Part V: Algorithms & Data Structs

Goal: Focus more closely on scalable parallel techniques, both computation and data

Announcement

- Notice on the calendar that next week's class (normally 5/4) is **rescheduled** for Thursday (5/6), same time, same place

Commentary on Homework

- Are there any further comments on the Red /Blue thread program?
- How was the Peril-L sample sort exercise?
  - Randomizing
  - Finding Cut-points
  - Global Exchange
  - Scooch

Recovering A Missed Chance

- Recall from last week … the balanced ( ) code

```
for (i=start; i<start+len_per_th; i++) {
    temp = symb[i];
    if (temp == "(" )
       o++;
    if (temp == ")" ) {
       o--;
         if (o < 0) {
            c++; o = 0;
         }
     }
```

- The question was raised, could we move symb[i] into a local variable before the if's

Can it?

- The answer was 'yes, though a modern compiler could do this for us'
- That answer's correct, but I missed the opportunity to say why
  - This move would not be legal in our assumed sequentially consistent shared memory model UNLESS the compiler could establish the global fact that the array is read only
  - It is legal in the Peril-*L* model, which has no coherency commitments at all

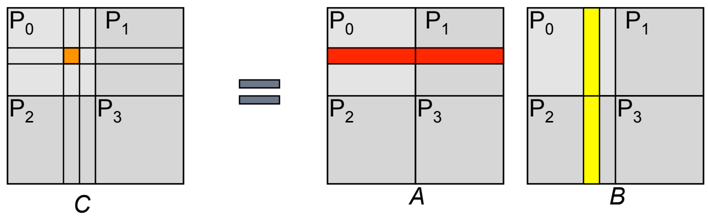
Reconceptualizing a Computation

- Good parallel solutions result from rethinking a computation ...
  - Sometimes that amounts to reordering scalar operations
  - Sometimes it requires starting from scratch
- The SUMMA matrix multiplication algorithm is the poster computation for rethinking!

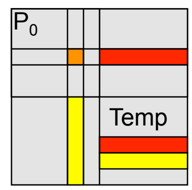
This computation is part of homework assignment

Return To A Lecture 1 Computation

- Matrix Multiplication on Processor Grid

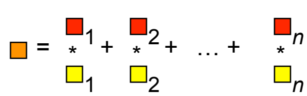


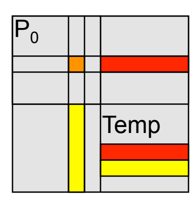
  - Matrices ***A*** and ***B*** producing *n* x *n* result ***C*** where $C_{rs} = \sum_{1 \le k \le n} A_{rk} * B_{ks}$



Applying Scalable Techniques

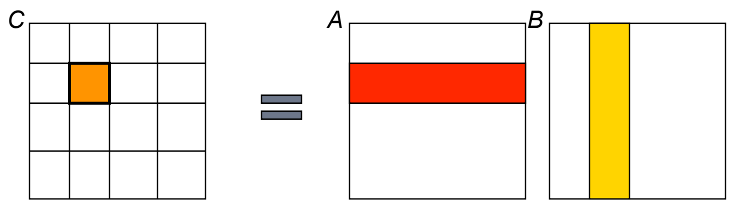
- Assume each processor stores block of ***C***, ***A***, ***B***; assume "can't" store all of any matrix
- To compute c_{rs} a processor needs all of row *r* of ***A*** and column *s* of ***B***
- Consider strategies for minimizing data movement, because that is the greatest cost -- what are they?





Grab All Rows/Columns At Once

- If all rows/columns are present, it's local



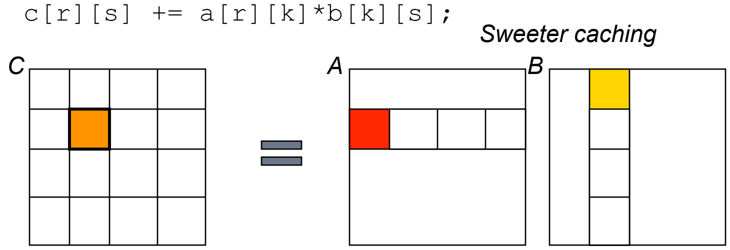
- •Each element requires O(n) operations
- •Modern pipelined processors benefit from large blocks of work
- •But memory space and BW are issues

Process *t* x *t* Blocks

- Use that solution, but incrementally
- Referring to local storage

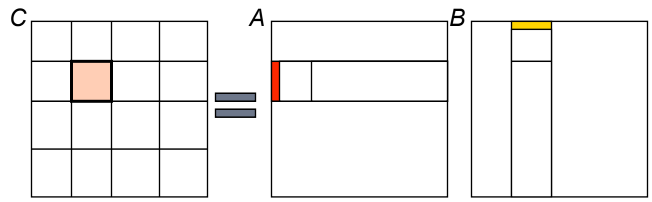
```
for (r=0; r < t; r++){
  for (s=0; s < t; s++){
      c[r][s] = 0.0;
      for (k=0; k < n; k++){
          c[r][s] += a[r][k]*b[k][s];
      }
  }
}
```

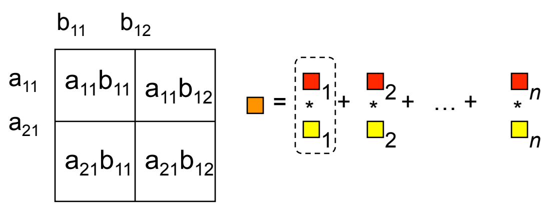
Only move a *t* x *t* block at a time



Change Of View Point

- Don't think of row-times-column





Switch orientation -- by using a *column* of *A* and a *row* of *B* compute all 1st terms of the dot products

