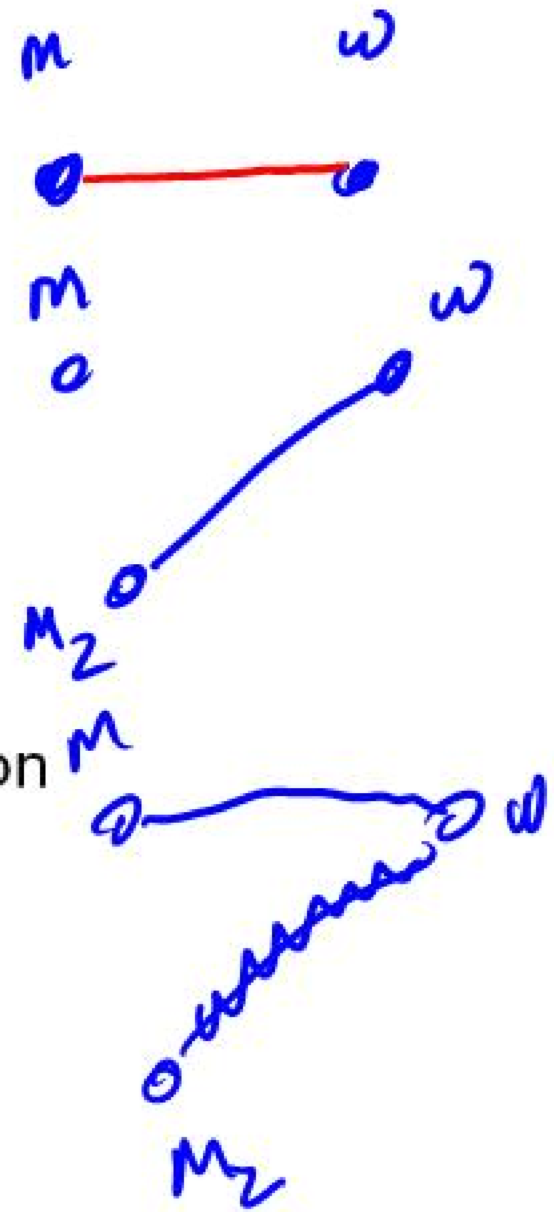
- Input
  - Preference lists for $m_1, m_2, \ldots, m_n$
  - Preference lists for $w_1, w_2, \ldots, w_n$
- Output
  - Perfect matching M satisfying stability property:

If (m', w') $\in$ M and (m'', w'') $\in$ M then
(m' prefers w' to w'') or (w'' prefers m'' to m')

Idea for an Algorithm

- m proposes to w
  - If w is unmatched, w accepts
  - If w is matched to m_2
    - If w prefers m to m_2 w accepts m, dumping m_2
    - If w prefers m_2 to m, w rejects m

Unmatched m proposes to the highest w on its preference list that it has not already proposed to



Algorithm

Initially all m in M and w in W are free
While there is a free m
    w highest on m's list that m has not proposed to
    if w is free, then match (m, w)
    else
        suppose (m_2, w) is matched
        if w prefers m to m_2
            unmatch (m_2, w)
            match (m, w)

Example

m_1: w_1 w_2 w_3

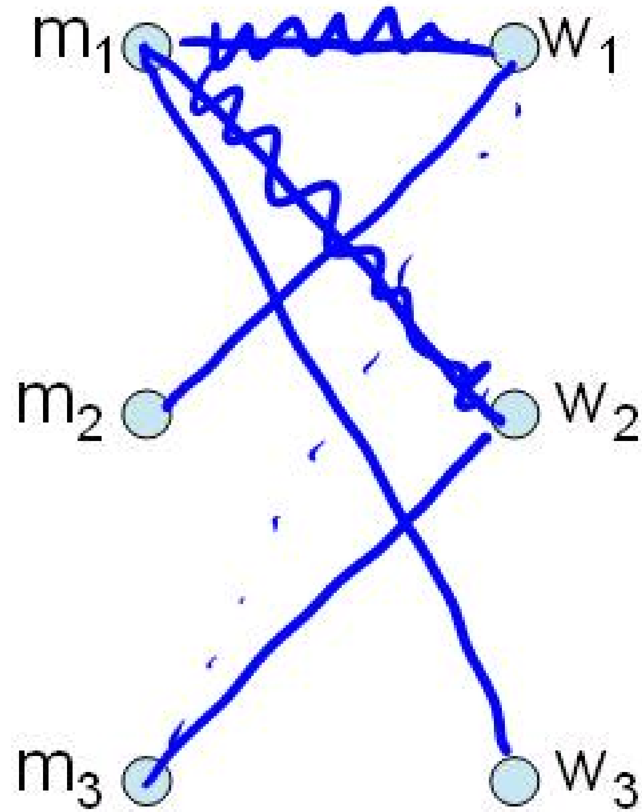
m_2: w_1 w_3 w_2

m_3: w_1 w_2 w_3

w_1: m_2 m_3 m_1

w_2: m_3 m_1 m_2

w_3: m_3 m_1 m_2



Does this work?

- Does it terminate?
- Is the result a stable matching?

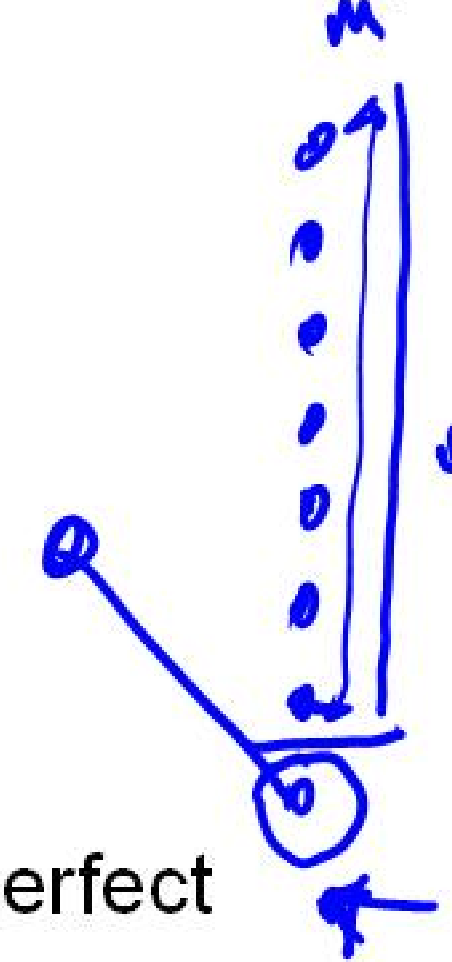
- Begin by identifying invariants and measures of progress
  - m's proposals get worse (have higher m-rank)
  - Once w is matched, w stays matched
  - w's partners get better (have lower w-rank)

Claim: The algorithm stops in at most n^2 steps

At each step
some m advance
in its preference list.

