

CSE524 Parallel Computation

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Announcements

- New homework assigned at end of class
- Discuss this week's assignment in 2 parts
 - n Red/Blue, now
 - n Essay, later this class
- Project description circulated at break

Project thinking begins this week

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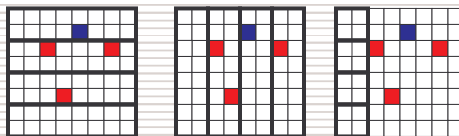
Red/Blue Problem

- Write Peril-L code to implement Red/Blue
- Goal is to get a scalable program
- Inputs:
 - n $n \times n$ board, initialized and allocated
 - t t , a small region used to sample
 - Termination condition: some $t \times t$ on left $> 90\%$
- Rules of the game
 - n Red moves 1 right on 1st half step if cell free
 - n Blue moves 1 down on 2nd half step if cell free

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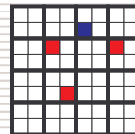
View Possible Solutions Globally

- Shared allows threads to process rows/cols



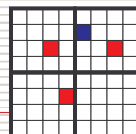
All references to
global memory

- Allocate $t \times t$ threads



Must be
multiplexed

- Scalable blocks



Maximize local work;
multiplexing possible

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Red-Blue ... One Solution

```

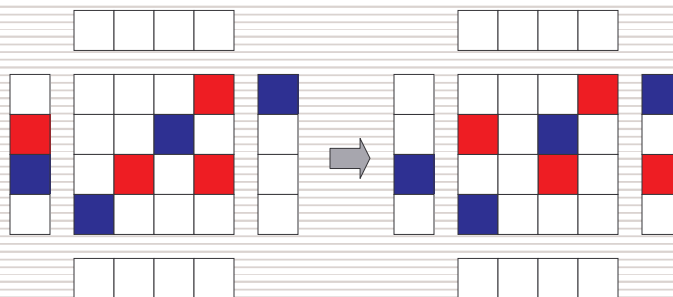
int RB_G[n][n];           t/n, colors 0=w,1=r,2=b
/* Initialize RB with data */ Get external data
int size=sqrt(P); my=n/size; Procs along row/col
int bin_GO[size][size][my]; MM for passing cols
int bou_GO[size][size][my]; MM for passing rows
int thresh=.9;           90% threshold
forall thr,thc in (0:size-1,0..size-1) { 2D threads
  int Lrb[my][my]=is_local(RB_G); Work on local assigned
  int lft[my],rgt[my],top[my],bot[my]; Local neighbr storage
  int xl = RB_G_myLo_1; yl = RB_G_myHi_1; Global index
  int xh = RB_G_myLo_2; yh = RB_G_myHi_2; Global index
  int done=0,countR, countB; Termination variables

```

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Cartoon of Red Move

- Get adjacent context from neighbors



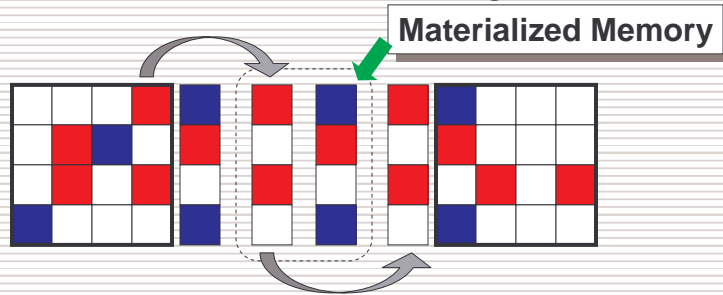
- Compute local state

Similarly for blue moves

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Cartoon of State Transfer

- Save values in materialized global memory



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Solution (Continued)

```
while (done==0) { Go till threshold
  bou_GO[thr][thc][0:my-1]=Lrb[xl][yl:yh]; pass left col
  bin_GO[trh][thc][0:my-1]=Lrb[xh][yl:yh]; pass right col
  rgt[0:my-1]=bou_GO[thr][(thc+1)%my][0:my-1]; last col+1
  lft[0:my-1]=bin_GO[trh][(thc-1)%my][0:my-1]; first col-1
  moveRed(Lrb[[]],rgt[],lft[]); barrier;
  bou_GO[thr][thc][0:my-1]=Lrb[xl:xh][yl]; pass top row
  bin_GO[trh][thc][0:my-1]=Lrb[xl:xh][yh]; pass bot row
  top[0:my-1]=bin_GO[(thr-1)%my][thc][0:my-1]; top row-1
  bot[0:my-1]=bou_GO[(thr+1)%my][thc][0:my-1]; last row+1
  moveBlue(Lrb[[]],bot[],top[]);
```

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Solution Continued

```
if (thc == 0)
  {for (k=0; k<my/t; k++) {
    countR = 0; countB = 0;
    for (i=0; i<t; i++) {
      for (j=0; j<t; j++) {
        if (Lrb[i][j]== 1)
          countR++;
        if (Lrb[i][j]== 2)
          countB++;
      }
    }
  }
```

Am I on left edge?