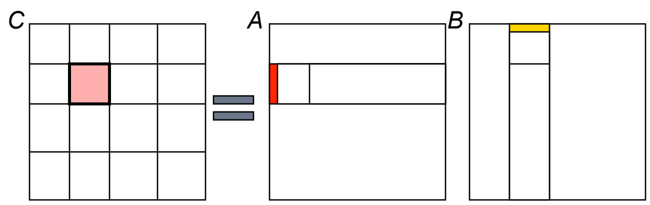
SUMMA

- Scalable Universal Matrix Multiplication Alg
  - Invented by van de Geijn & Watts of UT Austin
  - *Claimed to be the best machine independent MM*
- Whereas MM is usually A row x B column, SUMMA is A column x B row because computation switches sense
  - Normal: Compute all terms of a dot product
  - SUMMA: Computer a term of all dot products

Strange. But fast!

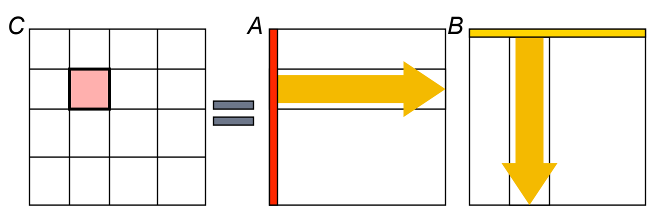
SUMMA Assumptions

- Threads have two indices, handle t x t block
- Let $p = P^{1/2}$, then thread u,v
  - reads all columns of A for indices u*t:(u+1)*t-1,j
  - reads all rows of B for indices i,v*t:(v+1)*t-1
  - The arrays will be in "global" memory and referenced as needed



Higher Level SUMMA View

- See SUMMA as an iteration multicasting columns and rows
- Each processor is responsible for sending/recving its column/row portion at proper time
- Followed by a step of computing next term locally



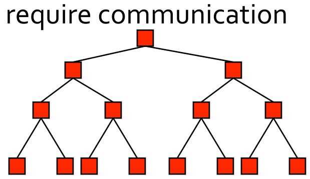
www.cs.utexas.edu/users/rvdg/abstracts/SUMMA.html

Summary of SUMMA

- Facts:
  - vdG & W advocate blocking for msg passing
  - Works for ***A*** being *m* x *n* and ***B*** being *n* x *p*
  - Works fine when local region is not square
  - Load is balanced esp. of Ceiling/Floor is used
  - Fastest machine independent MM algorithm!
- Key algorithm for 524: Reconceptualizes MM to handle high λ, balance work, use BW well, exploit efficiencies like multicast, …

Schwartz's Algorithm

- Jack Schwartz (NYU) asked: What is the optimal number of processors to combine *n* values?
  - Reasonable Answer: binary tree w/ values at leaves has O(*log n*) complexity
  - To this solution add *log n* values into each leaf
  - Same complexity (O(log n)), but *nlog n* values!
  - Asymptotically, the advantage is small, but the tree edges require communication



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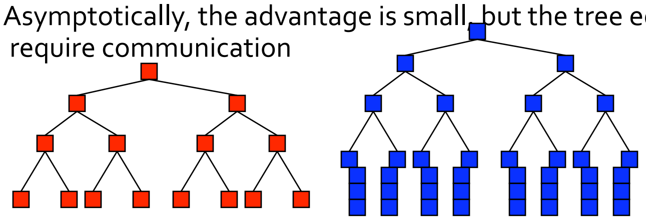
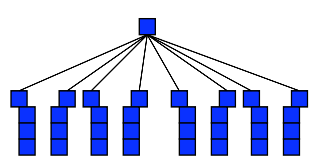


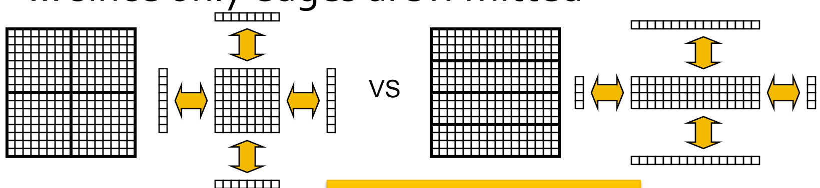
Schwartz

- Generally *P* is not a variable, and *P* << *n*
- Use Schwartz as heuristic: Prefer to work at leaves (no matter how much smaller *n* is than *P*) rather than enlarge (make a deeper) tree, implying tree will have no more than $log_2 P$ height
- Also, consider higher degree tree -- in cases of parallel communication (CTA) some of the communication may overlap



Block Allocations

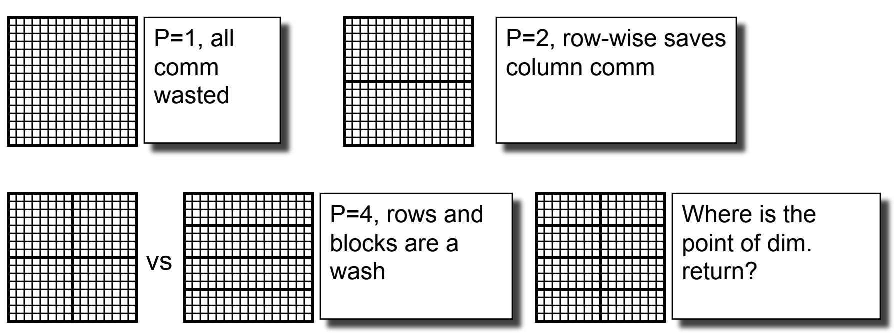
- The Red/Blue computation illustrated a 2D -block data parallel allocation of the problem
- Generally block allocations are better for data transmission: surface to volume advantage … since only edges are x-mitted