When the algorithms halts, every w is matched

Why?



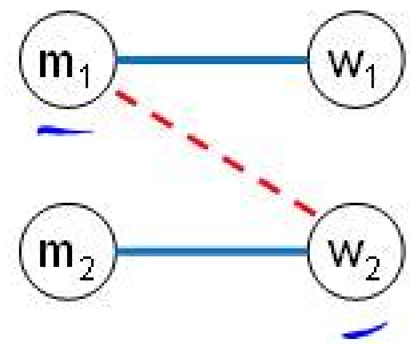
Hence, the algorithm finds a perfect matching

The resulting matching is stable

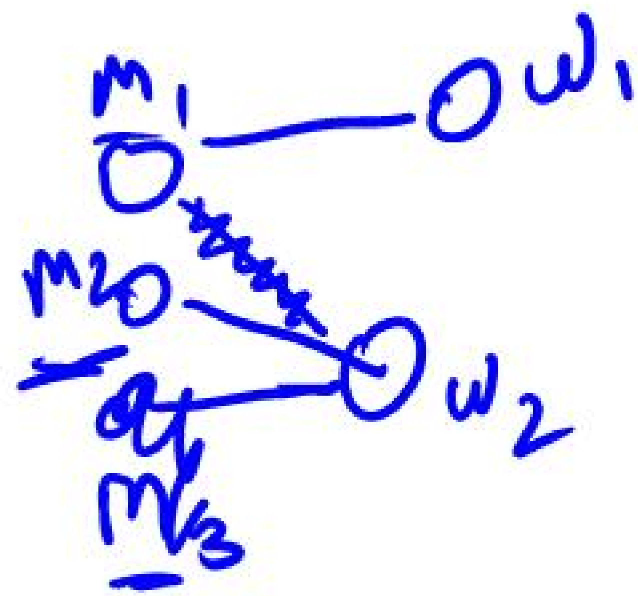
Suppose

$(m_1, w_1) \in M$, $(m_2, w_2) \in M$

m_1 prefers w_2 to w_1



How could this happen?



Result

- Simple, $O(n^2)$ algorithm to compute a stable matching
- Corollary
  - A stable matching always exists

A closer look

Stable matchings are not necessarily fair

m_1: w_1 w_2 w_3

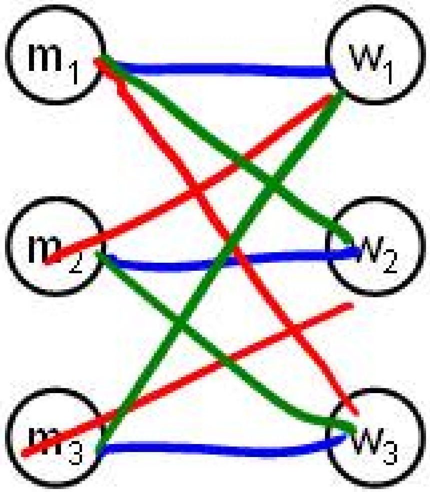
m_2: w_2 w_3 w_1

m_3: w_3 w_1 w_2

w_1: m_2 m_3 m_1

w_2: m_3 m_1 m_2

w_3: m_1 m_2 m_3



How many stable matchings can you find?

Algorithm under specified

- Many different ways of picking m's to propose
- Surprising result
  - All orderings of picking free m's give the same result
- Proving this type of result
  - Reordering argument
  - Prove algorithm is computing something mores specific
    - Show property of the solution – so it computes a specific stable matching

Proposal Algorithm finds the best possible solution for M

Formalize the notion of best possible solution:

- (m, w) is valid if (m, w) is in some stable matching
- best(m): the highest ranked w for m such that (m, w) is valid
- S* = {(m, best(m))}
- Every execution of the proposal algorithm computes S*

Proof

See the text book – pages 9 – 12

Related result: Proposal algorithm is the worst case for W

Algorithm is the M-optimal algorithm

Proposal algorithms where w's propose is W-Optimal

Best choices for one side may be bad for the other

Design a configuration for problem of size 4:

- M proposal algorithm:
  - All m's get first choice, all w's get last choice
- W proposal algorithm:
  - All w's get first choice, all m's get last choice

m_1:

m_2:

m_3:

m_4:

w_1:

w_2:

w_3:

w_4:

But there is a stable second choice

Design a configuration for problem of size 4:

- M proposal algorithm:
  - All m's get first choice, all w's get last choice
- W proposal algorithm:
  - All w's get first choice, all m's get last choice
- There is a stable matching where everyone gets their second choice

m_1: w_1 w_3 w_4 w_2

m_2: w_2 w_4 w_3 w_1

m_3: w_3 w_1 w_2 w_4

m_4: w_4 w_2 w_1 w_3

w_1: m_2 m_3 m_4 m_1

w_2: m_1 m_4 m_3 m_2

w_3: m_4 m_1 m_2 m_3

w_4: m_3 m_2 m_1 m_4

Suppose there are n m's, and n w's

- What is the minimum possible M-rank?

n

- What is the maximum possible M-rank?

n^2

- Suppose each m is matched with a random w, what is the expected M-rank?

Random Preferences

Suppose that the preferences are completely random

m_1: w_8 w_3 w_1 w_5 w_9 w_2 w_4 w_6 w_7 w_{10}
m_2: w_7 w_{10} w_1 w_9 w_3 w_4 w_8 w_2 w_5 w_6
…
w_1: m_1 m_4 m_9 m_5 m_{10} m_3 m_2 m_6 m_8 m_7
w_2: m_5 m_8 m_1 m_3 m_2 m_7 m_9 m_{10} m_4 m_6
…

If there are n m's and n w's, what is the expected value of the M-rank and the W-rank when the proposal algorithm computes a stable matching?