Do all my txt blocks

Count up colors

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Solution Continued

```
/* After a t x t block ... */
  if (countR/(countR+countB)>=thresh ||
      countB/(countR+countB)>=thresh)
    done = 1;
  }
}
done=+/done;
}
}
```

I color is over thresh

End of t x t tiles

End of if for term test

Find out how others did;synchs

End of while

End of thread

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Recap Solution

- Scaled by available processors; $t \times t$ is min
- Materialized memory implements a bulk exchange, benefiting from fast transfer
- Constructing context allows local compute
- Termination performed by reduction
- Why was the barrier needed?
- Is less data motion possible?

A good exercise: Analyze R/B complexity for different P

Schwartz's Algorithm

- Jack Schwartz (NYU) asked: What is optimal number of processors to combine n values?
 - n Reasonable Answer: binary tree w/ values at leaves has $O(\log n)$ complexity
 - n To this solution add $\log n$ more values to leaves
 - n Same complexity ($O(\log n)$), but $n \log n$ values!
- Generally P is not a variable, and $P \ll n$
- Use Schwartz as heuristic: Prefer to work at leaves rather than enlarge (make a deeper) tree, implying tree will have $\log P$ height

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Expressing A Tree

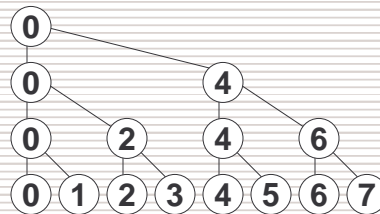
- Consider writing tree-based sum in Peril-L
 - n Processors will have several roles: leaf, intr node
 - n Solve synchronization with materialized memory
- Stages:
 - n Compute result at leaves
 - n Threads have IDs in $0 - 2^p - 1$, pass if $0 \neq \text{ID} \% 2$
 - n Even threads add, pass value if $0 \neq \text{ID} \% 4$
 - n Etc.

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Peril-L Code for Collective Ops

- Thread logic is easy ... initialize `stride = 1`

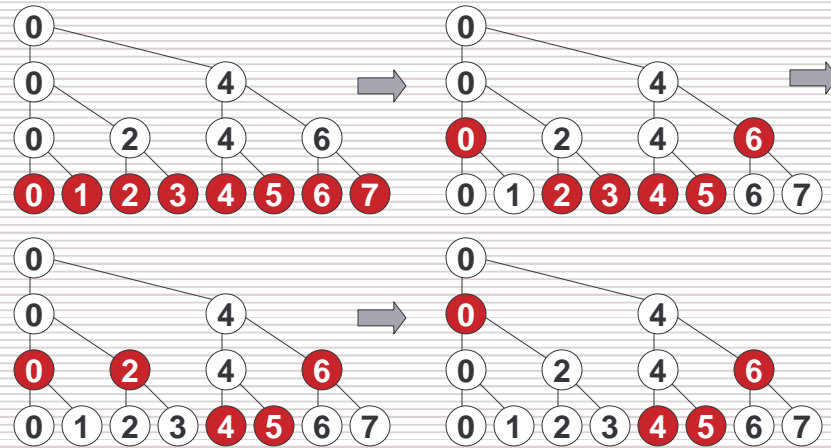
```
nodeval_GO[tID] = tally;           Send local val to tree node
while (stride < P) {               Begin logic for tree
  if (tID %(2*stride) == 0) {
    nodeval_GO[tID]= nodeval_GO[tID] +
    nodeval_GO[tID + stride]);
    stride = 2*stride;
  }
  else
    break;
}
```



Reverse to broadcast

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Materialized Memory Self-Synchs



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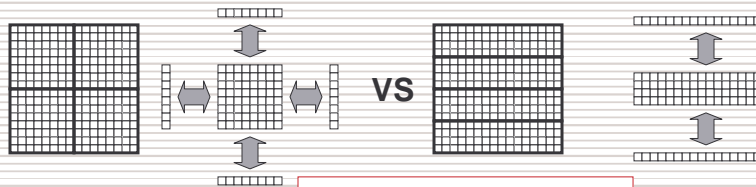
Asynchronous Trees

- MM gives asynchronous tree, preferred over a “leveled” tree using barriers
- Notice that once a thread’s role is complete it can continue computing
- For collective op + broadcast, a tree implemented with materialized memory allows half of the threads to continue
- **The Principle:** Relax synchronization rules wherever possible!