Now scale problem 4x

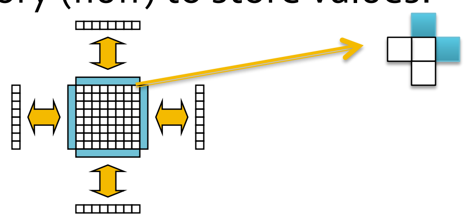
Different Regimens

- Though block is generally a good allocation it's not absolute:



Shadow Regions/Fluff

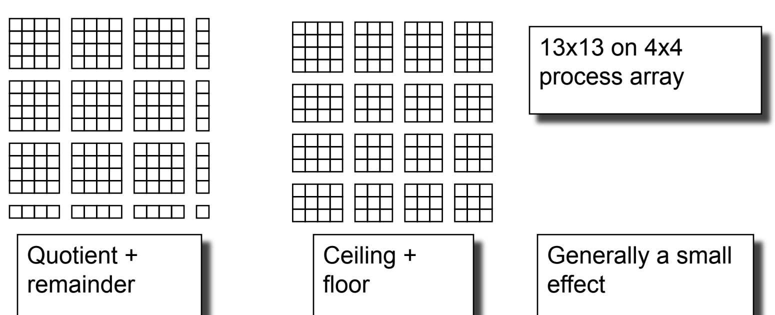
- To simplify local computation in cases where nearest neighbor's values x-mitted, allocate in-place memory (fluff) to store values:



- Array can be referenced as if it's all local

Aspect Ratio

- Generally *P* and *n* do not allow for a perfectly balanced allocation …
- Several ways to assign arrays to processors



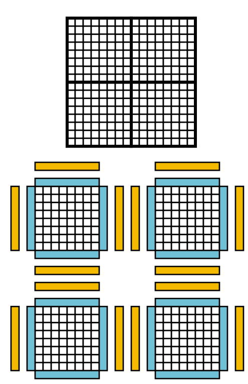
Assigning Processor 0 Work

- p_0 is often assigned "other duties", such as
  - Orchestrate I/O
  - Root node for combining trees
  - Work Queue Manager …
- Assigning p_0 the smallest quantum of work helps it avoid becoming a bottleneck
  - For either quotient + remainder or ceiling/floor p_0 should be the last processor

This is a late-stage tuning matter

Locality Always Matters

- Array computations on CMPs
  - Dense Allocation vs Fluff
  - Issue is cache invalidation
  - Keeping MM managed intermediate buffers keeps array and fluff local (L1)
  - Sharing causes elements at edge to repeatedly invalidate harming locality



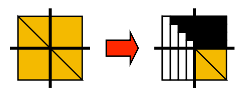
False sharing an issue, too

Load Balancing

- Certain computations are inherently imbalanced … LU Decomposition is one

gray is balanced work, white & black are finished

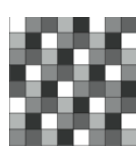
- Standard block decomposition quickly becomes very biased



  - Cyclic and block cyclic allocation are one fix

Cyclic & Block Cyclic

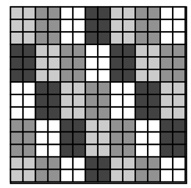
- Cyclic allocation means "to deal" the elements to the processes like cards
  - Allocating 64 elements to five processes: black, white, three shades of gray

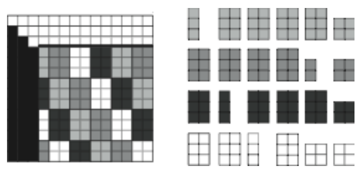


  - Block cyclic is the same idea, but rather with regular shaped blocks

Block Cyclic

- Consider the LU matrix allocated in 3x2 blocks to four processes:
- Then check it midway in the computation





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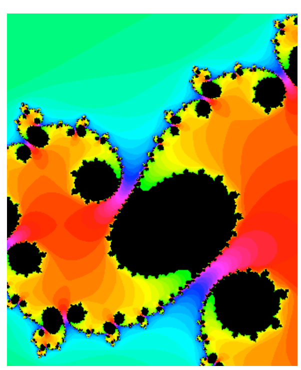
Opportunities To Apply Cyclic

- The technique applies to work allocation as well as memory allocation



Julia Set from http://alepho.clarku.edu/~djoyce/

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Break

Generalized Reduce and Scan

- The importance of reduce/scan has been repeated so often, it is by now our mantra
- In nearly all languages the only available operators are `+`, `*`, `min`, `max`, `&&`, `||`
- The concepts apply much more broadly
- Goal: Understand how to make user-defined variants of reduce/scan specialized to specific situations

Seemingly sequential looping code can be UD-scan

An Important Detail

- Recall scan specifics

  \+ scan of: 1 2 3 4 5 6 7 8

  is either: 1 3 6 10 15 21 28 36 [*inclusive*]

  or it is: 0 1 3 6 10 15 21 28 [*exclusive*]

- Important fact about standard scans

  $\alpha\text{-scan}_{inclusive}(x) = \alpha\text{-scan}_{exclusive}(x)\ \alpha\ x$