What is the run time of the Stable Matching Algorithm?

Initially all m in M and w in W are free

While there is a free m **Executed at most n^2 times**

w highest on m's list that m has not proposed to

if w is free, then match (m, w)

else

suppose (m_2, w) is matched

if w prefers m to m_2

unmatch (m_2, w)

match (m, w)

O(1) time per iteration

- Find free m
- Find next available w
- If w is matched, determine m_2
- Test if w prefers m to m_2
- Update matching

What does it mean for an algorithm to be efficient?

Key ideas

- Formalizing real world problem
  - Model: graph and preference lists
  - Mechanism: stability condition
- Specification of algorithm with a natural operation
  - Proposal
- Establishing termination of process through invariants and progress measure
- Under specification of algorithm
- Establishing uniqueness of solution

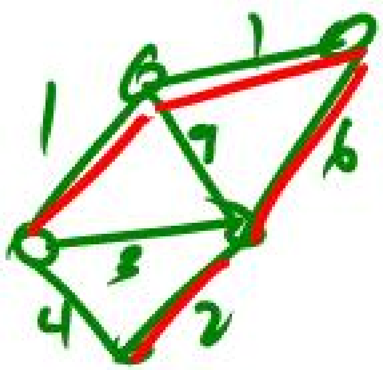
Introduction of five problems

- Show the types of problems we will be considering in the class
- Examples of important types of problems
- Similar looking problems with very different characteristics
- Problems
  - Scheduling
  - Weighted Scheduling
  - Bipartite Matching
  - Maximum Independent Set
  - Competitive Facility Location

Minimum Spanning Tree

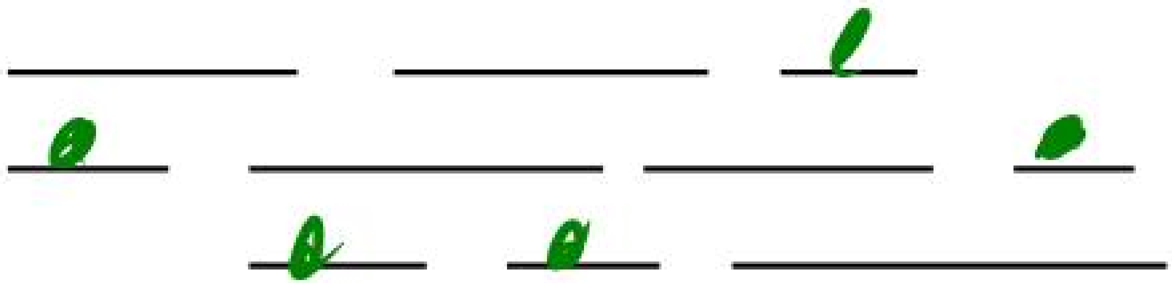
What is a problem?

- Instance — Graph + weights on edges
- Solution — set of edges
- Constraints on solution — edges form a spanning tree
- Measure of value — sum of the edge costs.



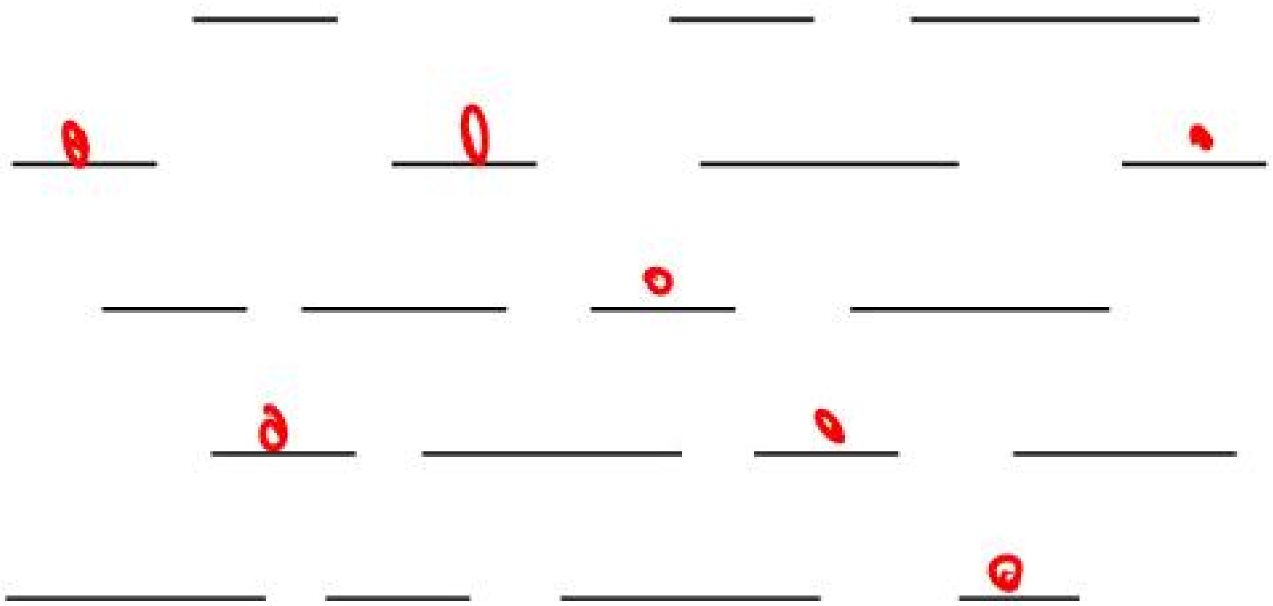
Problem: Scheduling

- Suppose that you own a banquet hall
- You have a series of requests for use of the hall: $(s_1, f_1), (s_2, f_2), \ldots$



- Find a set of requests as large as possible with no overlap

What is the largest solution?