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Block Allocations

- The Red/Blue computation illustrated a **2D-block** data parallel allocation of the problem
- Generally block allocations are better for data transmission: surface to volume advantage ... since only edges are x-mitted

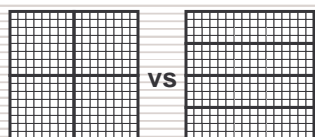
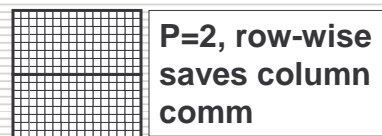


Now scale problem 4x

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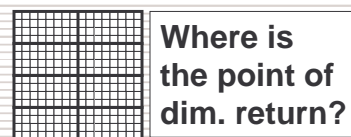
Different Regimens

- Though block is generally a good allocation it's not absolute:



vs

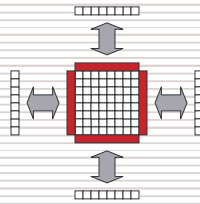
P=4, rows and blocks are a wash



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Shadow Buffers/Fluff

- To simplify local computation in cases where nearest neighbors values x-mitted, allocate in-place memory to store values:



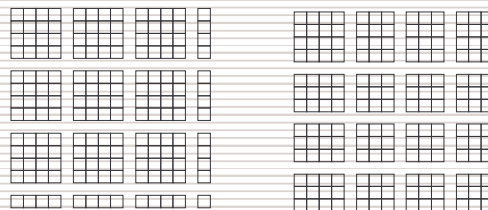
- Array can be referenced as if it's all local

Edge storage (rgt,...) illustrated in R/B

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Aspect Ratio

- Generally P and n do not allow for a perfectly balanced allocation ...
- Several ways to assign arrays to processors



Quotient +
remainder

Ceiling +
floor

Generally a
small effect

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Assigning Processor 0 Work

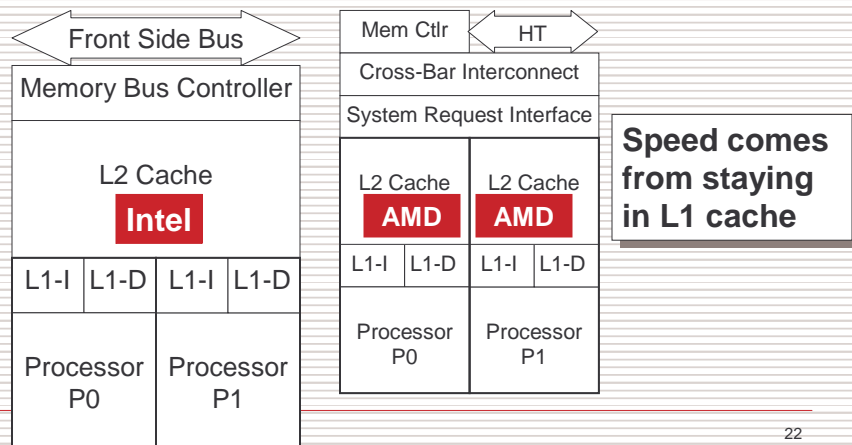
- P_0 is often assigned “other duties”, such as
 - Orchestrate I/O
 - Root node for combining trees
 - Work Queue Manager ...
- Assigning P_0 the smallest quantum of work helps it avoid becoming a bottleneck
 - For either quotient + remainder or ceiling/floor P_0 should be the last processor

This is a late-stage tuning matter

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Solving Array Problems on CMPs

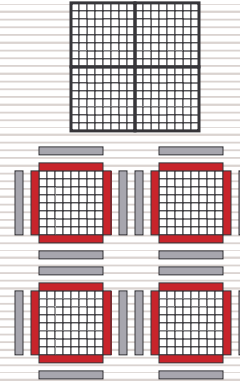
- How important are these topics for CMPs?