- For technical reasons prefer exclusive, for today, think inclusive

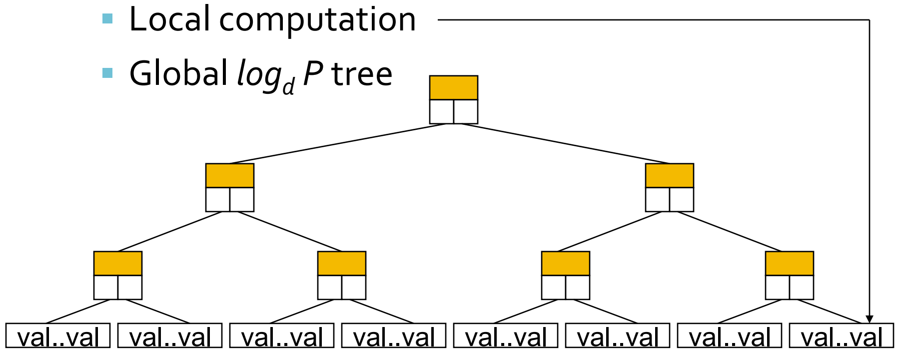
Examples Applicable Computations

- Reduce
  - Second smallest, or generally, kth smallest
  - Histogram, counts items in k buckets
  - Length of longest run of value 1s
  - Index of first occurrence of x
- Scan
  - Team standings
  - Find the longest sequence of 1s
  - Index of most recent occurrence

Associativity, but not commutativity, is key

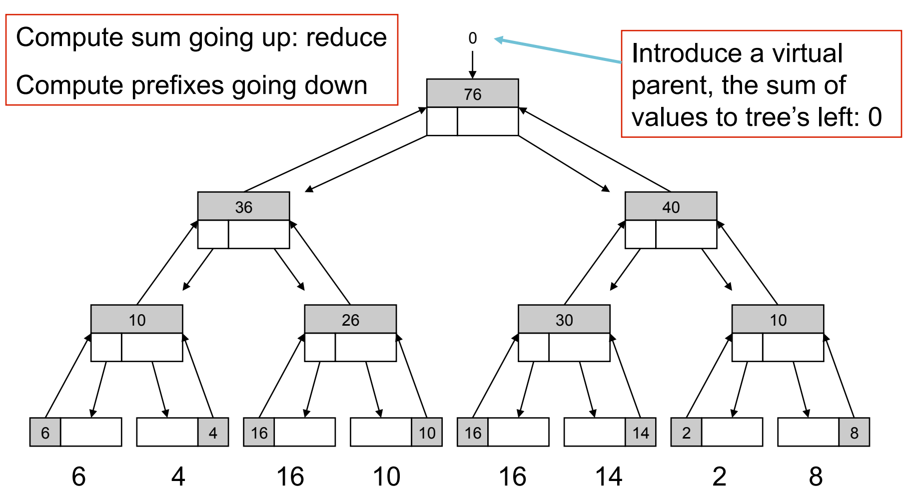
Structure of Computation

- Begin by applying Schwartz idea to problem
  - Local computation
  - Global $log_d\, P$ tree