choose min overlap

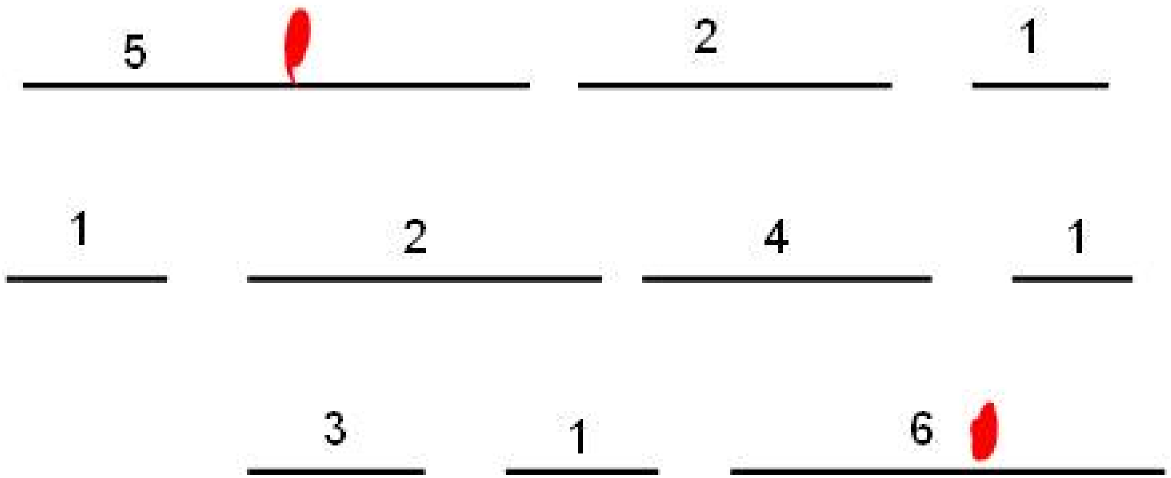
choose short duration

Greedy Algorithm

- Test elements one at a time if they can be members of the solution
- If an element is not ruled out by earlier choices, add it to the solution
- Many possible choices for ordering (length, start time, end time)
- For this problem, considering the jobs by increasing end time works

Suppose we add values?

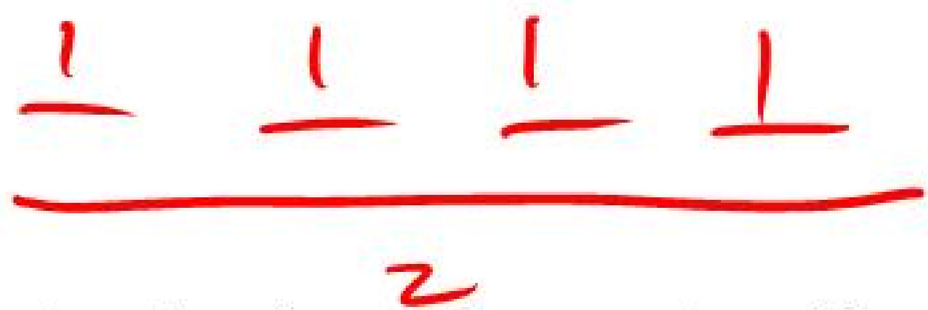
- (s_i, f_i, v_i), start time, finish time, payment
- Maximize value of elements in the solution



Greedy Algorithms

- Earliest finish time

- Maximum value



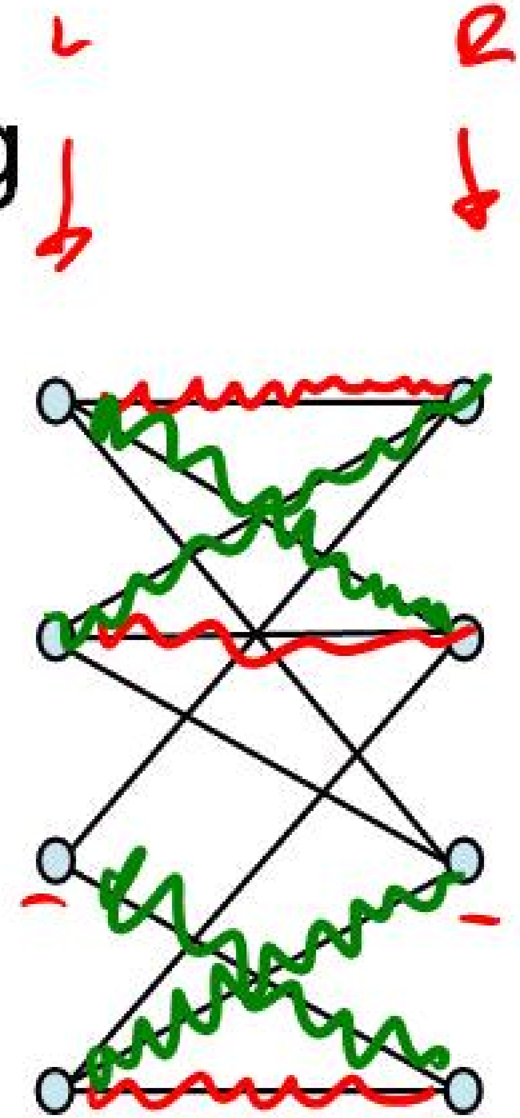
- Give counter examples to show these algorithms don't find the maximum value solution