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Locality Always Matters

- Array computations on CMPs
 - n Dense Allocation vs Fluff
 - n Issue is cache invalidation
 - n Keeping MM managed intermediate buffers keeps array and fluff local (L1)
 - n Sharing causes elements at edge to repeatedly invalidate killing locality



False sharing an issue, too

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Reduce For Global Sum

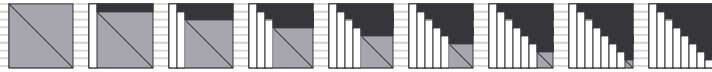
- Reduce abstraction vs `sum_G += 1`
 - n Reduce raises level of abstraction
 - n Reduce provides private var implementation
- Tree implementation of reduce
 - n Irrelevant for tiny numbers of processors

Bottom Line: CTA machine model not strictly required for CMPs. Using it yields scalable programs that benefit CMPs in most ways without causing any harm.

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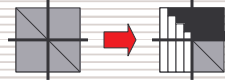
Load Balancing

- Certain computations are inherently imbalanced ... LU Decomposition is one



gray is balanced work, white & black are finished

- Standard block decomposition quickly becomes very biased



- n Cyclic and block cyclic allocation are one fix

Details in the appendix

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Work Queue, A Balancing Act(ivity)

- When work is generated dynamically, a work queue leads to reasonable balance
- Solution components:
 - n Queue -- often a cyclically managed global array
 - n Work or task descriptors -- succinct statements of the task to perform, often only index or bound
- Process:
 - n As work is created, *append* to queue
 - n On task completion, *remove* work from Q-head

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Work Queue Considerations

- Synchronization
 - n Threads modifying the queue do so exclusively
 - n Independent modification of head (*remove*) & tail (*append*) is dangerous, tough to do right



- Work quantum (grain) must exceed the overhead of queue manipulation and comm
 - n Large quantum may lead to poor “end game”

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Considerations (Continued)

- Ordering, though strict at queue *remove*, may not be strict when viewed at completion
- Work assignment interacts with locality
 - n Want strategies where descriptor embodies all data needed for task: chess move search
 - n Avoid global memory reference by caching data: floor planning, place and route, etc.
 - n When global reference inevitable, grain size should must cover comm costs too

Work quantum must be variable, not scalable

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Break

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Essay On 'Quicksort'

- Discuss

“Quicksort is an example of a sequential algorithm that is a good candidate to be incrementally transformed into a parallel algorithm”

- Where does the belief come from?
- What aspects are compatible/incompatible with the CTA?

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Reconceptualizing a Computation

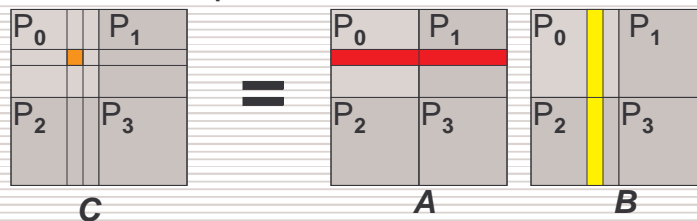
- Good parallel solutions result from rethinking a computation ...
 - n Sometimes that amounts to ordering scalar operations
 - n Sometimes it requires starting from scratch
- The SUMMA matrix multiplication algorithm is the poster computation for rethinking!

This computation is part of homework assignment

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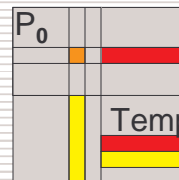
Return To A Lecture 1 Computation

- Matrix Multiplication on Processor Grid



- n Matrices **A** and **B** producing $n \times n$ result **C** where

$$C_{rs} = \sum_{k=1}^n A_{rk} * B_{ks}$$

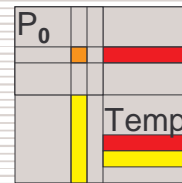


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Applying Scalable Techniques

- Assume each processor stores block of \mathbf{C} , \mathbf{A} , \mathbf{B} ; assume “can’t” store all of any matrix
- To compute c_{rs} a processor needs all of row r of \mathbf{A} and column s of \mathbf{B} .
- Consider strategies for minimizing data movement, because that is the greatest cost -- what are they?