More computation at nodes is OK

Recall Parallel Prefix Algorithm

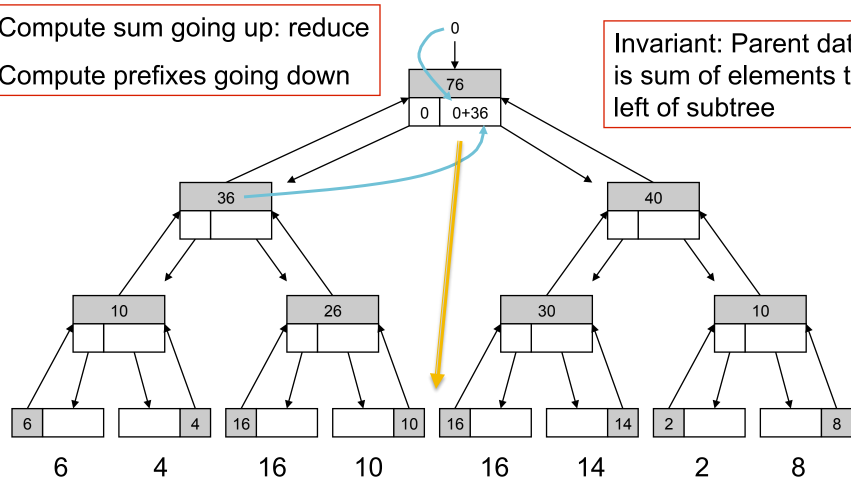


Parallel Prefix Algorithm

Compute sum going up: reduce

Compute prefixes going down

Invariant: Parent data is sum of elements to left of subtree

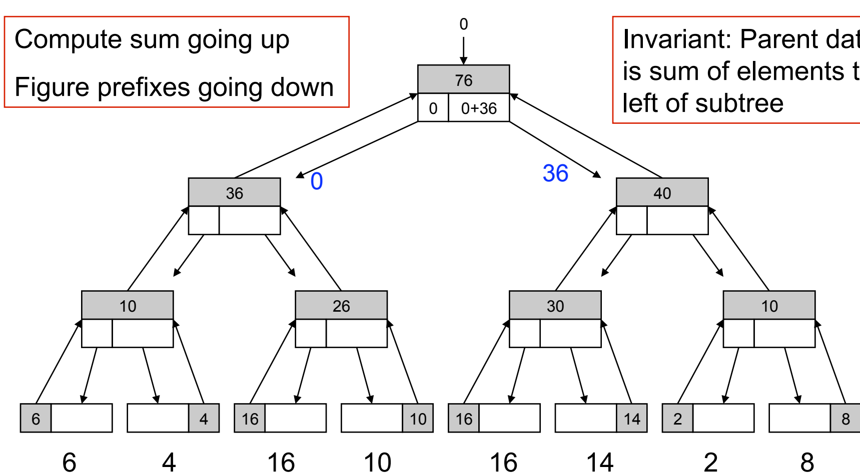


Parallel Prefix Algorithm

Compute sum going up

Figure prefixes going down

Invariant: Parent data is sum of elements to left of subtree



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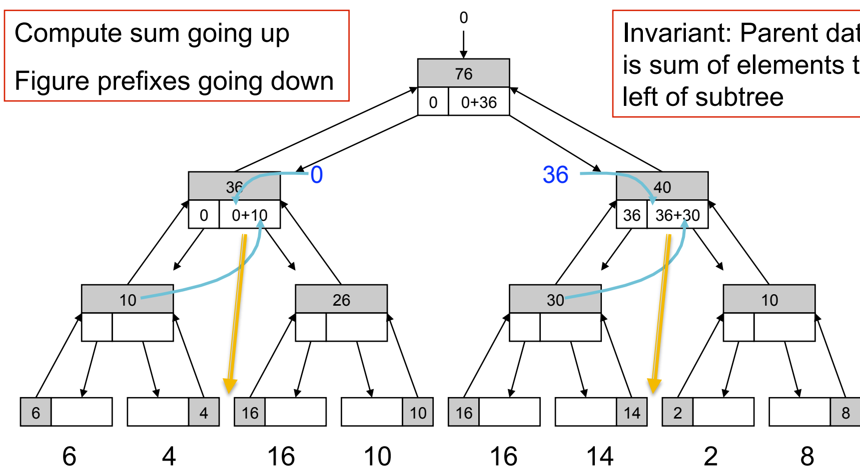
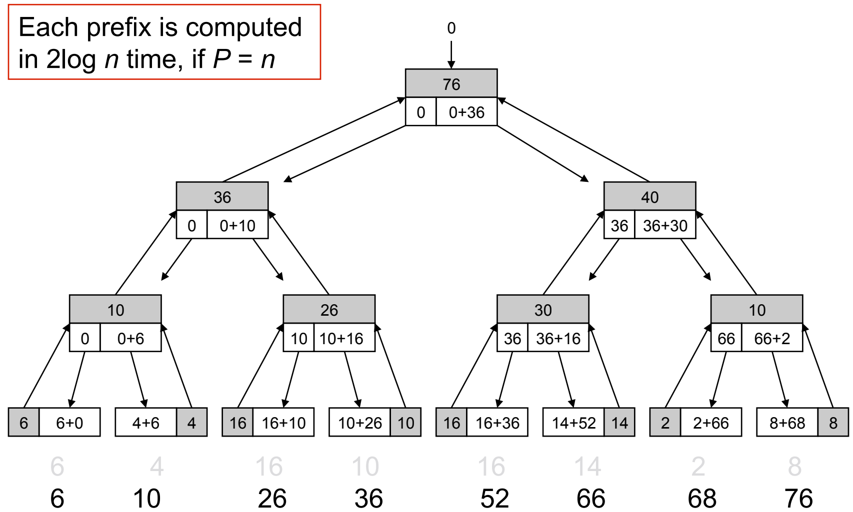


Parallel Prefix Algorithm

Each prefix is computed in 2log *n* time, if *P* = *n*



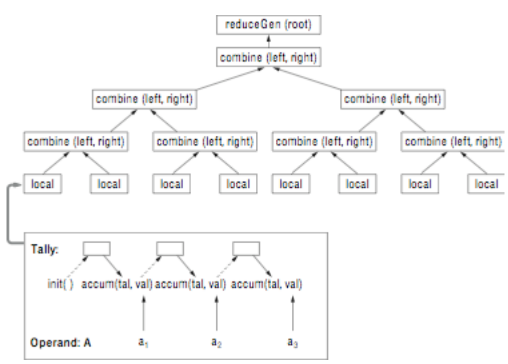
Introduce Four Functions

- Make four non-communication operations
  - `init()` initialize the reduce/scan
  - `accum()` perform local computation
  - `combine()` perform tree combining
  - *x*`_gen()` produce the final result for either op
    - *x* = reduce
    - *x* = scan
- Incorporate into Schwartz-type logic

Think of: `reduce(fi, fa, fc, fg)`

Assignment of Functions

- Init: Each leaf
- Accum: Aggregate each array value
- Combine: Each tree node
- reduceGen: Root



Example: +<<A Definitions

- Sum reduce uses a temporary value, called a `tally`, to hold items during processing
- Four reduce functions:
  - `tally init() {tal = new tally; tal=0;`
    `return tal;}`
  - `tally accum(int op_val, tally tal)`
    `{tal += op_val; return tal; }`
  - `tally combine(tally left, tally right)`
    `{return left + right; }`
  - `int reduce_gen(tally ans) {return ans;}`

More Involved Case

- Consider Second Smallest -- useful, perhaps for finding smallest nonzero among non-negative values
- **`tally`** is a struct of the smallest and next smallest found so far {float sm, nsm}
- Four functions:

```
tally init() {
   pair = new tally;
   pair.sm = maxFloat;
   pair.nsm = maxFloat;
   return pair; }
```