Dynamic Programming

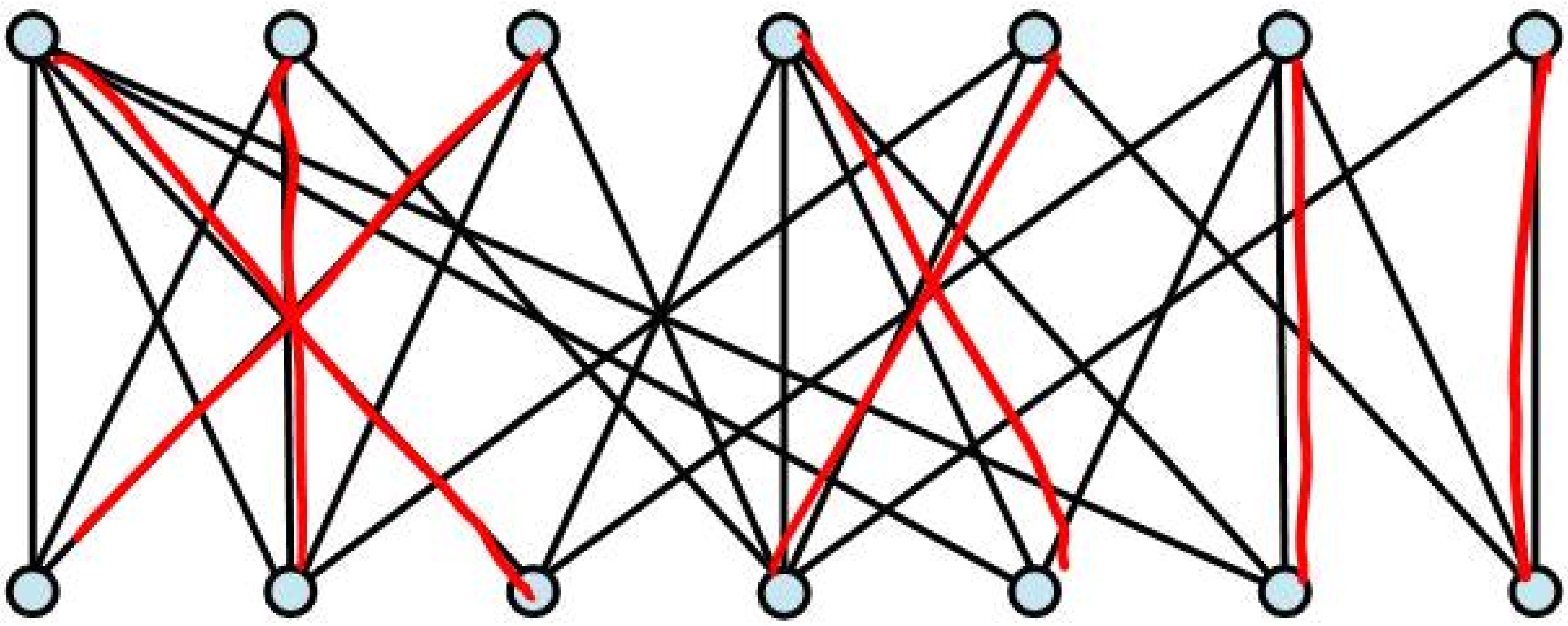
- Requests $R_1, R_2, R_3, \ldots$
- Assume requests are in increasing order of finish time ($f_1 < f_2 < f_3 \ldots$)
- Opt_i is the maximum value solution of $\{R_1, R_2, \ldots, R_i\}$ containing R_i
- $Opt_i = Max\{ j \mid f_j < s_i \}[Opt_j + v_i]$

Matching

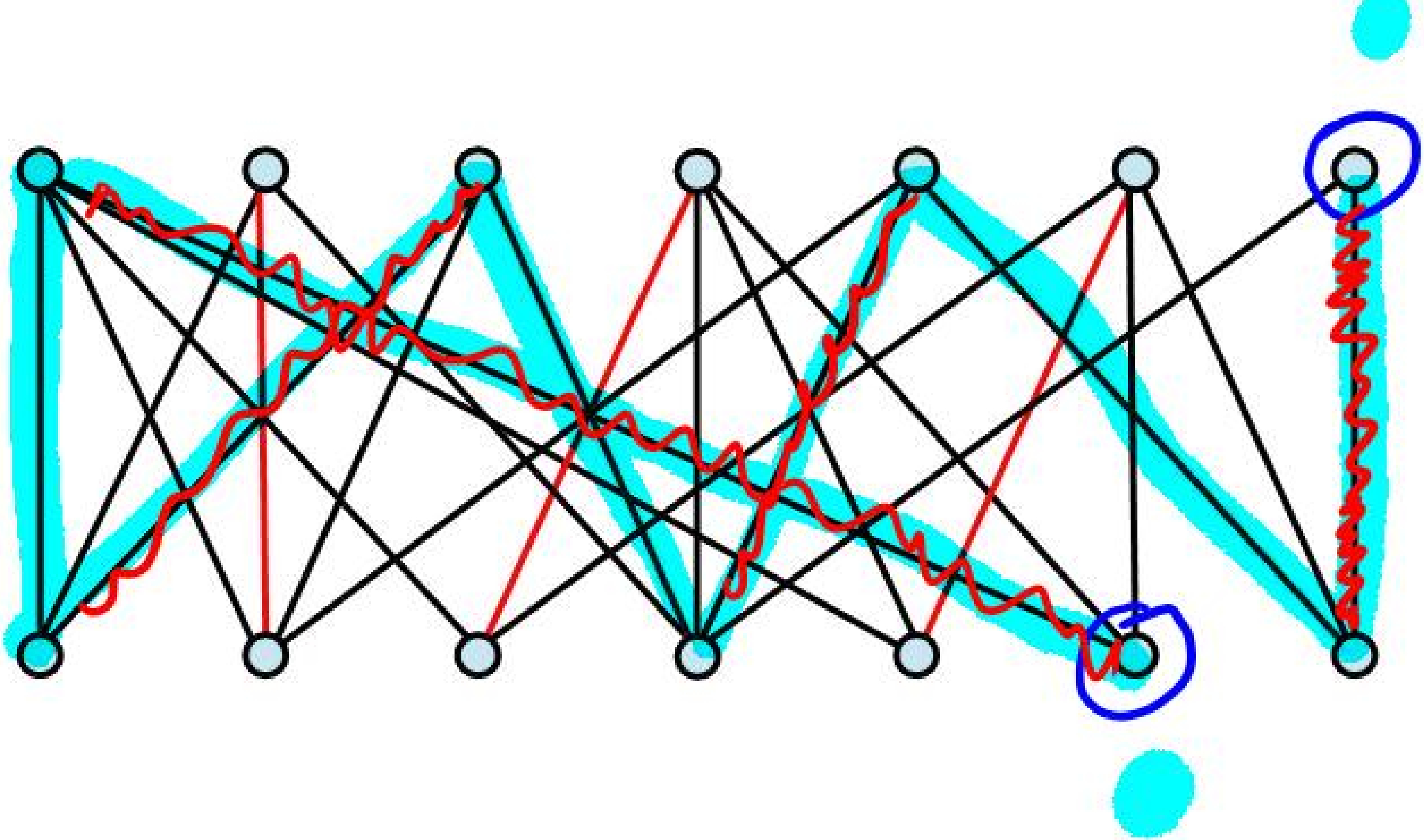
- Given a bipartite graph G=(U,V,E), find a subset of the edges M of maximum size with no common endpoints.
- Application:
  - U: Professors
  - V: Courses
  - (u,v) in E if Prof. u can teach course v