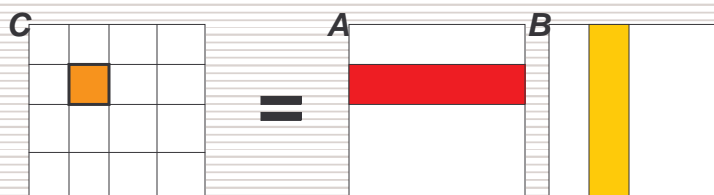
$$c_{rs} = a_{r1}b_{1s} + a_{r2}b_{2s} + \dots + a_{rn}b_{ns}$$



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Grab All Rows/Columns At Once

- If all rows/columns are present, it's local



- Each element requires $O(n)$ operations
- Modern pipelined processors benefit from large blocks of work
- But memory space and BW are issues

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Process $t \times t$ Blocks

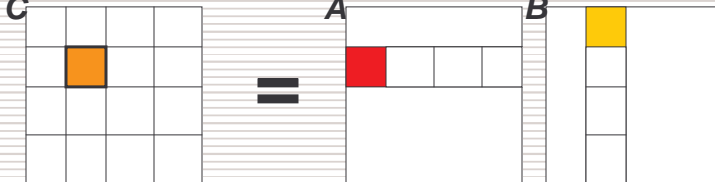
- Referring to local storage

```

for (r=0; r < t; r++){
  for (s=0; s < t; s++){
    c[r][s] = 0.0;
    for (k=0; k < n; k++){
      c[r][s] += a[r][k]*b[k][s];
    }
  }
}

```

Sweeter caching

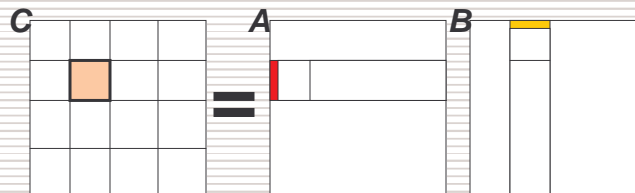


Only move a $t \times t$ block at a time

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Change Of View Point

- Don't think of row-times-column



$$\begin{array}{cc}
 & b_{11} & b_{12} \\
 a_{11} & a_{11}b_{11} & a_{11}b_{12} \\
 a_{21} & a_{21}b_{11} & a_{21}b_{12}
 \end{array}
 \quad
 \begin{array}{c}
 \text{orange} \\
 = \\
 * \\
 + \\
 * \\
 + \dots + \\
 * \\
 + \\
 * \\
 + \\
 *
 \end{array}
 \begin{array}{c}
 \text{red} \\
 \text{red} \\
 \dots \\
 \text{red} \\
 \text{yellow} \\
 \text{yellow} \\
 \dots \\
 \text{yellow}
 \end{array}$$

Switch orientation -- by using a *column* of *A* and a *row* of *B* compute all 1st terms of the dot products

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SUMMA

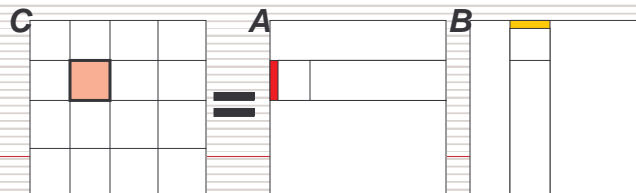
- Scalable Universal Matrix Multiplication Alg
 - n Invented by van de Geijn & Watts of UT Austin
 - n Claimed best machine independent MM
- Whereas MM is usually A row x B column, SUMMA is A column x B row because computation switches from sense
 - n Normal: Compute all terms of dot product
 - n SUMMA: Computer first term of all dot products

Strange. But fast!

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SUMMA Assumptions

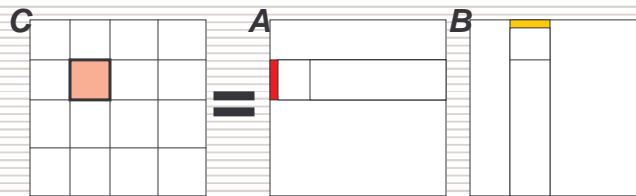
- Threads have two indices, handle $t \times t$ block
- Let $p = P^{1/2}$, then thread u, v
 - n reads all columns of A for indices $u^*t:(u+1)^*t-1, j$
 - n reads all rows of B for indices $i, v^*t:(v+1)^*t-1$
 - n The arrays will be in global memory and referenced as needed



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Solve SUMMA In Peril-L

- Arrays in Global Memory
- Consider how processors reference arrays
 - n Where do operands come from?
 - n Where are results saved?



www.cs.utexas.edu/users/rvdg/abstracts/SUMMA.html

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Homework

- Read the remainder of Chapter 5
- Write a version of SUMMA in Peril-L
- Decide on Project Topic -- write a paragraph description of the computation that you expect to solve for the term project
- We will use a cluster making MPI and ZPL the most amenable languages we'll discuss

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