Second Smallest (Continued)

- Accumulate

```
tally accum(float op_val, tally tal) {
   if (op_val < tal.sm) {
     tal.nsm = tal.sm;
     tal.sm = op_val;
   } else {
      if (op_val > tal.sm && op_val < tal.nsm)
        tal.nsm = op_val;
   }
   return tal;
}
```

Finds 2nd smallest *distinct* value

Second Smallest (Continued)

```
tally combine(tally left, tally right){
  return
   accum(left.nsm, accum(left.sm, right));}

int reduce_gen(tally ans) {return ans.nsm;}
```

- Notice that the signatures are all different
- Conceptually easy to write equivalent code, but reduction abstraction clarifies

Custom Use of Parallel Prefix

- PoPP presents the state of the art of user-defined scans
- The conclusion must be, that generally it is
  - inconvenient, cumbersome, difficult
  - requires low-level knowledge and interface
- But, custom scan has wide application

- Take a moment to think "outside the box" on adding UD Scan to a programmer's tool belt

Essential Feature of || Prefix

- Because the definition of the computation is in terms of prefixes we usually see scan as a ***sequential left to right operation***
- But studying the implementational or compiler view of the computation, we notice …

From the backbone logic of the tree evaluation that the crux is combining adjacent sequences

The Main Idea

Add scan to languages with semantics of a *user defined* INFIX operator rather than as a LEFT ASSOCIATIVE operator, i.e. prefer

$$((\oplus)\oplus(\oplus))\oplus((\oplus)\oplus(\oplus))$$

to

$$(((((((\oplus)\oplus)\oplus)\oplus)\oplus)\oplus)\oplus)\oplus)$$

Rethinking Scan As Combining

- Accordingly, think of the operation as
-
  - $x_r \ldots x_s \oplus x_{s+1} \ldots x_t$
  - where
    - the sequences are contiguous
    - begin anywhere, end anywhere
    - any nonzero length
- Additionally, think about
  - The data to be merged from the two halves
  - The basis case starting with initial data
  - The completion processing

Consequences of ⊕ view

- To make the new view concrete, notice that
  - The substrings need a descriptor for state: **tally**
  - The basis case is an initial tally value: Initial($inval_i$) in each position *i*
  - The result of $x_1 \ldots x_s \oplus x_{s+1} \ldots x_n$ is the root value of the implementation tree, but the computation may not be finished [down sweep] implying that there is a finalize step: $outval_i$=Final( )
- Defining the tally, `Initial( )`, `ltally⊕rtally` and `Finalize()` suffices

Three Parts of + reduce

- The tally is a single float

Initialize:
- float tally = inval; //initialize

Complete:
- outval = tally; //final output from root

Combine: ltally ⊕ rtally
- float tally = ltally + rtally; //sum is left+right

Three Parts of + Scan

Initialize [each item in sequence]:

- pair tally = new Pair() //descriptor is a pair
- float tally.pre = 0; float tally.sum = inval; //initialize

Complete [each item in sequence]:

- outval = tally.pre + tally.sum //final output

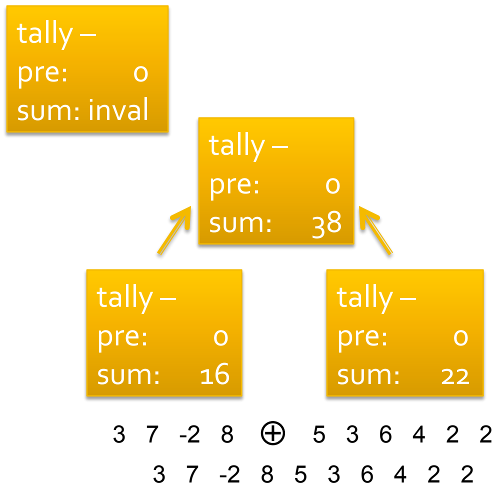
Combine: ltally ⊕ rtally

- pair tally = new Pair() //describe combin'n
- float tally.pre = ltally.pre; //prefix is left prefix
- float tally.sum=ltally.sum+rtally.sum; //sum is left+right
- THEN: ltally.pre = tally.pre; //left prefix is prefix
- rtally.pre = tally.pre+left.sum //right is prefix+l.sum

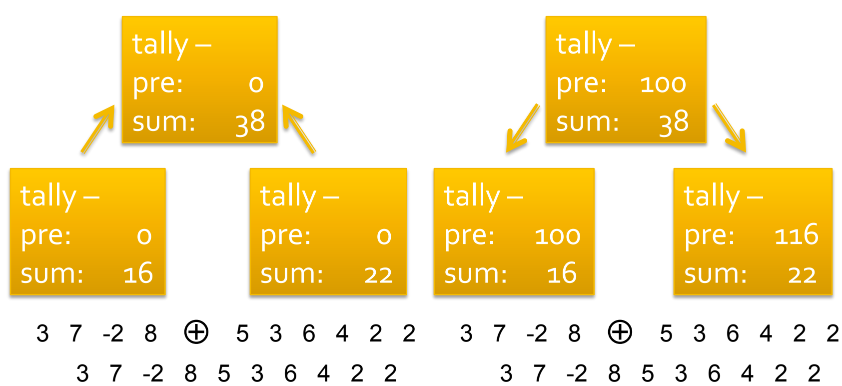
Three Parts of +scan [cartoon]

tally –
pre: 0
sum: inval

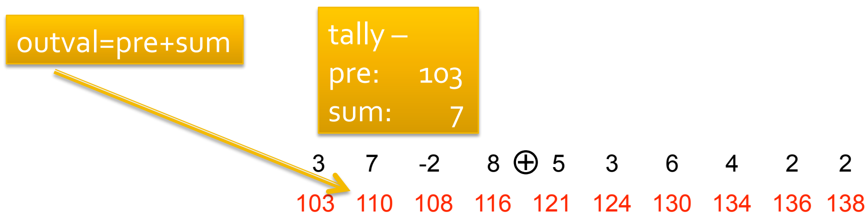
Three Parts of +scan [combine]