Find a maximum matching

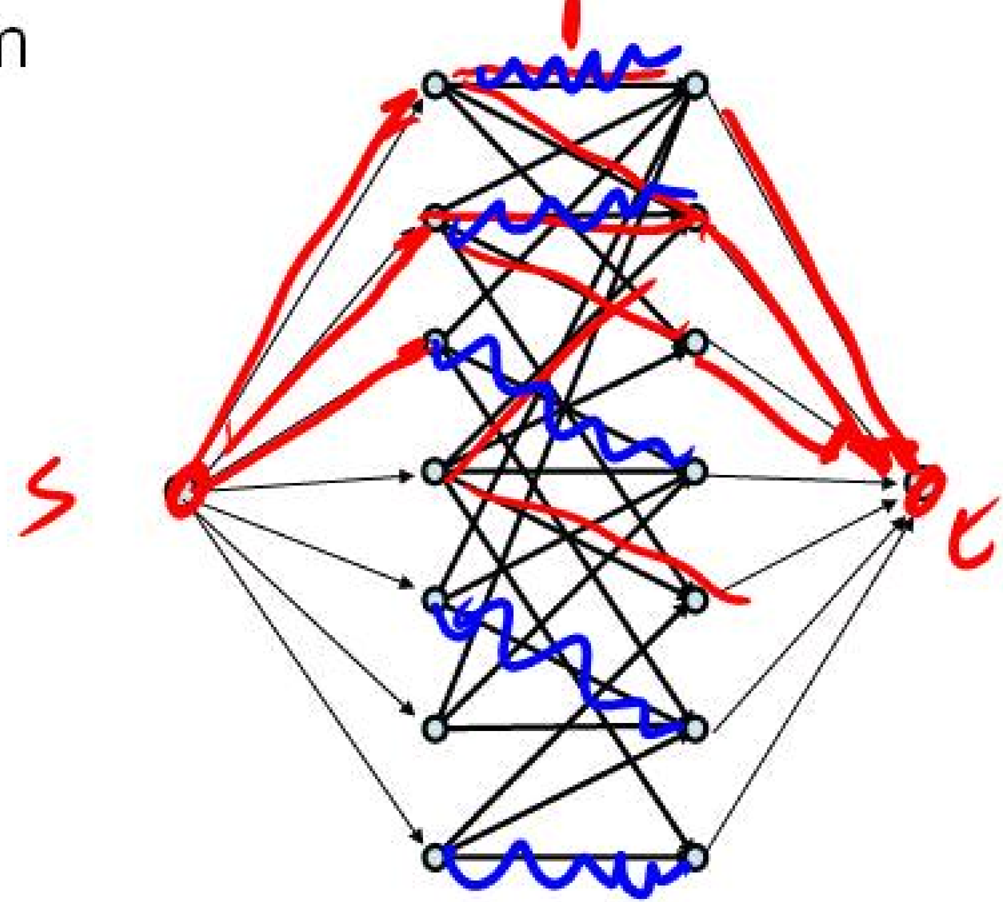


Augmenting Path Algorithm



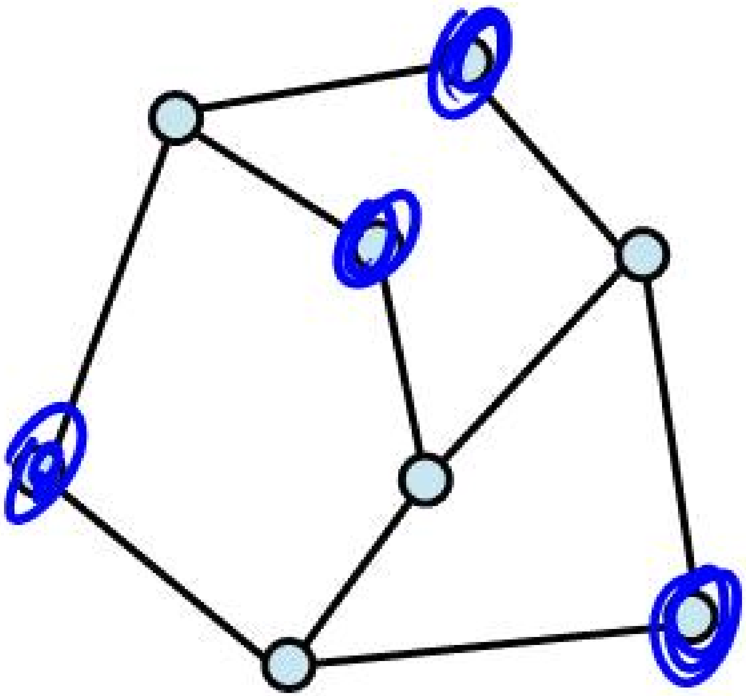
Reduction to network flow

- More general problem
- Send flow from source to sink
- Flow subject to capacities at edges
- Flow conserved at vertices
- Can solve matching as a flow problem