Three Parts of +scan [downsweep]



Three Parts of +scan [final]



Parts of + Scan

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Another Ex.: Longest Run of x

- How do we think of this computation as combining two subcomputations

  xx0000x0xxxx ⊕ x0xxxxxx000
- Obviously
  - x runs can be at the start, interior, or end
  - Combining will merge a start and end run
  - … Making it an interior run
- The tally needs to keep this information

Longest Run of x [a *reduce* cartoon]

tally – in == x
from start: 1
inside: 0
from end: 1

tally – in != x
from start: 0
inside: 0
from end: 0

xx0000x0xxxx ⊕ x0xxxxxx000

xx0000x0xxxxx0xxxxxx000

Longest Run of x [a *reduce* cartoon]

tally – in == x
from start: 1
inside: 0
from end: 1

tally – in != x
from start: 0
inside: 0
from end: 0

tally --
from start: 2
inside: 6
from end: 0

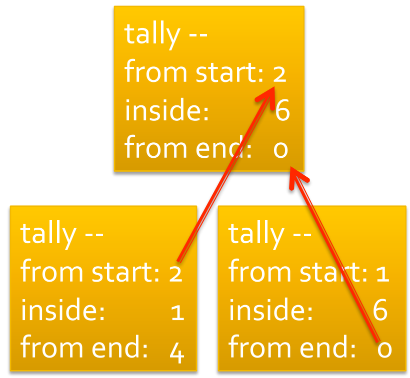
tally --
from start: 2
inside: 1
from end: 4

tally --
from start: 1
inside: 6
from end: 0

xx0000x0xxxx ⊕ x0xxxxxx000

xx0000x0xxxxx0xxxxxx000

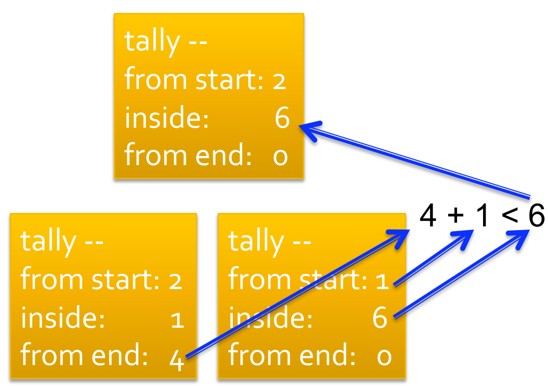
Longest Run of x [a *reduce* cartoon]



xx0000x0xxxx ⊕ x0xxxxxx000

xx0000x0xxxxx0xxxxxx000

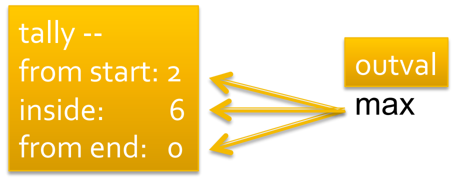
Longest Run of x [a *reduce* cartoon]



xx0000x0xxxx ⊕ x0xxxxxx000

xx0000x0xxxxx0xxxxxx000

Longest Run of x [a *reduce* cartoon]



xx0000x0xxxx ⊕ x0xxxxxx000

xx0000x0xxxxx0xxxxxx000

Balanced Parentheses...

- Illustrate for the matching parentheses
  - Carry along the count of excess of opens/closes
  - Cancel if matched, else record the excess
  - Output "yes" if excess is 0

  - Descriptor for "balanced parens" is two ints, excess open parens `opCount` and excess closed parents `clCount`

A || Prefix Solution

- Visualize a processor per point (not really)
  - Each point is initialized to its data structure
  - Pairs are combined in some way
  - Process continues until there is one descriptor
  - Compute the final result
- Illustrate on this problem: `a-f(c)*(d+f(e))`

a	-	f	(	c	)	*	(	d	+	f	(	e	)	)
0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	1	1

Tri-Partite Parallel Prefix

Create a tally:

```
if (inval == '(' )
    int tally.opCount = 1;
else
    int tally.opCount = 0;
if (inval == ')' ) {
    int tally.clCount = 1;
else
    int tally.clCount = 0;
```

Combine two tallies:

```
tally.clCount = ltally.clCount;
tally.opCount = rtally.opCount;
int temp = ltally.opCount - rtally.clCount;
if (temp < 0)
   tally.clCount += abs(temp);
else
   tally.opCount += temp;
```

Finalize result from tally:

```
outval = (tally.opCount == 0) && (tally.clCount == 0);
```

Matching Parens

- Working out the details
  Matching

a	-	f	(	c	)	*	(	d	+	f	(	e	)	)
0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	1	1