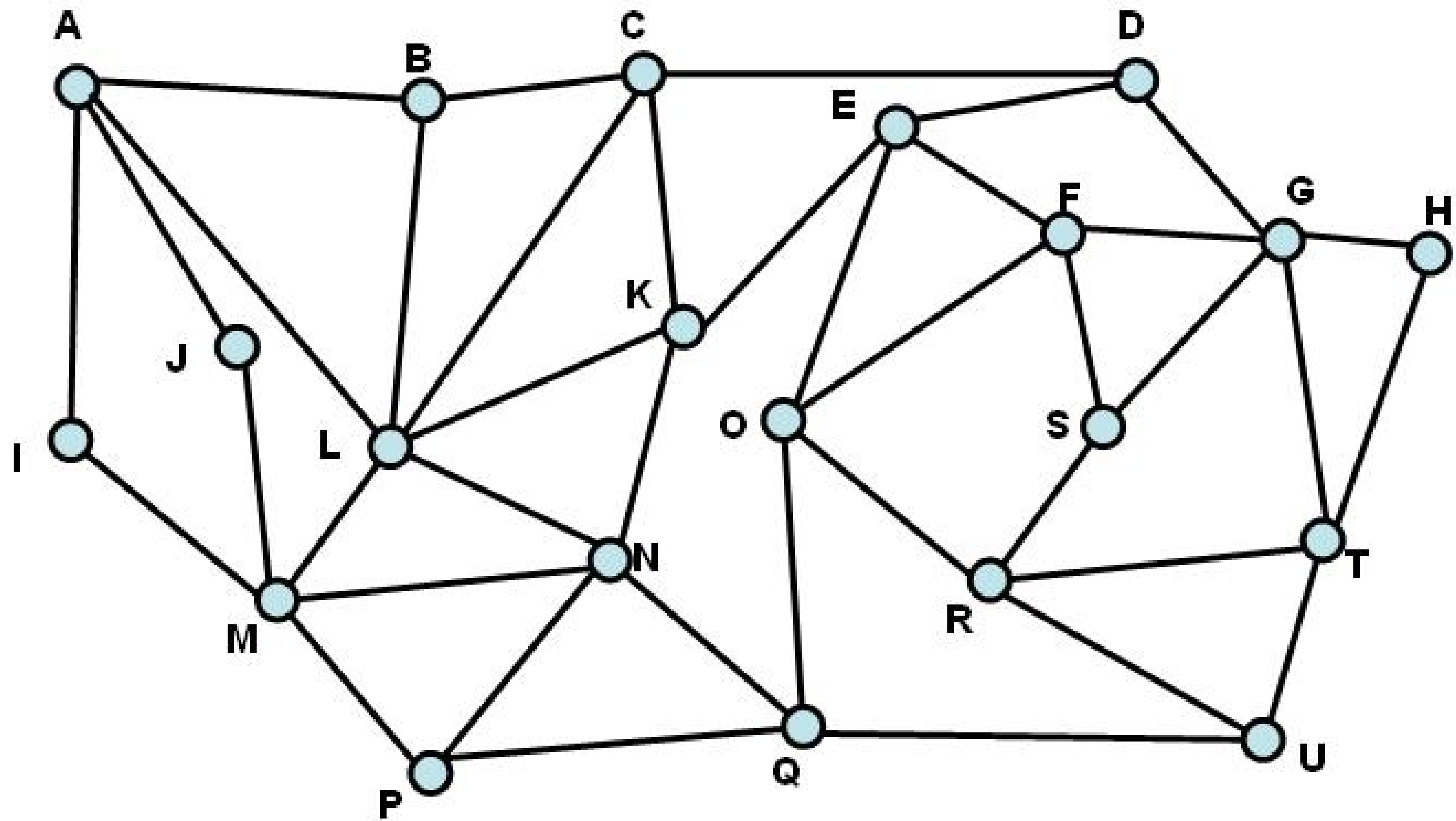


Maximum Independent Set

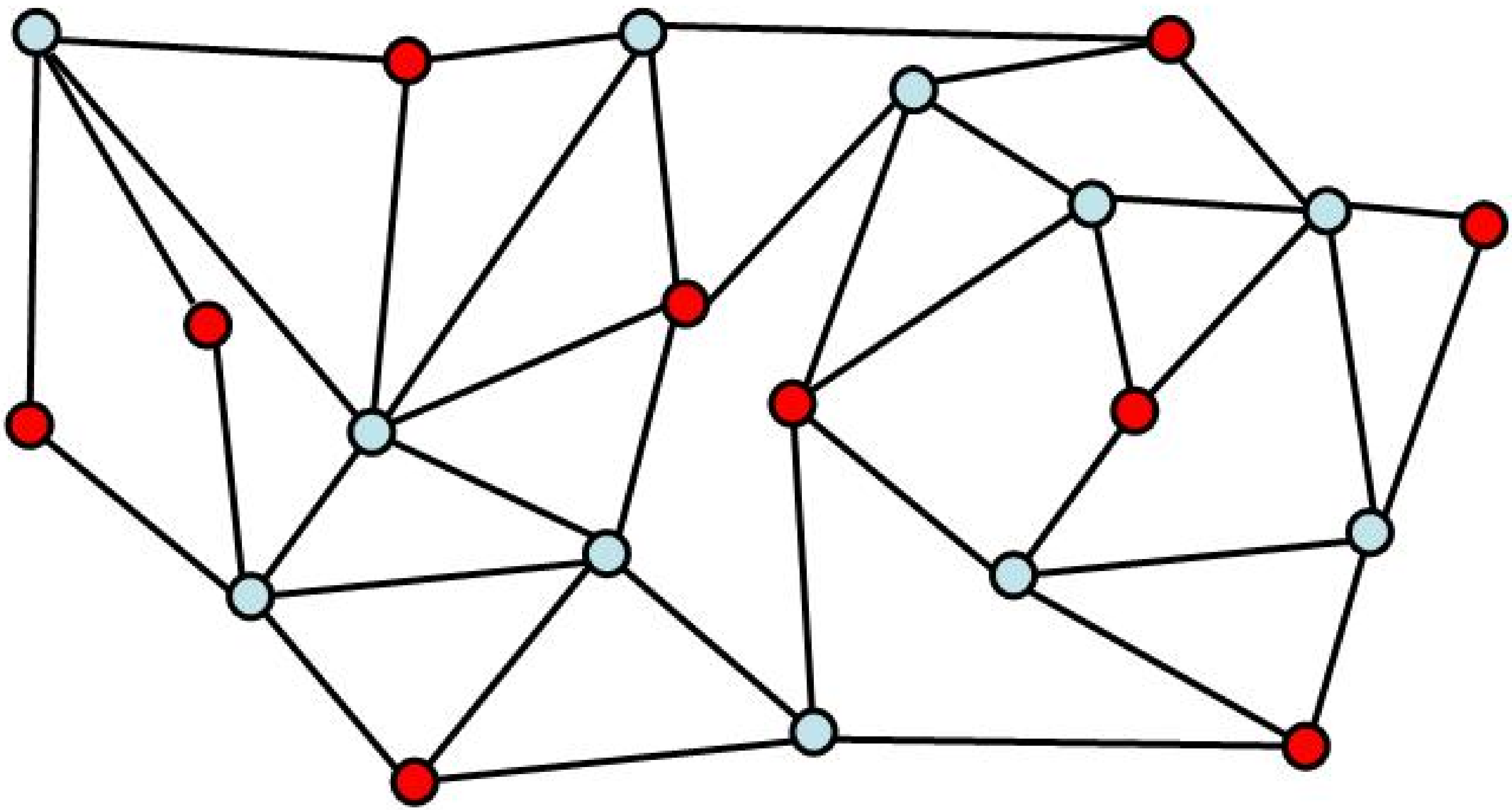
- Given an undirected graph G=(V,E), find a set I of vertices such that there are no edges between vertices of I
- Find a set I as large as possible



Find a Maximum Independent Set



Verification: Prove the graph has an independent set of size 10



Key characteristic

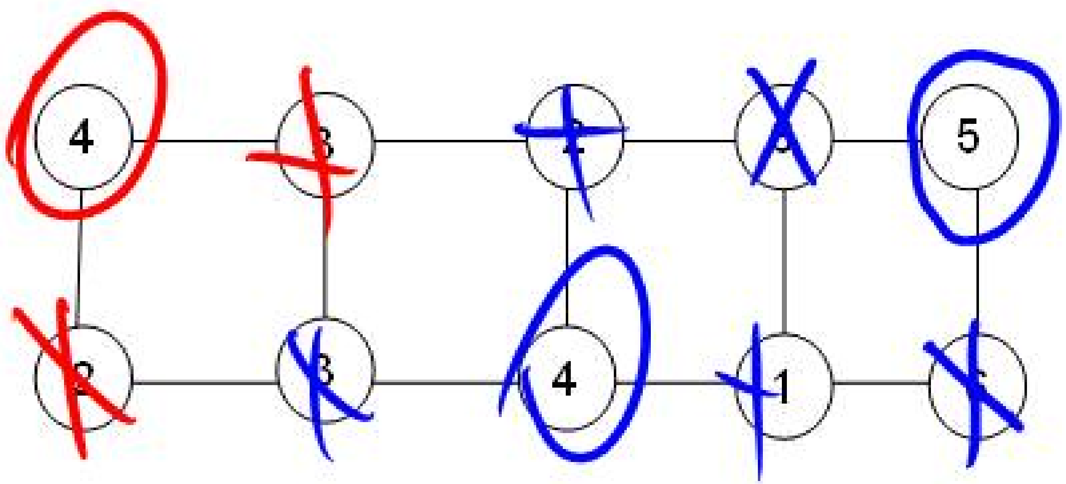
- Hard to find a solution
- Easy to verify a solution once you have one
- Other problems like this
  - Hamiltonian circuit
  - Clique
  - Subset sum
  - Graph coloring

NP-Completeness

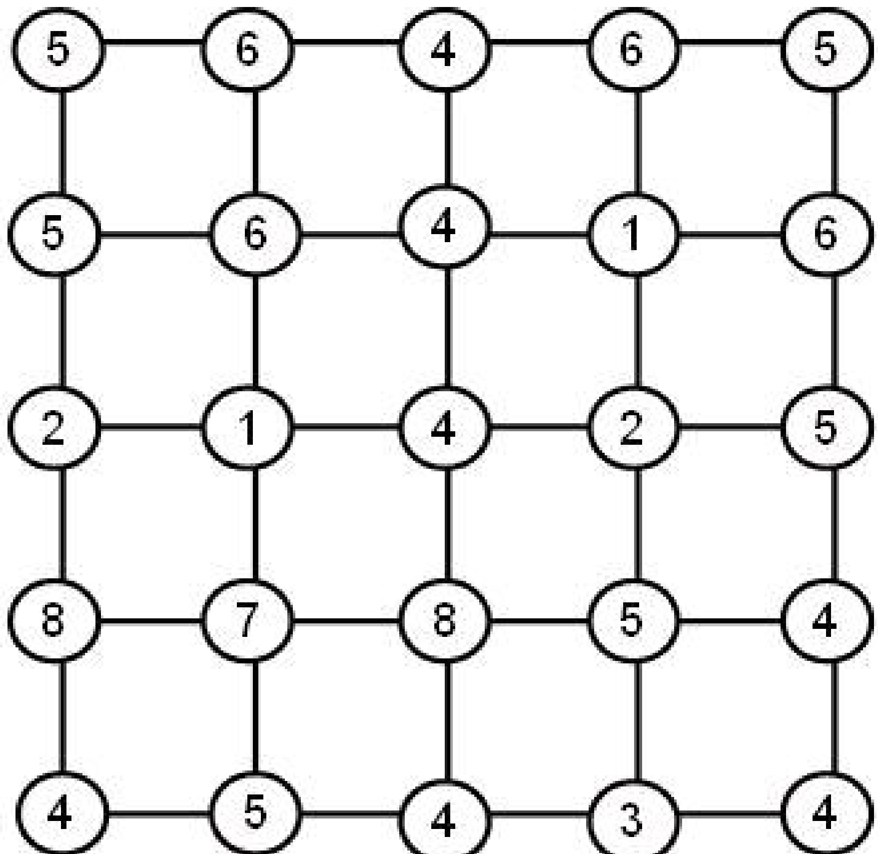
- Theory of Hard Problems
- A large number of problems are known to be equivalent
- Very elegant theory

Are there even harder problems?

- Simple game:
  - Players alternating selecting nodes in a graph
    - Score points associated with node
    - Remove nodes neighbors
  - When neither can move, player with most points wins



P- space complete.



Competitive Facility Location

- Choose location for a facility
  - Value associated with placement
  - Restriction on placing facilities too close together
- Competitive
  - Different companies place facilities
    - E.g., KFC and McDonald's

Complexity theory

- These problems are P-Space complete instead of NP-Complete
  - Appear to be much harder
  - No obvious certificate
    - G has a Maximum Independent Set of size 10
    - Player 1 wins by at least 10 points

An NP-Complete problem from Digital Public Health

- ASHAs use Pico projectors to show health videos to Mothers' groups
- Limited number of Pico projectors, so ASHAs must travel to where the Pico projector is stored
- Identify storage locations for k Pico projectors to minimize the maximum distance an ASHA must travel

Summary

- Scheduling
- Weighted Scheduling
- Bipartite Matching
- Maximum Independent Set
- Competitive Scheduling