Matching Parens

- Working out the details
  Matching

a	-	f	(	c	)	*	(	d	+	f	(	e	)	)
0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	1	1

a-	f(	c)	*(	d+	f(	e)	)
0	1	0	1	0	1	0	0
0	0	1	0	0	0	1	1

Matching Parens

- Working out the details
  Matching

```
a - f ( c ) * ( d + f ( e ) )
0 0 0 1 0 0 0 1 0 0 0 1 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 1 1
a-  f(  c)  *(  d+  f(  e)  )
0   1   0   1   0   1   0   0
0   0   1   0   0   0   1   1
```

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Matching Parens

- Working out the details
  Matching

```
a - f ( c ) * ( d + f ( e ) )
0 0 0 1 0 0 0 1 0 0 0 1 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 1 1
a-  f(  c)  *(  d+  f(  e)  )
0   1   0   1   0   1   0   0
0   0   1   0   0   0   1   1
a-f(    c)*(    d+f(    e))
1       1       1       0
0       1       0       2
```

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Matching Parens

- Working out the details
  Matching

```
a - f ( c ) * ( d + f ( e ) )
0 0 0 1 0 0 0 1 0 0 0 1 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 1 1
a-  f(  c)  *(  d+  f(  e)  )
0   1   0   1   0   1   0   0
0   0   1   0   0   0   1   1
a-f(    c)*(    d+f(    e))
1       1       1       0
0       1       0       2
a-f(c)*(        d+f(e))
1               0
0               1
a-f(c)*(d+f(e))
0
0
```


Matching Parens

- Working out the details
  Mismatching

```
a - f ) c ) * ( d + f ( e ) )
0 0 0 0 0 0 0 1 0 0 0 1 0 0 0
0 0 0 1 0 1 0 0 0 0 0 0 0 1 1
```

Matching Parens

- Working out the details
  Mismatching

```
a - f ) c ) * ( d + f ( e ) )
0 0 0 0 0 0 0 1 0 0 0 1 0 0 0
0 0 0 1 0 1 0 0 0 0 0 0 0 1 1
a-  f)  c)  *(  d+  f(  e)  )
0   0   0   1   0   1   0   0
0   1   1   0   0   0   1   1
```


Matching Parens

- Working out the details
  Mismatching

```
a - f ) c ) * ( d + f ( e ) )
0 0 0 0 0 0 0 1 0 0 0 1 0 0 0
0 0 0 1 0 1 0 0 0 0 0 0 0 1 1
a-  f)  c)  *(  d+  f(  e)  )
0   0   0   1   0   1   0   0
0   1   1   0   0   0   1   1
a-f)    c)*(    d+f(    e))
0       1       1       0
1       1       0       2
```

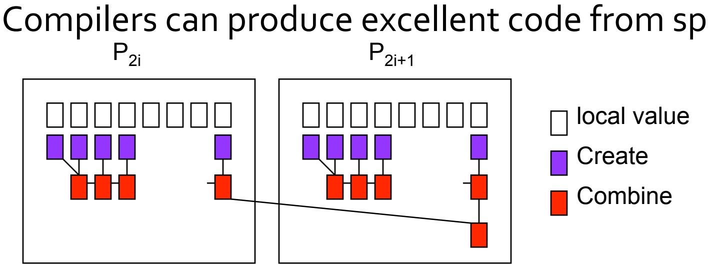
Matching Parens

- Working out the details
  Mismatching

```
a - f ) c ) * ( d + f ( e ) )
0 0 0 0 0 0 0 1 0 0 0 1 0 0 0
0 0 0 1 0 1 0 0 0 0 0 0 0 1 1
a-  f)  c)  *(  d+  f(  e)  )
0   0   0   1   0   1   0   0
0   1   1   0   0   0   1   1
a-f)    c)*(    d+f(    e))
0       1       1       0
1       1       0       2
a-f)c)*(        d+f(e))
1               0
2               1
a-f)c)*(d+f(e))
0
2
```


Compiling The || Prefix

- One last question concerned how the 3 parts of the || prefix specification fit into the tree model shown for prefix sum & Schwartz?
  - Short answer, they don't have to
  - Compilers can produce excellent code from spec



Emphasizing the Point

- At the start of class we cited bal-parens – the leaf code for a Schwartz approach

```
for (i=start; i<start+len_per_th; i++) {
  if (symb[i] == "(" )
     o++;
  if (symb[i] == ")" ) {
    o--;
    if (o < 0) {
      c++; o = 0;
    }
  }
```

- Combining required entirely different code
- The Infix approach captures the whole thing, except for pre- and post-operations

Summary on || Prefix

- By thinking abstractly of carrying along information that describes the sequence, combining adjacent subsequences, and finally extracting a value, it is possible to move directly to a || prefix solution
- Using the abstraction is an intellectually different way of thinking about sequential computations

HW 5, Part I … for Tuesday

- Think of a "sequential computation" that can be expressed as a UD reduce or scan
  - Examples from this lecture are off limits
  - Prefer a scan; it's often easy to convert a reduce into a scan: A 10-bucket histogram (a reduce) is related to a 10-team "league standings" (a scan) that gives won/loss for game input, team *t* beat *u*
- Turn in a document giving an infix formulation of the computation together with a worked example

HW 5, Part II … for Thursday

- Write an MPI program for the SUMMA alg
  - Create rectangular arrays A, B, C, filling A, B
  - Send portions of A, B to worker processes
  - Iterate over common dimension,
    - send columns of A, rows of B to other processes
    - for each, multiply A elements times B elements and accumulate into local portion of C
  - Measure time, except for initialization, and report the "usual stuff" for different numbers of processes